# Study on the Dynamic Performance of Heavy-duty Forging Manipulator

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*Abstract:* - Cooperating with heavy forging press, the heavy-duty forging manipulator is used for handling the workpiece during the free-forging process. Therefore, the size accuracy of the forgings is related to the dynamics performance of the forging manipulator. This paper aims to study the dynamic performance of DDS heavy-duty forging manipulator. The kinematics model is firstly built to predict the trajectories of the tong. Then, the dynamics models are derived by using rigid-body method and flexible-body method respectively. Finally, the simulation results indicate that partially flexible-body model is well predicted the trajectory of the tong.

*Key-Words:* -Forging manipulator, Dynamics model, Partially Flexible-body method, Trajectory

## 1 Introduction

Heavy-duty forging manipulators are not only the basic equipment for precision manufacture of heavy-forgings, but also the key to assure the forging quality of heavy-forgings [1-2]. Fig. 1 shows a typical forging manipulator, with the basic motions in operation process: walking, motion of the tong and buffering. While the accuracy of those motions realized by lifting mechanism of manipulator, has direct effects on the quality of forgings. So, the kinematics and dynamic analysis of lifting mechanism has always been the focus of manipulator studies. Ge*et al.* [3] succeeded to establish the relationship between outputs of end-effectors and actuator inputs and study the output kinematic characteristics for heavy-payload forging manipulators. Wang *et al.*[4]evaluated the mutual reaction loads between the forging process and the assisting manipulator by combining the forging finite element method simulation and the kinematics analysis. Yan *et al.* [5] built the kinematic model of the forging manipulator, and derived the closed-form inverse kinematic solution by using the homogeneous coordinate transformation method.

The researchers [6,7]analyzed the compliance process by building dynamic modeling for forging manipulator. Parikh *et al.*[8] address the forward kinematics problem of a parallel manipulator and propose an iterative neural network strategy for its real-time solution to a desired level of accuracy. Wu *et al.* [9]studied the dynamic characteristics of the two degree-of freedom planar parallel manipulator of a heavy duty hybrid machine tool. Xu *et al.*[10] built up kinematics of a typical DDS forging manipulator. In typical condition, the load of manipulator mechanism was achieved through building visual 3-D dynamic simulation of multirigid-body system by Ren*et al.*[11]. The literatures mentioned above present the rigid-body dynamics model of manipulator. However, seldom have researchers studied elastodynamics to introduce the elastic transmutation of mechanism. In contrast with elastodynamics model, rigid-body dynamics model has a greater error of movement analysis as a result of ignoring the flexibility of mechanism. Therefore, it's necessary to study the elastodynamics of manipulator becomes more important and meaningful for production process.

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(a) Photo
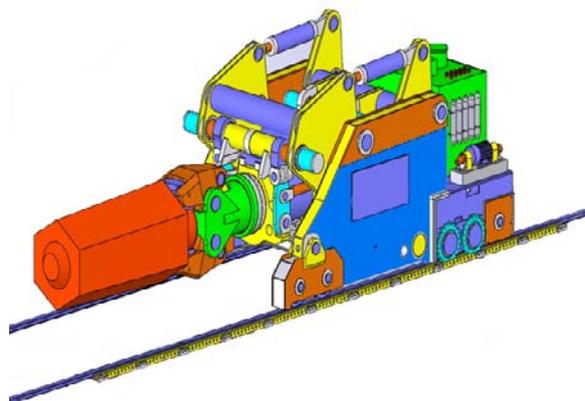


(b) CAD model

Fig.1:A typical forging manipulator

Thus, on the basis of the rigid-body dynamics analysis, this study establishes the elastodynamics model of forging manipulator and analyzes the effect of flexible-body. Because of the obvious differences in stiffness, the influence varies from component to component. The flexibility of some members, such as linkage, lazy arm and hydraulic cylinders, is only considered in this study to simplify calculation process and release motion low truly. The rest of this paper is organized as follow. In the second section, rigid-body dynamics model of forging manipulator will be established to calculate the inertial forces and external loads acting on components in every moment. Section 3 will set up partially flexible-body dynamics model by using the methods for manipulator mechanism form literatures [12-16]. And then, the simulation algorithm and the flowchart of entire analysis process will be introduced in section 4. Section 5 will discuss the calculation results with different working conditions. Eventually, section 6 states the conclusions.

## 2 Rigid-body dynamics model
### 2.1 Kinematics model

The manipulator in this study is a DDS forging manipulator, as the latest and largest tonnage forging manipulator all over the world at present, whose components are large-sizeand the main moving is designed as a parallelmechanism.

