Determination of onboard coplanar orbital maneuvers with orbits determined using GPS

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Abstract - In the present paper a study is made in order to find an algorithm that can calculate coplanar orbital maneuvers for an artificial satellite. The idea is to find a method that is fast enough to be combined with onboard orbit determination using GPS data collected from a receiver that is located in the satellite. After a search in the literature, three algorithms are selected to be tested. Preliminary studies show that one of them (the so called “Minimum Delta-V Lambert Problem”) has several advantages over the two others, both in terms of accuracy and time required for processing. So, this algorithm is implemented and tested numerically combined with the orbit determination procedure. Some adjustments are performed in this algorithm in the present paper to allow its use in real-time onboard applications. Considering the whole maneuver, first of all a simplified and compact algorithm is used to estimate in real-time and onboard the artificial satellite orbit using the GPS measurements. By using the estimated orbit as the initial one and the information of the final desired orbit (from the specification of the mission) as the final one, a coplanar bi-impulsive maneuver is calculated. This maneuver searches for the minimum fuel consumption. Two kinds of maneuvers are performed, one varying only the semi major axis and the other varying the semi major axis and the eccentricity of the orbit, simultaneously. The possibilities of restrictions in the locations to apply the impulses are included, as well as the possibility to control the relation between the processing time and the solution accuracy. Those are the two main reasons to recommend this method for use in the proposed application.

Keywords: Astrodynamics, Orbital Maneuver, Artificial Satellite, GPS, Lambert Problem.

1 Introduction

For the general problem of orbital maneuvers, many alternatives are studied in the literature considering different conditions. An important field of research considers the so called low thrust maneuver. In this model, a force with low magnitude is applied during a finite time. To find the trajectory of the spacecraft is necessary to integrate the equations of motion. There are many results in the literature considering this model, beginning with the works of Lawden ([1], [2]). Many other more recent researches are available dealing with this model, like references [3] to [8]. A second approach uses the idea of an impulsive thrust, which is the case where the thrust is assumed to have an infinity magnitude. Several papers used this approach, like references [9] to [13].

Later, the idea of gravitational capture has been considered. In this situation the perturbation of a third-body [14] can be used to decrease the fuel consumption of an orbital maneuver. References [15] to [17] explain this idea in some detail.

Another approach that appeared in the literature, to find alternatives to reduce fuel expenditure in space missions, is the swing by maneuvers. References [18] to [32] show more details, as well as missions using this technique.

The problem of autonomous satellite maneuvers can be defined as the problem of searching for a solution of the orbital transfer problem that does not require any command from an Earth's control center. To develop this task is necessary that the satellite is able to determine its orbit by itself, which will be described in next sections. After getting this information, the satellite needs to solve the problem of the orbital maneuver with minimum fuel consumption and, after that, generates the commands to perform this change in its orbit autonomously.

So, after having its orbit determined and considering that the information of the final expected orbit is available from the definition of the mission, a coplanar bi-impulsive maneuver is calculated, searching for the minimum fuel consumption. This sequence of calculations is based in the assumption that the control available to modify the orbit of the spacecraft is a bi-impulsive maneuver that is a very popular condition in space missions. The two orbits involved are assumed to be coplanar, which covers many important situations. It is also known that the CPU time allowed for calculating the optimal transfer is of the order of a few seconds. In this way, the orbit is estimated in intervals from 10 to 30 seconds and the initial point of the transfer is supposed to be the position of the satellite in that time. Next, the final orbit is divided in a certain number of points and, in each point, the optimal bi-impulsive orbital transfer is calculated. The consumption of each maneuver is stored and, from these data, the global minimum can be found.

Three algorithms selected in the literature were tested to choose a method that can be used for calculating the orbital maneuver. From these three, two of them allow the calculation of the impulsive tri-dimensional maneuvers [33, 34], but they do not perform the calculations fast enough to be executed on board and in real-time for this application. In this way, the algorithm developed by Prado [35] for coplanar maneuvers is chosen. After this choice, some adjustments in the program code were made in order to make it fast enough to be compatible with real-time applications.

Prado's method solves the bi-impulsive coplanar transfer problem with variations in the orbital elements semi-major axis, eccentricity, and argument of perigee, the three possibilities for a coplanar maneuver. The restriction of coplanar maneuvers does not compromise the use of the algorithm, because the planar maneuvers have important applications, since there are several missions that do not require orbital plane change. The method uses the "Minimum Delta-V Lambert Problem" [35] that will be shown later in this paper.