The manipulator can hold workpiece up to 300t, is a complex multi-body mechanism system which always works under heavy-load or extreme environments. As shown in Fig.2, the main mechanism of DDS forging manipulator is the lifting mechanism. In order to simplify calculation

process, the lifting mechanism can be projected into a plane denoted as $x0y$, as shown in Fig.3.
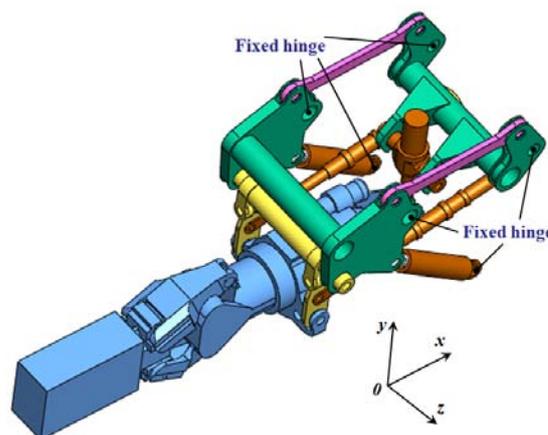


Fig.2: CAD model of lifting mechanism
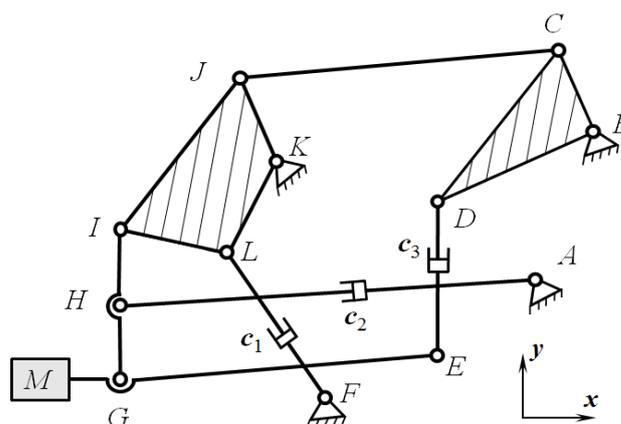


Fig. 3: Lifting mechanism

The lifting mechanism consists of several parts including linkages, hydraulic drives and motion

pairs. Hydraulic drives are with the lifting hydraulic cylinder, the buffer hydraulic cylinder and the leaning hydraulic cylinder, which are individually denoted by $c_1$, $c_2$ and $c_3$. In lifting process, the cylinder $c_1$ controls the vertical movement of workpiece through inputting lifting signal. At the same time, the cylinders $c_2$ and $c_3$ are perfectly closed. While, the cylinders $c_1$ and $c_2$ are closed and the cylinder $c_3$ realizes leaning movement by inputting leaning signal in leaning condition.

With the same mechanism, the liftingoperation and leaning operation are similar. Therefore, the kinematics model is built only for lifting operation in this study. As illustrated in Fig. 3, the length of lifting hydraulic cylinder denoted $l_1$ changes with Eq. (1) in lifting process.

$$l_1 = f(t) + l_{1\min} \tag{1}$$

where, $l_{1\min}$ is the minimum length of lifting hydraulic cylinder, $f(t)$ represent input signal and $t$ denotes the lifting time.

On kinematics modeling, Cartesian coordinate origin is established at motion pair denoted $F$, and the matrix relationship, which includes position, velocity and acceleration for other motion pairs. Taking the pair $m$ and $n$ of a member $r$ for example, the coordinate relationship of the pairs is written Eq. (2).

$$\begin{bmatrix} u_{mx} \\ u_{my} \end{bmatrix} = \begin{bmatrix} k_{11}^r & k_{12}^r & k_{13}^r \\ k_{21}^r & k_{22}^r & k_{23}^r \end{bmatrix} \begin{bmatrix} u_{nx} \\ u_{ny} \\ l_{mn} \end{bmatrix} \tag{2}$$

Where $u_{mx}$, $u_{my}$, $u_{nx}$ and $u_{ny}$ represent the coordinate of kinematic pairs $m$ and $n$ in vertical and horizontal directions, respectively; $k^r = \begin{bmatrix} k_{11}^r & k_{12}^r & k_{13}^r \\ k_{21}^r & k_{22}^r & k_{23}^r \end{bmatrix}$ is position matrix, $l_{mn}$ is the distance from pairs$m$to $n$.

By calculating the First-Order derivative and the Second-Order derivative of Eq. (2) individually, the velocity and the acceleration of the pair $m$ and $n$ can be given as the following equations.

$$\begin{bmatrix} \dot{u}_{mx} \\ \dot{u}_{my} \end{bmatrix} = \begin{bmatrix} k_{11}^{r\,\prime} & k_{12}^{r\,\prime} & k_{13}^{r\,\prime} \\ k_{21}^{r\,\prime} & k_{22}^{r\,\prime} & k_{23}^{r\,\prime} \end{bmatrix} \begin{bmatrix} \dot{u}_{nx} \\ \dot{u}_{ny} \\ \dot{l}_{mn} \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \ddot{u}_{mx} \\ \ddot{u}_{my} \end{bmatrix} = \begin{bmatrix} k_{11}^{r\,\prime\prime} & k_{12}^{r\,\prime\prime} & k_{13}^{r\,\prime\prime} \\ k_{21}^{r\,\prime\prime} & k_{22}^{r\,\prime\prime} & k_{23}^{r\,\prime\prime} \end{bmatrix} \begin{bmatrix} \ddot{u}_{nx} \\ \ddot{u}_{ny} \\ \ddot{l}_{mn} \end{bmatrix} \tag{4}$$

Where $\dot{u}_{mx}$, $\dot{u}_{my}$, $\ddot{u}_{mx}$ and $\ddot{u}_{my}$ represent the velocity and acceleration of kinematic pair m in vertical and horizontal directions, respectively, kinematic pair n is similar, $\dot{l}_{mn}$ and $\ddot{l}_{mn}$ are the velocity and the acceleration of pair m relative to pair n in the direction of $l_{mn}$.

## 2.2 Dynamics model

By analyzing elastodynamics of DDS forging manipulator, the rigid-body dynamics model is without consideration of flexibility of all components. The position, velocity and acceleration are directly calculated form kinematics analysis, with consideration of the inertial forces and external loads acting on members in every moment. According to the dynamic statics method from literatures [3-5], rigid-body dynamics equation can be set up for each member, such as the dynamics Eq. (5) of member $r$. Finally, dynamics equations of entire mechanism are gained by combining every member equation.

$$\begin{cases} \sum_{i=1}^{n} N_{xi}^r d_{xi} + N_{yi}^r d_{yi} + m^r g r_i - J^r \varepsilon^r = 0 \\ N_{x1}^r + N_{x2}^r + \cdots + N_{xn}^r - m^r a_x^r = 0 \\ N_{y1}^r + N_{y2}^r + \cdots + N_{yn}^r - m^r g - m^r a_y^r = 0 \end{cases} \tag{5}$$

where, $N_{xn}^r$ and $N_{yn}^r$ represent load acting on $n$-$th$ pair of member denoted $r$ in vertical and horizontal respectively, $m^r$, $J^r$ and $\varepsilon^r$ are mass, rotational inertia and angle acceleration of member $r$, while $a_x^r$ and $a_y^r$ are accelerations of centroid in two directional (vertical and horizontal) for member $r$, separately. $d_{xi}$, $d_{yi}$ and $r_i$ donated the force arm of $N_{xn}^r$, $N_{yn}^r$ and gravity.

# 3 Partially flexible-bodydynamics model

## 3.1 Elastodynamics analysis for linkages

As the lifting mechanism of DDS forging manipulator is parallel four connection rods of mechanism, it can be researched as planar linkage for movement of in vertical plane. Due to the effect of partially flexible-body, like linkage, lazy arm and hydraulic cylinders, is relatively obvious. Therefore, elastodynamics analysis in this paper only takes those members as flexible-body, is called partially flexible-body dynamics model. What's more, this model consists of elastodynamics analysis of lazy arm, linkage and hydraulic cylinders.

It is an effectiveapproach to analyseelastodynamics for lazy arm and linkage by finite element method (FEM). The basic idea of FEM is that mechanism is seen as a series of instantaneous linkage in every moment; then, rigid-body dynamics and elastodynamics are calculated separately for allinstantaneous linkages; finally, elastic-motion response of mechanism can be obtained by way of combining with the two results [14-15]. According to the FEM, lazy arm and linkage, denoted as*IG* and *JC* individually, are flexible-body simplified as discrete lumped-mass model, and then, their element dynamics equations are built by means of the FEM theory and Lagrange's equations, as follows.

$$m_{\mathrm{I}}\ddot{\delta}_{\mathrm{I}} + k_{\mathrm{I}}\delta = Q \qquad (6)$$

$$m_{\mathrm{II}}\ddot{\delta}_{\mathrm{II}} + k_{\mathrm{II}}\delta = Q \qquad (7)$$

Besides,Eqs. (6) and (7) are rewritten in matrix as shown in below:

$$\begin{bmatrix} m_{\mathrm{I}} & \\ & m_{\mathrm{II}} \end{bmatrix}\begin{bmatrix} \ddot{\delta}_{\mathrm{I}} \\ \ddot{\delta}_{\mathrm{II}} \end{bmatrix} + \begin{bmatrix} k & \\ & k \end{bmatrix}\begin{bmatrix} \delta \\ \delta \end{bmatrix} = \begin{bmatrix} Q \\ Q \end{bmatrix} \qquad (8)$$

Where, $m_{\mathrm{I}}$, $m_{\mathrm{II}}$, $k_{\mathrm{I}}$ and $k_{\mathrm{II}}$ are shown in *Appendix*.

Furthermore, the dynamics formulation can be achieved by transforming from local coordinator togobble coordinator system. Ultimately, Eq. (8) can be simplified as,

$$M\ddot{q} + Kq = Q_s \qquad (9)$$

Where, $M$, $K$ and $Q_s$ are the mass matrix, the stiffness matrix and generalized force; $\ddot{q}$ and $q$ are theresponses of entire system.