2 Onboard orbit determination

The Global Positioning System (GPS) is a satellite navigation system used to determine positions, velocities and time with high accuracy. The GPS system allows a GPS receiver to determine its position and time at any place using data from at least four GPS satellites. Using such a system, an algorithm to determine onboard the satellite orbit in real-time using the GPS system and Kalman filtering was developed by Chiaradia et. al. [36]. It is a simplified and compact model with low computational cost. The extended Kalman filter (EKF) estimates the state vector, composed by the position and velocity components, bias, drift, and drift rate of the GPS receiver clock. A simple fixed step size fourth order Runge-Kutta numerical integrator is found to be suitable to integrate the differential equations of orbital motion. The algorithm uses a large 30 seconds step-size of propagation (10 second step-size can be used as well). The force model in the equations of motion considers the perturbations due to the geopotential up to order and degree 10 of the spherical harmonics. The state error covariance matrix is computed through the transition matrix, which is calculated analytically in an optimized way [37]. The raw single frequency pseudorange GPS measurements are used as observations by the Kalman filter. They are modeled taking into account most of the GPS satellite and receiver clock offsets. To analyze the developed algorithm, the Topex/Poseidon satellite (T/P) data were chosen, because it carries a dual frequency receiver GPS on board experimentally to test the ability of the GPS to provide precise orbit determination (POD). All T/P GPS data set and GPS navigation message are easily found in the Internet in Rinex format [38].

The satellite orbit is estimated using the developed algorithm with a good accuracy and minimum computational cost. It can be noted that, for all the days tested, the real position and velocity errors are less than the estimated position and velocity errors given by the filter covariance. It shows the conservative behavior of the filter. In all cases, the filtering takes around one hour to converge. Before achieving the convergence, the onboard computer can use the GPS navigation solution provided by a GPS receiver at 30-meter level error. The position accuracy with SA off or on varies from 15 to 20 m with standard deviation that goes from 6 to 10 m. The velocity accuracy goes from 0.014 to 0.018 m/s with standard deviation varying from 0.006 to 0.009 m/s.

3 Minimum Delta-V Lambert problem

The original Lambert's problem is one of the most important and popular topics in celestial mechanics. It can be defined as: "A Keplerian orbit, about a given gravitational center of force, is to be found connecting two given points (P_1 and P_2) in a given time Δt".

Prado [35] formulated and proposed several forms to solve a problem that is related to the Lambert's problem. His formulation is slightly different from the original one, but it also has many important applications. This problem is called "Minimum Delta-V Lambert's Problem" and it is formulated as follows: "A Keplerian orbit, about a given gravitational center of force, is to be found connecting two given points (P_1 and P_2), such that the ΔV for the transfer is minimum".

To solve this problem, the analytical expressions for the ΔV (as a function of only one independent variable) and for its first derivative with respect to this variable are obtained. Furthermore, a numerical scheme to get the root of the first derivative and the numeric value of the ΔV at this point is used. From this information it is possible to get all the other parameters involved, like the components of the impulses, their locations, etc. Its method is closely connected to the search for a minimum two-impulse transfer between two given coplanar orbits.

2.1 Definition of the Problem

Suppose that there is a spacecraft in a Keplerian orbit that is called O_0 (the initial orbit). It is desired to transfer this spacecraft to a final Keplerian orbit O_2 that is coplanar with the orbit O_0. The orbits are not co-axial and are not located in any specific position. To develop this transfer, the spacecraft departures from the point $P_1(r_1,\theta_1)$, where one impulse is applied with magnitude ΔV_1 that makes an angle ϕ_1 with the transverse local direction (perpendicular to the radius vector). The transfer orbit crosses the final orbit in the point $P_2(r_2,\theta_2)$, where a second impulse is applied with magnitude ΔV_2 that makes an angle ϕ_2 with transverse local direction. To define the basic problem (the "Minimum Delta-V Lambert's Problem"), it is necessary to specify the true anomaly (θ_1) of the departure point in the orbit O_0 (P_1) and the true anomaly (θ_2) of the point of arrival in the orbit O_2 (P_2). With these two values given and all the Keplerian elements of both orbits known, it is possible to determine both radius vectors $\vec{r}_1$ and $\vec{r}_2$ at the beginning and at the end of the transfer. Then the problem is to find which transfer orbit connecting these two vectors and using only two impulses is the one that requires the minimum ΔV for the maneuver. This problem is what is called "Minimum ΔV Lambert's Problem". The sketch of the transfer and the variables used are shown in Fig. 1. It shows the initial, transfer and final orbit, as well as the variables used to describe the problem: ω_0 (the argument of the periapsis of the initial orbit), P_1 (the point where the first impulse is applied), r_1 (the distance between P_1 and the central body), Θ_1 (the angular position of P_1, measured from the X-axis), ΔV_1 (the magnitude of the first impulse), ϕ_1 (the angle that specify the direction of the first impulse, measured from the perpendicular to the position vector), ω_2 (the argument of the periapsis of the final orbit), P_2 (the point where the second impulse is applied), r_2 (the distance between P_2 and the central body), Θ_2 (the angular position of P_2, measured from the X-axis), ΔV_2 (the magnitude of the second impulse), ϕ_2 (the angle that specify the direction of the first impulse, measured from the perpendicular to the position vector).