## 3.2 Equivalent stiffness for cylinder of manipulator

Because of the manipulator in this study which the total mass of tong and workpiece exceed 700t, the influence of hydraulic stiffness for mechanism motion should not be neglected. In lifting process, horizontal buffer cylinder and leaning cylinder are locked and seen as nonlinear spring having static stiffness; what's more, equivalent stiffness is leaded into established dynamics model in this study. According to liquid capacity effect of hydraulic cylinder,

$$\Delta Q = \frac{V}{\beta_e} \cdot \frac{dP}{dt} \qquad (10)$$

Where, $\Delta Q$, $V$, $P$ and $\beta_e$ represent the variation of flow, liquid volume in cylinder, hydraulic pressure and the equivalent elastic modulus of liquid. Eq. (10) is multiplied by $dt$ and then is integrated, as shown below,

$$V = \frac{V}{\beta_e} \cdot P \qquad (11)$$

By substituting $P = \frac{F}{A}$ into EQ.(11), it can be rewritten as:

$$F = \frac{\beta_e A^2}{V} x \qquad (12)$$

Where, $F$ and $A$ denote external load acting on cylinder and effective area of cylinder, $K_q = \frac{\beta_e A^2}{V}$ represents the equivalent stiffness of hydraulic cylinder and is lead into dynamics model of buffer cylinder and leaning cylinder, and elastodynamics model can be derived in the end, as shown below.

$$M\ddot{q} + K_q q = P(t) \qquad (13)$$

## 4 Simulation algorithm

In elastodynamics equations, mass matrix and stiffness matrix of system is function with structure parameter and position, so the modal frequency is changeable according to varied position of mechanism. System elastodynamics equations of manipulator are solved by using the mode superposition method which is a semi-discrete FEM and combines with FEM in space domain and difference-method in time domain.

Based on the canonical modes matrix of system obtained from natural frequency and modes, the elastodynamics equations of DDS forging manipulator are derived as *2nd* order differential equations without coupling in normal coordinate.

$$\begin{cases} \ddot{z}_1 + \omega_1^2 z_1 = p_1(t) \\ \ddot{z}_2 + \omega_2^2 z_2 = p_2(t) \\ \vdots \qquad \vdots \qquad \vdots \\ \ddot{z}_n + \omega_n^2 z_n = p_n(t) \end{cases} \qquad (14)$$

With the integration of Eq. (14), Duhamel integral formula of elastodynamics response for system is written as,

$$z_n(t) = \frac{1}{\omega_n} \int_0^t p_n(t) \sin\omega_n(t-\tau)d\tau + A_n \sin\omega_n t + B_n \cos\omega_n t$$

(15)

BY taking the derivation and integration operation to Eq. (15) with introduced initial conditions as given in Eq. (16), the elastodynamics response can be obtained as shownin Eq. (17):

$$\begin{cases} z_n \mid_{t=0} = \{\phi_n\}^{\mathrm{T}}[M]\{q_0\} \\ \dot{z}_n \mid_{t=0} = \{\phi_n\}^{\mathrm{T}}[M]\{\dot{q}_0\} \end{cases}$$

(16)

$$\begin{cases} z_n = \frac{\dot{z}_{n0}}{\omega_n}\sin\omega_n t + z_{n0}\cos\omega_n t + \frac{p_n(t)}{\omega_n^2}[1-\cos\omega_n t] \\ \dot{z}_n = \dot{z}_{n0}\cos\omega_n t - \omega_n z_{n0}\sin\omega_n t + \frac{p_n(t)}{\omega_n^2}\sin\omega_n t \end{cases}$$

(17)

In elastodynamics analysis of DDS forging manipulator, mass matrix, stiffness matrix and generalized force are variable with time, and it is difficult to indicate mode displacement in expressions about time $t$. As a result, recursive form of system response is derived by way of dispersing time $t$ into $n$ equidistant time segments denoted $\Delta t$ which all parameters are constant. Recursive form is shown in the following equations, and the elastodynamics result of manipulator can be achieved by solving Eq. (18) in the end.

$$\begin{cases} z_{n0}^k = \{\phi_n^k\}^{\mathrm{T}}[M^k]\{q_0^k\} \\ \dot{z}_{n0}^k = \{\phi_n^k\}^{\mathrm{T}}[M^k]\{\dot{q}_0^k\} \\ z_{n0}^{k+1} = \frac{\dot{z}_{n0}^k}{\omega_n^k}\sin\omega_n^k\Delta t + z_{n0}^k\cos\omega_n^k\Delta t + \frac{p_n^k(t)}{(\omega_n^k)^2}[1-\cos\omega_n^k\Delta t] \\ \dot{z}_n^{k+1} = \dot{z}_{n0}^k\cos\omega_n^k\Delta t - \omega_n^k \dot{z}_{n0}^k\sin\omega_n^k\Delta t + \frac{p_n^k(t)}{\omega_n^k}\sin\omega_n^k\Delta t \\ q^{k+1} = \{\phi_n^{k+1}\}\{z_{n0}^{k+1}\} \\ \dot{q}^{k+1} = \{\phi_n^{k+1}\}\{\dot{z}_{n0}^{k+1}\} \end{cases}$$