Using basic equations from the two-body celestial mechanics, it is possible to write an analytical expression for the total ΔV (= ΔV_1 + ΔV_2) required for this maneuver. To specify each of the three orbits involved in the problem, the elements D, h, and k are used. They are defined by the following equations:

$$D = \frac{\mu}{C}; \quad k = e\cos\omega; \quad h = e\sin\omega , \tag{1}$$

where μ is the gravitational parameter of the central body (in km^3/s^2); C is the angular momentum of the orbit (in km^2/s), e is the eccentricity, and ω is the argument of the perigee.

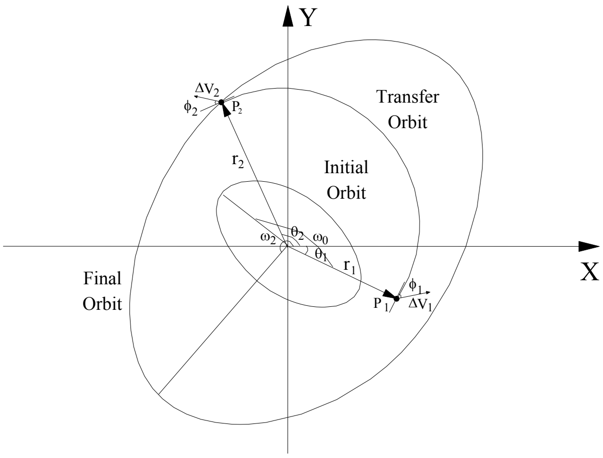

Figure 1 - Geometry of the Minimum ΔV Lambert's Problem.

The subscripts "*0*" for the initial orbit, "*1*" for the transfer orbit, and "2" for the final orbit are also used. In those variables, the expressions for the radial (subscript *r*) and transverse (subscript *t*) components of the two impulses are:

$$\Delta V_{r1} = \left(D_1 k_1 - D_0 k_0\right) sin\theta_1 - \left(D_1 h_1 - D_0 h_0\right)\cos\theta_1 , \tag{2}$$

$$\Delta V_{t1} = D_1 - D_0 + \left(D_1 k_1 - D_0 k_0\right)\cos\theta_1 + \left(D_1 h_1 - D_0 h_0\right) sin\theta_1 , \tag{3}$$

$$\Delta V_{r2} = \left(D_2 k_2 - D_1 k_1\right) sin\theta_2 - \left(D_2 h_2 - D_1 h_1\right)\cos\theta_2 , \tag{4}$$

$$\Delta V_{t2} = D_2 - D_1 + \left(D_2 k_2 - D_1 k_1\right)\cos\theta_2 + \left(D_2 h_2 - D_1 h_1\right) sin\theta_2 . \tag{5}$$

Now, the problem is to find the transfer orbit that minimizes the total ΔV and satisfies the two following constraints shown in Eqs. (6) and (7), expressing the fact that the orbits intersect:

$$g_1 = D_0^2\left(1 + k_0 \cos\theta_1 + h_0\, sin\theta_1\right) - D_1^2\left(1 + k_1 \cos\theta_1 + h_1 \sin\theta_1\right) = 0, \tag{6}$$

$$g_2 = D_2^2\left(1 + k_2 \cos\theta_2 + h_2\, sin\theta_2\right) - D_1^2\left(1 + k_1 \cos\theta_2 + h_1\, sin\theta_2\right) = 0. \tag{7}$$

The above equations are obtained from references [39] and [40]. The problem is then reduced to the one of finding the D_1 that gives the minimum value for the expression $\Delta V = \sqrt{\Delta V_{r1}^2 + \Delta V_{t1}^2} + \sqrt{\Delta V_{r2}^2 + \Delta V_{t2}^2}$.

2.2 Using the chain rule to obtain the derivatives

The constraints (6) and (7) can be used to solve this system of equations for two of the variables, making the equation for the ΔV a function of only one independent variable. The system formed by these two equations is symmetric and linear in the variables h_1 and k_1, so the system is solved for these two variables. The results are the Eqs. (8) and (9):

$$k_1 = -\csc\left(\theta_1 - \theta_2\right)\left[\left(\left(\frac{D_0^2}{D_1^2}\right)\left(1 + k_0 \cos\theta_1 + h_0\, sin\theta_1\right) - 1\right) sin\theta_2 - \left(\left(\frac{D_2^2}{D_1^2}\right)\left(1 + k_2\, cos\theta_2 + h_2 \sin\theta_2\right) - 1\right) sin\theta_1\right] \tag{8}$$

$$h_1 = -csc\left(\theta_1 - \theta_2\right)\left[\left(\left(\frac{D_2^2}{D_1^2}\right)\left(1 + k_2 \cos\theta_2 + h_2 \sin\theta\right) \text{-} 1\right) cos\theta_1 - \left(\left(\frac{D_0^2}{D_1^2}\right)\left(1 + k_0\, cos\theta_1 + h_0 \sin\theta_1\right) - 1\right) cos\theta_2\right] \tag{9}$$