(18)

Finally, Eq. (19) indicates the dynamics response of whole machine and tong on the basis of summing up responses of elastodynamics and rigid-body dynamics. Furthermore, the flowchart of entire analysis process could be achieved, as shown in Fig.4.

$$\{X\} = \{U\} + \{Z\}$$

(19)

where $\{X\}$, $\{U\}$ and $\{Z\}$ represent the dynamics response, the elastodynamics response and rigid-body dynamics response, respectively.
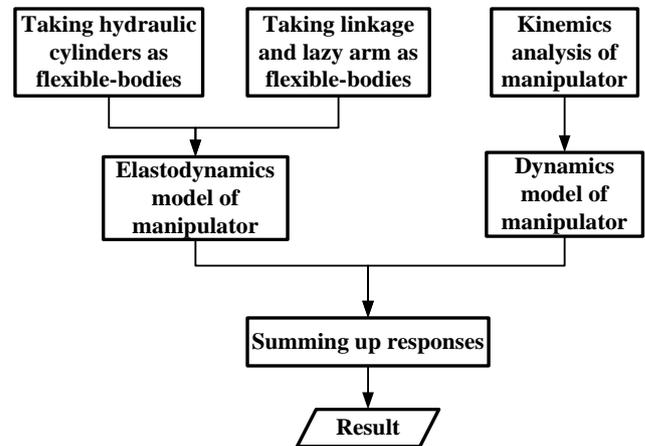


Fig.4 Flowchart ofcalculation

# 5 A case study and discussions

In this study, DDS forging manipulator with the load limit is 300t, and the load condition, input signal (Eq. (1)) and primary parameters of manipulator mechanism are shown in Table 1. To investigate the effect of flexibilities both in lifting condition and leaning condition, different lifting total-time denoted $\tau$ and positions including the lowest, the middle andthe highest position are applied to the lifting movement and the leaning movement for manipulator mechanism, respectively. In lifting process, the larger value of $\tau$ represents the lower speed of lifting cylinder and the values are 20s, 50s, 100s and 200s in this paper. While the speed of leaning cylinder denoted $v_0$ is 35mm/s at each leaning position.

Fig. 5 and Fig.6 show the trajectory of tong rigid-body dynamics response and flexible-body dynamics response for lifting condition by considering pair F as coordinate origin in Fig.2. According to compare with the responses, the error can be seen obviously at the initial stage of lifting process. Furthermore, Fig.5 reverls that the effect of flexible-body which hydraulic cylinders are significantly greater than linkage and laze arm whose effection can be ignored almost completely. Meanwhile, the higher the speed of lifting cylinder, the greater the effect of flexible-body, as shown in Fig.6. However, Fig.7 and Fig.8 illustrate the

flexible-body affect the displacement of tong mainly in horizontal direction owing to the smaller stiffness of horizontal hydraulic cylinder shown in Fig.2.

Table1 Crucialparameters of DDS forging manipulator

| | |
|---|---|
| The limit load of manipulator $M_{max}$ / t | 300 |
| The stroke of tong $\Delta l_m$ /mm | 3800 |
| The stroke of lifting hydraulic cylinder $\Delta l_1$ /mm | 2630 |
| The stroke of leaning hydraulic cylinder $\Delta l_3$ /mm | 1764 |
| $l_{AH}$ /mm | 9679 |
| $l_{JC}$ /mm | 6355 |
| $l_{DE}$ /mm | 3315 |
| $\beta_e$ /Mpa | 1000 |
| $P$ /Mpa | 32 |
| Input signal in lifting condition | $l_1 = l_{1min} + \dfrac{\Delta l_1}{2}\left[1+\sin\left(\dfrac{\pi}{\tau}t - \dfrac{\pi}{2}\right)\right]$ |
| Input signal in leaning condition | $l_1 = v_0 t + l_{1min}$ |

For the leaning condition, Fig.9 show the trajectory of tong when manipulator mechanicsm is at the lowest, the middle and the highest position individually.Nevertheless, the effect of flexible-body is not notable, in contrast to the lifting condition, as a result of the postion of manipulator mechanicsm is fxed and each pair force changes a little. In addition, the performance of the influence appear mainly in the second half of leaning process.
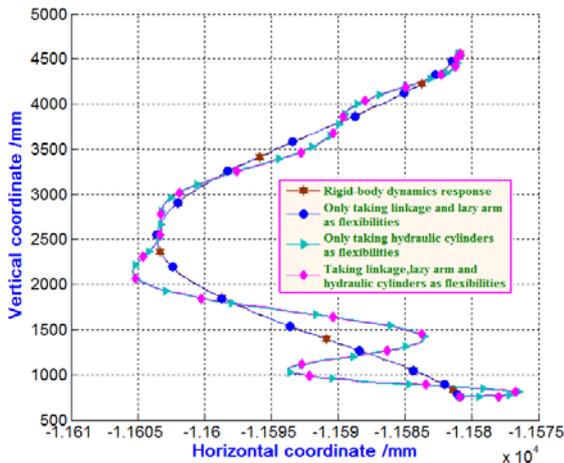


Fig.5 The trajectory comparison of rigid-body dynamics and flexible-body dynamics for tong
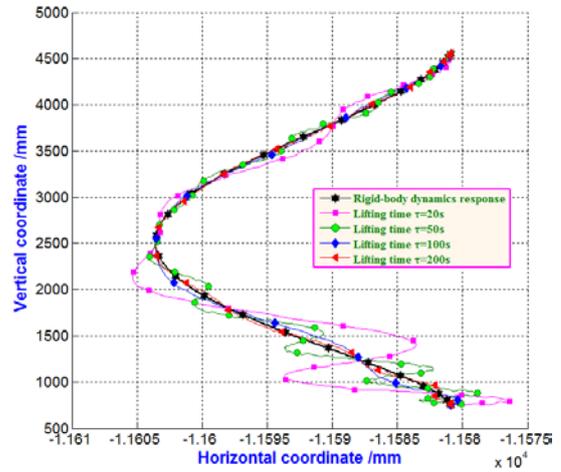


Fig.6 The trajectory of flexible-body dynamics for tong in different conditions
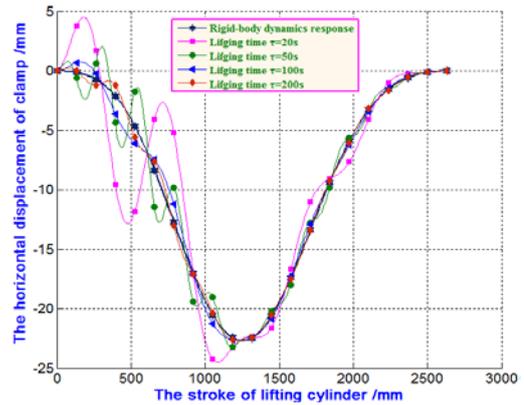


Fig.7 The horizontal displacement of tong in different conditions
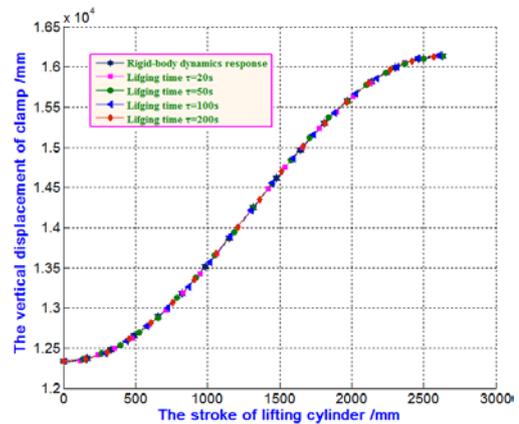
Fig.8 The vertical displacement of tong in different conditions

Fig.9 The trajectory of tong in leaning condition at different positions
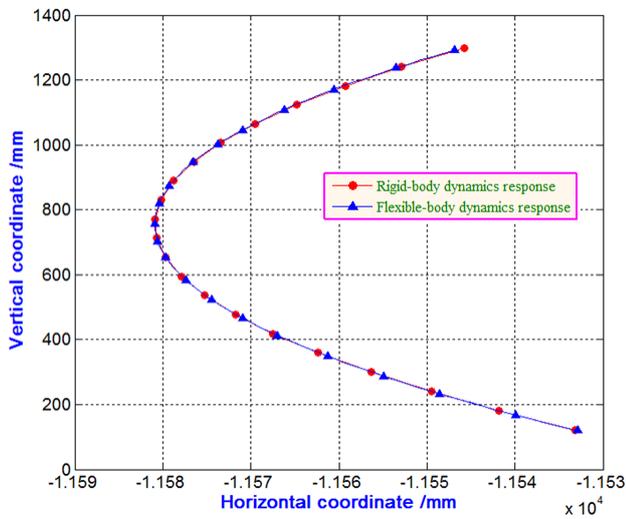