Now that the ΔV is a function of only one variable (D_1) (remember that θ_1 and θ_2 are fixed values for the Lambert's problem), elementary calculus can be used to find its minimum. All that has to be done is to search for the root of the expression $\frac{\partial(\Delta V)}{\partial D_1}$. From the definition of ΔV it is possible to write:

$$\frac{\partial(\Delta V)}{\partial D_1} = 0 = \frac{1}{\Delta V_1}\left[\Delta V_{r1}\frac{\partial(\Delta V_{r1})}{\partial D_1} + \Delta V_{t1}\frac{\partial(\Delta V_{t1})}{\partial D_1}\right] + \frac{1}{\Delta V_2}\left[\Delta V_{r2}\frac{\partial(\Delta V_{r2})}{\partial D_1} + \Delta V_{t2}\frac{\partial(\Delta V_{t2})}{\partial D_1}\right]. \quad (10)$$

Now, the chain rule for derivatives is applied to obtain expressions for the quantities $\frac{\partial(\Delta V_{r1})}{\partial D_1}$; $\frac{\partial(\Delta V_{t1})}{\partial D_1}$; $\frac{\partial(\Delta V_{r2})}{\partial D_1}$; and $\frac{\partial(\Delta V_{t2})}{\partial D_1}$.

A general expression for them is:

$$\frac{\partial(\Delta V_{ij})}{\partial D_1} = \left.\frac{\partial(\Delta V_{ij})}{\partial D_1}\right|_{Direct} + \frac{\partial(\Delta V_{ij})}{\partial k_1}\frac{\partial k_1}{\partial D_1} + \frac{\partial(\Delta V_{ij})}{\partial h_1}\frac{\partial h_1}{\partial D_1}, \quad (11)$$

where $i = r,t$; $j = 1,2$ and the word "Direct" stands for the part of the derivative that comes from the explicit dependence of ΔV_{ij} in the variable D_1. The expressions for $\frac{\partial(\Delta V_{ij})}{\partial k_1}$ and $\frac{\partial(\Delta V_{ij})}{\partial h_1}$ can be obtained from Eqs. (2) to (5) and the expressions for $\frac{\partial k_1}{\partial D_1}$ and $\frac{\partial h_1}{\partial D_1}$ can be obtained from the Eqs. (8) and (9).

With all those equations available, a numerical algorithm was built by Prado [35] to iterate in the variable D_1 to find the only real root of the equation $\frac{\partial(\Delta V)}{\partial D_1} = 0$. To obtain the value of $\frac{\partial(\Delta V)}{\partial D_1}$ for a given D_1, necessary for the iteration process required, the following steps can be used:

1. Evaluate k_1 and h_1 from Eqs. (8) and (9) for the given D_1;
2. With D_1, h_1 and k_1 the Eqs. (2) to (5) are used to evaluate ΔV_{r1}, ΔV_{t1}, ΔV_{r2}, ΔV_{t2}, ΔV_1($\sqrt{\Delta V_{r1}^2 + \Delta V_{t1}^2}$), and ΔV_2 ($\sqrt{\Delta V_{r2}^2 + \Delta V_{t2}^2}$);
3. With all those quantities known, it is possible to evaluate $\frac{\partial(\Delta V_{ij})}{\partial k_1}$ and $\frac{\partial(\Delta V_{ij})}{\partial h_1}$ (obtained from Eqs. (2) to (5)) and (10) to finally obtain $\frac{\partial(\Delta V)}{\partial D_1}$ for the given D_1.

2.3 Solving the equation $\frac{\partial(\Delta V)}{\partial D_1} = 0$

At this point it is important to remark that the function $\frac{\partial(\Delta V)}{\partial D_1}$ is very sensitive to small variations in D_1, especially when close to the real root. Its curve is almost a straight line with a slope that goes to infinity when θ_2 - θ_1 goes to 180°. Fig. 2 shows the detail for a transfer where θ_2 - θ_1 = 3.14 rad, considering $D_0 = \sqrt{3}$; $h_0 = 0$; $k_0 = 1/3$, $D_2 = \sqrt{2}$; $h_2 = 1/4$; $k_2 = 0.4333$.
. From that figure, it is easy to see that this fact comes from the sharpness of the curve $\Delta V \times D_1$, when close to the minimum. This characteristic is particular for the set of variables used and is not a physical problem. If another independent variable is used, like the argument of the perigee of the transfer orbit, the curve for the $\Delta V \times D_1$ has a much less sharp minimum and, in consequence, its derivative has no big jumps.