(a) clamp arm at the lowest position
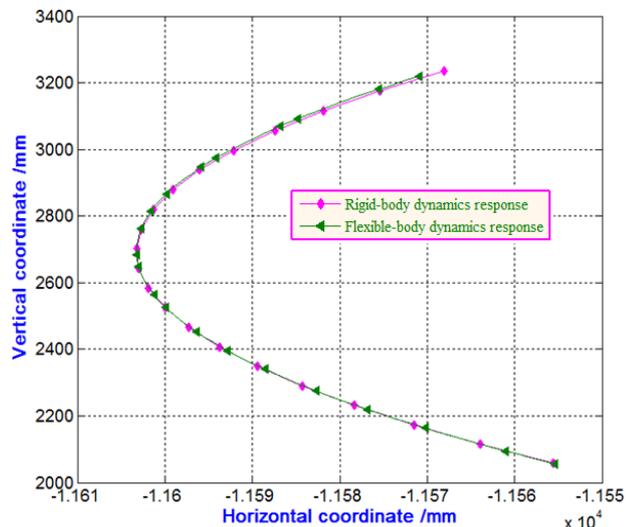


(b) clamp arm at the middle position
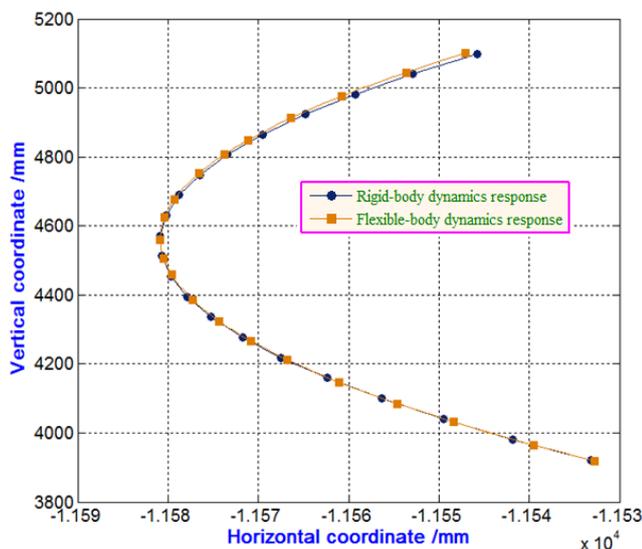


(c) clamp arm at the highest position

# 6 Conclusions

A coupling dynamics model considering partially components as flexible-body is proposed in this research for evaluating the effect of flexible-body on forging manipulator. Then a case with different conditions is introduced into this model to simulate the lifting and leaning movements. The main conclusions are as follows:

(1) This research suggested that the coupling model in this paper is effective and the time of calculation is short. What's more, flexible-body, particularly hydraulic cylinder, have obvious effect on the movement of tong in the first half of lifting process. Another important aspect, the higher the speed of lifting cylinder, the greater the effect of flexible-body.With the leanig process, the flexible-body has a increasing influence on the trajectory of tong, while the error of displacment is just insignificant.

(2) In order to obtain a good control strategy for manipulator in lifting condition, it is reasonable that hydraulic cylinders should work at a lower speed, especially in the first half of lifting process and the second half of leaning process individually. So, this approach provides a method to set down control parameters and design manipulator mechanism.

With Further analysis, the dynamics model can introduce other influential factors such as redundant actuation, joint clearance and the flexibility of other components. Even, the dynamics model can be set up in spatial coordinate or elastodynamics of manipulator can be analyzed for forging process.

## Acknowledgement

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## Appendix

In Eq. (8), $m_{\mathrm{I}}$ and $m_{\mathrm{II}}$ are the mass matrix while $k_{\mathrm{I}}$ and $k_{\mathrm{II}}$ are the stiffness matrix for element dynamics equations (Eq. (6) and Eq. (7)), those matrix is computed with the following equations:

$$m_{\mathrm{I}} = \frac{m_{IG}}{420}\begin{bmatrix} 140 & & & 70 & & & & & & & & \\ & 156 & 22l_{GH} & & 54 & -13l_{GH} & & & & & & \\ & 22l_{GH} & 4l_{GH}^2 & & 13l_{GH} & -3l_{GH}^2 & & & & & & \\ 70 & & & 140 & & & & & & & & \\ & 54 & 13l_{GH} & & 156 & -22l_{GH} & & & & & & \\ & -13l_{GH} & -3l_{GH}^2 & & -22l_{GH} & 4l_{GH}^2 & & & & & & \\ & & & & & & 140 & & & 70 & & \\ & & & & & & & 156 & 22l_{IH} & & 54 & -13l_{IH} \\ & & & & & & & 22l_{IH} & 4l_{IH}^2 & & 13l_{IH} & -3l_{IH}^2 \\ & & & & & & 70 & & & 140 & & \\ & & & & & & & 54 & 13l_{IH} & & 156 & -22l_{IH} \\ & & & & & & & -13l_{IH} & -3l_{IH}^2 & & -22l_{IH} & 4l_{IH}^2 \end{bmatrix}$$

$$
\boldsymbol{m}_{\mathrm{II}} = \frac{m_{JC}}{420}
\begin{bmatrix}
140 & & & 70 & & \\
& 156 & 22l_{JC} & & 54 & -13l_{JC} \\
& 22l_{JC} & 4l_{JC}^2 & & 13l_{JC} & -3l_{JC}^2 \\
70 & & & 140 & & \\
& 54 & 13l_{JC} & & 156 & -22l_{JC} \\
& -13l_{JC} & -3l_{JC}^2 & & -22l_{JC} & 4l_{JC}^2
\end{bmatrix}
$$