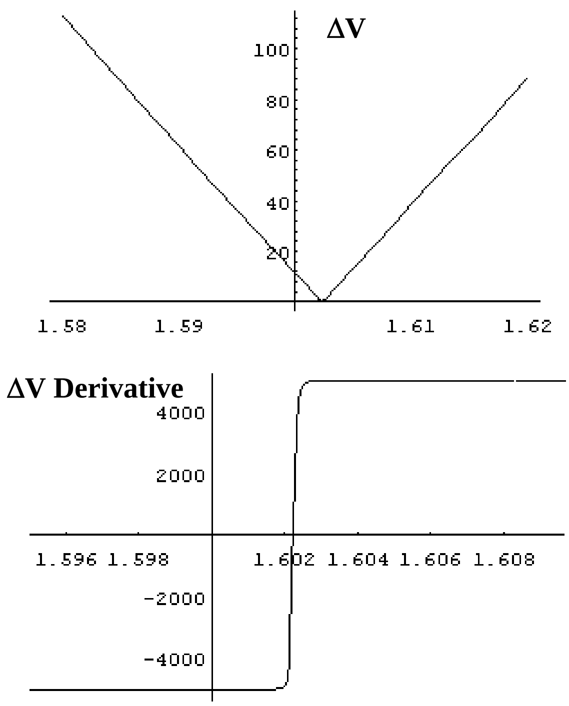

Figure 2 - ΔV and its derivative as a function of *D₁* (Adapted from Prado [35]).

This behavior makes numerical methods to find the root based on derivatives (like the popular Newton-Raphson) not very adequate. The method of dividing the interval in two parts in each iteration shows to be very adequate. Expressions for the components of the variation of the velocity are shown below.

$$\Delta V_{r1} = -\frac{\csc(\theta_1-\theta_2)}{2D_1}\Big[2\left(D_1^2-D_2^2\right)+ 2\left(D_0^2-D_1^2\right)\cos(\theta_1-\theta_2)+ \left(D_0^2k_0-D_0D_1k_0\right)\cos(2\theta_1-\theta_2)+ \left(D_0^2k_0+D_0D_1k_0-2D_2^2k_2\right)\cos(\theta_2)+ \left(D_0^2h_0-D_0D_1h_0\right)\sin(2\theta_1-\theta_2)+ \left(D_0^2h_0+D_0D_1h_0-2D_2^2h_2\right)\sin(\theta_2)\Big] \quad (12)$$

$$\Delta V_{t1} = \frac{D_0}{D_1}\left[(D_0-D_1)(1+k_0\cos\theta_1+h_0\sin\theta_1)\right], \quad (13)$$

$$\Delta V_{r2} = -\frac{\csc(\theta_1-\theta_2)}{2D_1}\Big[2\left(D_1^2-D_0^2\right)+ 2\left(D_2^2-D_1^2\right)\cos(\theta_1-\theta_2)+ \left(D_2^2k_2-D_1D_2k_2\right)\cos(\theta_1-2\theta_2)+ \left(D_2^2k_2+D_1D_2k_2-2D_0^2k_0\right)\cos\theta_1+ \left(D_1D_2h_2-D_2^2h_2\right)\sin(\theta_1-2\theta_2)+ \left(D_2^2h_2+D_1D_2h_2-2D_0^2h_0\right)\sin\theta_1\Big], \quad (14)$$

and

$$\Delta V_{t2} = \frac{D_2}{D_1}\left[(D_1-D_2)(1+k_2\cos\theta_2+h_2\sin\theta_2)\right]. \quad (15)$$

Those equations allow the calculation of the expression for $\frac{\partial(\Delta V)}{\partial D_1}$ that is given by Eq. (10). The partial derivatives involved are given by:

$$\frac{\partial(\Delta V_{r1})}{\partial D_1} = -\frac{\csc(\theta_1-\theta_2)}{2D_1^2}\Big[2\left(D_1^2+D_2^2\right)- 2\left(D_0^2+D_1^2\right)\cos(\theta_1-\theta_2)- D_0^2k_0\cos(2\theta_1-\theta_2)+ \left(2D_2^2k_2-D_0^2k_0\right)\cos\theta_2- D_0^2h_0\sin(2\theta_1-\theta_2)+ \left(2D_2^2h_2-D_0^2h_0\right)\sin\theta_2\Big], \quad (16)$$

$$\frac{\partial(\Delta V_{t1})}{\partial D_1} = -\left(\frac{D_0^2}{D_1^2}\right)\left[1+k_0\cos\theta_1+h_0\sin\theta_1\right], \quad (17)$$

$$\frac{\partial(\Delta V_{r2})}{\partial D_1} = -\frac{\csc(\theta_1-\theta_2)}{2D_1^2}\Big[2\left(D_1^2+D_0^2\right)- 2\left(D_1^2+D_2^2\right)\cos(\theta_1-\theta_2)- D_2^2k_2\cos(\theta_1-2\theta_2)+ \left(2D_0^2k_0-D_2^2k_2\right)\cos\theta_1+ D_2^2h_2\sin(\theta_1-2\theta_2)+ \left(2D_0^2h_0-D_2^2h_2\right)\sin\theta_1\Big], \quad (18)$$

$$\frac{\partial(\Delta V_{t2})}{\partial D_1} = \left(\frac{D_2^2}{D_1^2}\right)\left[1+k_2\cos\theta_2+h_2\sin\theta_2\right]. \quad (19)$$

Now, the same technique of dividing the interval in two parts in each iteration is used, to find the root of the Eq.(10).

3 Simulations

To analyze the proposed algorithm, the Topex/Poseidon satellite (T/P) data are chosen, because it carries a dual frequency receiver GPS on board experimentally to test the ability of the GPS to provide precise orbit determination (POD). All T/P GPS data set and GPS navigation message are easily found in the Internet in Rinex format [38,41].

The mission of the Topex/Poseidon satellite is jointly conducted by the United States National Aeronautics and Space Administration (NASA) and the French space agency, Centre National d'Etudes Spatiales (CNES). The main goal of this mission is to improve the knowledge of the global ocean circulation. Other applications include studying the ocean tides, geodesy and geodynamics, ocean wave height, and winding speed.

The T/P spacecraft orbits the Earth at an altitude of 1336 km, inclination of 66°, and nearly zero eccentricity. The period of the orbit is 1.87 hrs. Table 1 shows the orbit in more detail [42]

TABLE 1 - NOMINAL ORBITAL PARAMENTER FOR THE T/P SATELLITE

PARAMETER	NOMINAL VALUES
Semi major axis	7 712.190 km
Eccentricity	0.0015
Inclination	66.039°
Orbital period	112 min
Perigee motion	0°/day
Node motion	-2.29°/day
Mean motion	4 613.6°/day

This satellite carries a total of five tracking systems including Satellite Laser Ranging (SLR), DORIS Doppler, GPS, TDRSS, and the satellite altimeter itself. The satellite orbit must be determined with a RMS radial accuracy of 13 cm. This is an extremely stringent accuracy requirement for a satellite of this shape and altitude [43]. The T/P receiver can track up to six GPS satellites at once on both frequencies if Anti-Spoofing is inactive.

The eccentricity is controlled to maintain the argument of perigee in 90º, producing a frozen orbit. This frozen orbit configuration minimizes height variations (with respect to the ellipsoid) and it is very favorable for altimeter mission [44].

After the T/P orbit is determined using the algorithm developed by Chiaradia et. al. [36], this orbit is considered the initial one for calculating the orbital maneuver. The final orbit is given as the initial conditions. Then, the optimized Prado's algorithm calculates the optimal transfer for this time. In each 30-second interval, this procedure is repeated. After 24 hours of integration, the procedure provides the optimal maneuver for the period.

The algorithm allows developing maneuvers varying the semi major axis, the eccentricity, and/or the argument of perigee. In this work, only maneuvers varying the semi major axis and eccentricity are performed. In Tables 2 to 7 are shown the optimal maneuvers performed varying only the semi major axis for two days of simulation. Tables 8 to 13 show the optimal maneuvers varying the semi major axis and the eccentricity, simultaneously, for the same periods. Three simulations are made for each table of results. They consider different values for the semi major axis of the final orbit.

The orbit determination is carried out using the osculating elements; however the algorithm for calculating the orbital maneuvers uses the mean elements. Therefore, it is necessary to perform a transformation from osculating to mean elements and to add it to algorithm. Then, the Keplerian elements shown in Tables 2 to 13 are mean elements.

TABLE 2 - MANEUVERS VARYING SEMI MAJOR AXIS FOR 11/18/1993 – SIMULATION 1

PARAMETERS	SIMULATION 1
a_0 (m)	7728608.9
e_0	0.002515
ω_0 (degree)	257.85°
a_2(m)	7730000.0
θ_1 (degree)	5.5°
θ_2 (degree)	185.96°
a_1(m)	7729303.8
e_1	0.002488
ω_1(degree)	-138.90°
r_1 (m)	7734460.1
r_2 (m)	7723907.8
ϕ_1 (degree)	0.61°
ϕ_2 (degree)	0.61°
ΔV_1 (m/s)	0.32
ΔV_2 (m/s)	0.32
ΔV_{Total} (m/s)	0.64
t (s)	77251.1

TABLE 3 - MANEUVERS VARYING SEMI MAJOR AXIS FOR 11/18/1993 – SIMULATION 2

PARAMETERS	SIMULATION 2
a_0 (m)	7728608.9
e_0	0.002515
ω_0 (degree)	257.85°
a_2(m)	7800000.0
θ_1 (degree)	5.5°
θ_2 (degree)	185.96°
a_1(m)	7764276.8
e_1	0.004511
ω_1(degree)	175.31°
r_1 (m)	7734460.1
r_2 (m)	7723907.8
ϕ_1 (degree)	0.61°
ϕ_2 (degree)	0.61°
ΔV_1 (m/s)	16.49
ΔV_2 (m/s)	16.45
ΔV_{Total} (m/s)	32.94
t (s)	77251.1