$$
\boldsymbol{k}_{\mathrm{I}} =
\begin{bmatrix}
\dfrac{(EA)_{GH}}{l_{GH}} & & & \dfrac{(EA)_{GH}}{l_{GH}} & & & & & & & & \\[2ex]
& \dfrac{12(EA)_{GH}}{l_{GH}^3} & \dfrac{6(EA)_{GH}}{l_{GH}^2} & & -\dfrac{12(EA)_{GH}}{l_{GH}^3} & \dfrac{6(EA)_{GH}}{l_{GH}^2} & & & & & & \\[2ex]
& \dfrac{6(EA)_{GH}}{l_{GH}^2} & \dfrac{4(EA)_{GH}}{l_{GH}} & & -\dfrac{6(EA)_{GH}}{l_{GH}^2} & \dfrac{2(EA)_{GH}}{l_{GH}} & & & & & & \\[2ex]
-\dfrac{(EA)_{GH}}{l_{GH}} & & & \dfrac{(EA)_{GH}}{l_{GH}} & & & & & & & & \\[2ex]
& -\dfrac{12(EA)_{GH}}{l_{GH}^3} & -\dfrac{6(EA)_{GH}}{l_{GH}^2} & & \dfrac{12(EA)_{GH}}{l_{GH}^3} & -\dfrac{6(EA)_{GH}}{l_{GH}^2} & & & & & & \\[2ex]
& \dfrac{6(EA)_{GH}}{l_{GH}^2} & \dfrac{2(EA)_{GH}}{l_{GH}} & & -\dfrac{6(EA)_{GH}}{l_{GH}^2} & \dfrac{4(EA)_{GH}}{l_{GH}} & & & & & & \\[2ex]
& & & & & & \dfrac{(EA)_{IH}}{l_{IH}} & & & \dfrac{(EA)_{IH}}{l_{IH}} & & \\[2ex]
& & & & & & & \dfrac{12(EA)_{IH}}{l_{IH}^3} & \dfrac{6(EA)_{IH}}{l_{IH}^2} & & -\dfrac{12(EA)_{IH}}{l_{IH}^3} & \dfrac{6(EA)_{IH}}{l_{IH}^2} \\[2ex]
& & & & & & & \dfrac{6(EA)_{IH}}{l_{IH}^2} & \dfrac{4(EA)_{IH}}{l_{IH}} & & -\dfrac{6(EA)_{IH}}{l_{IH}^2} & \dfrac{6(EA)_{IH}}{l_{IH}^2} \\[2ex]
& & & & & & -\dfrac{(EA)_{IH}}{l_{IH}} & & & \dfrac{(EA)_{IH}}{l_{IH}} & & \\[2ex]
& & & & & & & -\dfrac{12(EA)_{IH}}{l_{IH}^3} & -\dfrac{6(EA)_{IH}}{l_{IH}^2} & & \dfrac{12(EA)_{IH}}{l_{IH}^3} & -\dfrac{6(EA)_{IH}}{l_{IH}^2} \\[2ex]
& & & & & & & \dfrac{6(EA)_{IH}}{l_{IH}^2} & \dfrac{2(EA)_{IH}}{l_{IH}} & & -\dfrac{6(EA)_{IH}}{l_{IH}^2} & \dfrac{4(EA)_{IH}}{l_{IH}}
\end{bmatrix}
$$

$$
\boldsymbol{k}_{\mathrm{II}} =
\begin{bmatrix}
\dfrac{(EA)_{JC}}{l_{JC}} & & & \dfrac{(EA)_{JC}}{l_{JC}} & & \\[2ex]
& \dfrac{12(EA)_{JC}}{l_{JC}^3} & \dfrac{6(EA)_{JC}}{l_{JC}^2} & & -\dfrac{12(EA)_{JC}}{l_{JC}^3} & \dfrac{6(EA)_{JC}}{l_{JC}^2} \\[2ex]
& \dfrac{6(EA)_{JC}}{l_{JC}^2} & \dfrac{4(EA)_{JC}}{l_{JC}} & & -\dfrac{6(EA)_{JC}}{l_{JC}^2} & \dfrac{2(EA)_{JC}}{l_{JC}} \\[2ex]
-\dfrac{(EA)_{JC}}{l_{JC}} & & & \dfrac{(EA)_{JC}}{l_{JC}} & & \\[2ex]
& -\dfrac{12(EA)_{JC}}{l_{JC}^3} & -\dfrac{6(EA)_{JC}}{l_{JC}^2} & & \dfrac{12(EA)_{JC}}{l_{JC}^3} & -\dfrac{6(EA)_{JC}}{l_{IH}^2} \\[2ex]
& \dfrac{6(EA)_{JC}}{l_{JC}^2} & \dfrac{2(EA)_{JC}}{l_{JC}} & & -\dfrac{6(EA)_{JC}}{l_{JC}^2} & \dfrac{4(EA)_{JC}}{l_{JC}}
\end{bmatrix}
$$

Where, $m_{IG}$ and $l_{JC}$ are the mass and length of lazy arm, respectively; $E$, $A$ and $I$ denote elastic modulus, section area and inertia moment of member, remaining the same.