TABLE 4 - MANEUVERS VARYING SEMI MAJOR AXIS FOR 11/18/1993 – SIMULATION 3

PARAMETERS	SIMULATION 3
a_0 (m)	7728608.9
e_0	0.002515
ω_0 (degree)	257.85°
a_2(m)	7850000.0
θ_1 (degree)	5.5°
θ_2 (degree)	185.96°
a_1(m)	7789257.5
e_1	0.0074
ω_1(degree)	170.12°
r_1 (m)	7734460.1
r_2 (m)	7843813.8
ϕ_1 (degree)	0.61°
ϕ_2 (degree)	0.61°
ΔV_1 (m/s)	27.93
ΔV_2 (m/s)	27.82
ΔV_{Total} (m/s)	55.74
t (s)	77251.1

TABLE 5 - MANEUVERS VARYING SEMI MAJOR AXIS FOR 01/21/1994 – SIMULATION 4

PARAMETERS	SIMULATION 4
a_0 (m)	7726538.9
e_0	0.002169
ω_0 (degree)	235.23°
a_2(m)	7730000.0
θ_1 (degree)	5.14°
θ_2 (degree)	183.97°
a_1(m)	7728268.8
e_1	0.002144
ω_1(degree)	-123.40°
r_1 (m)	7729361.4
r_2 (m)	7727444.1
ϕ_1 (degree)	1.05°
ϕ_2 (degree)	1.05°
ΔV_1 (m/s)	0.80
ΔV_2 (m/s)	0.80
ΔV_{Total} (m/s)	1.60
t (s)	62081.0

TABLE 6 - MANEUVERS VARYING SEMI MAJOR AXIS FOR 01/21/1994 – SIMULATION 5

PARAMETERS	SIMULATION 5
a_0 (m)	7726538.9
e_0	0.002169
ω_0 (degree)	235.23°
a_2(m)	7800000.0
θ_1 (degree)	5.14°
θ_2 (degree)	183.97°
a_1(m)	7763256.6
e_1	0.004878
ω_1(degree)	136.23°
r_1 (m)	7729361.4
r_2 (m)	7797420.9
ϕ_1 (degree)	1.05°
ϕ_2 (degree)	1.05°
ΔV_1 (m/s)	16.98
ΔV_2 (m/s)	16.93
ΔV_{Total} (m/s)	33.91
t (s)	62081.0

TABLE 7 - MANEUVERS VARYING THE SEMIMAJOR AXIS FOR 01/21/1994 – SIMULATION 6

PARAMETERS	SIMULATION 6
a_0 (m)	7726538.9
e_0	0.002169
ω_0 (degree)	235.23°
a_2(m)	7850000.0
θ_1 (degree)	5.14°
θ_2 (degree)	183.97°
a_1(m)	7788247.9
e_1	0.0079
ω_1(degree)	137.89°

r_1 (m)	7729361.4
r_2 (m)	7847404.4
ϕ_1 (degree)	1.05°
ϕ_2 (degree)	1.05°
ΔV_1 (m/s)	28.41
ΔV_2 (m/s)	28.30
ΔV_{Total} (m/s)	56.71
t (s)	62081.0

TABLE 8 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 11/18/1993 – SIMULATION 7

PARAMETERS	SIMULATION 7
a_0 (m)	7726950.7
e_0	0.00505
ω_0(degree)	341.97°
a_2(m)	7730000.0
e_2	0.0001
θ_1 (degree)	314.96°
θ_2 (degree)	299.94°
a_1(m)	7729835.5
e_1	0.00048
ω_1 (degree)	-163.42°
r_1 (m)	7729835.5
r_2 (m)	7730772.9
ϕ_1 (degree)	73.59°
ϕ_2 (degree)	1.47°
ΔV_1 (m/s)	0.15
ΔV_2 (m/s)	1.38
ΔV_{Total} (m/s)	1.53
t (s)	53521.1

TABLE 9 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 11/18/1993 – SIMULATION 8

PARAMETERS	SIMULATION 8
a_0 (m)	7728608.8
e_0	0.002515
ω_0(degree)	257.85°
a_2(m)	7800000.0
e_2	0.0001
θ_1 (degree)	5.5°
θ_2 (degree)	155.97°
a_1(m)	7764773.0
e_1	0.0045
ω_1 (degree)	168.50°
r_1 (m)	7734460.1
r_2 (m)	7800160.3
ϕ_1 (degree)	0.92°
ϕ_2 (degree)	0.98°
ΔV_1 (m/s)	16.72
ΔV_2 (m/s)	16.24
ΔV_{Total} (m/s)	32.96
t (s)	77251.1

TABLE 10 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 11/18/1993 – SIMULATION 9

PARAMETERS	SIMULATION 9
a_0 (m)	7728608.8
e_0	0.002515
ω_0(degree)	257.85°
a_2(m)	7850000.0
e_2	0.0001
θ_1 (degree)	5.5°
θ_2 (degree)	167.97°
a_1(m)	7790744.3
e_1	0.0078
ω_1 (degree)	165.87°
r_1 (m)	7734460.1
r_2 (m)	7849998.3
ϕ_1 (degree)	0.12°
ϕ_2 (degree)	0.18°
ΔV_1 (m/s)	28.60
ΔV_2 (m/s)	27.15
ΔV_{Total} (m/s)	55.75
t (s)	77251.1

TABLE 11 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 01/21/1994 – SIMULATION 10

PARAMETERS	SIMULATION 10
a_0 (m)	7725673.5
e_0	0.0058
ω_0(degree)	53.38°
a_2(m)	7730000.0
e_2	0.0001
θ_1 (degree)	37.55°
θ_2 (degree)	231.97°
a_1(m)	7726015.8
e_1	0.00061
ω_1 (degree)	-128.8°
r_1 (m)	7721414.7
r_2 (m)	7730772.8
ϕ_1 (degree)	0.14°
ϕ_2 (degree)	0.15°
ΔV_1 (m/s)	0.16
ΔV_2 (m/s)	1.85
ΔV_{Total} (m/s)	2.01
t (s)	38401.0

TABLE 12 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 01/21/1994 – SIMULATION 11

PARAMETERS	SIMULATION 11
a_0 (m)	7726538.9
e_0	0.002169
ω_0(degree)	7.8°
a_2(m)	7800000.0
e_2	0.0001
θ_1 (degree)	5.2°
θ_2 (degree)	159.97°
a_1(m)	7762859.0
e_1	0.0048
ω_1 (degree)	-138.09°
r_1 (m)	7729361.4
r_2 (m)	7800206.4
ϕ_1 (degree)	0.09°
ϕ_2 (degree)	0.15°
ΔV_1 (m/s)	16.79
ΔV_2 (m/s)	17.12
ΔV_{Total} (m/s)	33.91
t (s)	62081.0

TABLE 13 - MANEUVERS VARYING SEMI MAJOR AXIS AND ECCENTRICITY FOR 01/21/1994 – SIMULATION 12

PARAMETERS	SIMULATION 12
a_0 (m)	7726538.9
e_0	0.002169
ω_0(degree)	7.8°
a_2(m)	7850000.0
e_2	0.0001
θ_1 (degree)	5.2°
θ_2 (degree)	169.97°
a_1(m)	7788568.9
e_1	0.0078
ω_1 (degree)	-139.14°
r_1 (m)	7729361.4
r_2 (m)	7850073.1
ϕ_1 (degree)	0.15°
ϕ_2 (degree)	0.09°
ΔV_1 (m/s)	28.55
ΔV_2 (m/s)	28.16
ΔV_{Total} (m/s)	56.71
t (s)	62081.0

All maneuvers studied varying the semi major axis are of the Hohmann type. It means that the difference between the true anomalies of the points, where the impulses are applied, is around 180º. The directions of all the impulses are nearly tangential, as expected. The instant to perform the maneuver, in each day, is shown in several tables. For example, in Tables 5 to 7 one notes that the optimal maneuver in the three cases occurs in the same instant (62081.0 s). The mean semi-major axis of the initial orbit obtains its maximum value in this time and this fact minimizes the consumption of the orbital maneuver that has as goal to increase the semi major axis of the satellite orbit.

The maneuvers varying the semi-major axis and the eccentricity are shown in Tables 8 to 13 and they are not of the Hohmann type. They are optimal bi-impulsive coplanar maneuvers, but the impulses are not always tangential to the orbit and the angle between the positions, where the impulses are applied, is not 180º.

3 CONCLUSIONS

Three algorithms were selected for a preliminary analysis of their capacity to perform autonomous orbital maneuvers. One of them is implemented and tested numerically in details, combined with an algorithm to determine the orbit of a spacecraft. Some adjustments in the code of this algorithm are performed for use in real-time and on board applications. Two kinds of maneuvers are executed, one varying the semi-major axis and the other one varying the semi-major axis and the eccentricity, simultaneously. The simulations were performed using the initial orbit as the one estimated by the Kalman filtering. The method converges to the optimal solution in times that are short enough to allow real-time and on board applications. The results obtained are always within the expectations based in related theories. This algorithm has the possibility of restricting the regions where the maneuver is applied, as well as to control the relation between the processing time and solution accuracy. These are two good reasons to recommend this method to be used in the proposed application.

ACKNOWLEDGEMENTS
The authors wish to express their appreciation for the support provided by UNESP (Universidade Estadual Paulista "Júlio de Mesquita Filho") of Brazil and INPE (Brazilian Institute for Space Research).

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