Development and Investigation of the Mathematical Models for Potentially Hazardous Nuclear Power Objects with Deviated Arguments

JAMSHID GHARAKHANLOU
Department of Heat and Power
National Technical University of Ukraine "KPI"
Polytekhnichna str. 37, 02056, Kyiv - UKRAINE
http://inter.kpi.ua/

IVAN V. KAZACHKOV (1,2)
(1) Department of Energy Technology, Royal Institute of Technology (KTH)
Brinellvägen, 68, Stockholm, 10044 - SWEDEN
Ivan.Kazachkov@energy.kth.se
http://www.kth.se/itm/inst?l=en_UK
(2) Department of Physics and Mathematics, Nizhyn Gogol State University
Kropyv'yans'koho 2, Nizhyn
Chernigiv region, 16602, UKRAINE
http://en.wikipedia.org/wiki/Nizhyn_Gogol_State_University

OLEKSANDR V. KONOVAL
Department of Physics and Mathematics
Nizhyn Gogol State University
Kropyv'yans'koho 2, Nizhyn, Chernigiv region
16602, UKRAINE
http://www.ndu.edu.ua/

Abstract: - The paper is devoted to one of the problems of system analysis for nuclear power plants (NPP) in touch with the modelling and simulation of the potentially hazardous objects in nuclear power safety including severe accidents at the nuclear power plants. Development and implementation of the mathematical models for analysis and computer simulations of the passive protection systems against severe accidents and mathematical modelling and simulation of the potentially hazardous objects (PHO) with account of different deviating arguments concerning the real features are considered. It has concern to a need of the planning and decision making in the environmental and other tasks.

Key-Words: - Potentially hazardous object, nuclear power, non-linear dynamical system, time shifts, control parameters, severe accident

1 Introduction

The development of the mathematical models for analysis and simulation of the different complex systems in nuclear power safety issues is of paramount interest for modern computer modelling and simulation tasks. Simulation of the potentially hazardous objects is considered in touch with a need of a tactical and strategic planning and decision making support in a lot of the environmental and other tasks to analyze peculiarities of the real systems with account of different delays and forecasting terms as it happens in a real world.

The reason for importance of such models and computer simulations and their use is due to prediction of any available negative environmental influence of the PHO. Development of the aggregate PHO models provided the possibilities to systematic investigation of the crucial situations in their evolution and peculiarities in control parameters, bifurcations of the regimes, catastrophes, etc. Certainly the results obtained have had concern to any other dynamical systems of similar type, and not only to potentially hazardous objects [1-4].

In this paper, the aggregate dynamical models for potentially hazardous objects are considered in touch with nuclear power safety estimation and other types of hazards. The differential equations of the model have been analyzed and computer simulation has been presented. Base for such investigation is statistical data on the objects functioning, as well as some information about available deviating arguments – time delays by any parameters and time forecasts. Both are of importance to get closer to the real systems with their specific features.

Also a number of physical processes, e.g. the ones having concern to severe accidents at the NPPs [5-9], peculiarities of the penetration dynamics of a liquid corium jet into the other liquid medium of coolant, vapour flow, etc. are dependent on some deviating arguments, for example delays in time by action of some parameters.

2 Statement of the Problem

Many complex systems of diverse nature including the potentially hazardous objects could be considered as non-linear statistical dynamical systems that consist of a number of different parameters, which are interconnected. As far as any PHO is a too complicated dynamical system to build its deterministic model, it seemed to be quite reasonable to construct its aggregate model based on the available statistical data.

The differential equations for different kind of such cases are analyzed and available time deviating arguments are found and disputed [1-9]. The aggregate models for potentially hazardous objects with time shifts are considered in touch with nuclear power safety estimation, forecasts and revealing the critical features of the objects to be studied and managed.

2.1 The aggregate model of PHO

The other types of hazardous, critical or specific situations and computer simulation has been presented for the case with time shifts (time delays and time forecasts for diverse parameters) and without them. The possibility for model application to tactical and strategic planning of the objects' development is discussed. The analysis of the shifted arguments and their influence on the model behaviors are presented.

Except the aggregate dynamical systems for modeling of the PHO based on general statistical information about the complex object [1-4], the time delays in the concrete thermal hydraulic problem about penetration of the corium melt jet into a coolant pool during severe accidents at the NPPs [5-9] is stated and studied too.

2.1.1 The equations of the aggregate model

According to the works [1, 2], in general case the aggregate dynamical model of the PHO may be represented as follows:

$$\frac{dx_i}{dt} = (N_i - x_i)\cdot(a_{i0} + \sum_{j=1}^{n} a_{ij}\cdot(N_j - x_j)), \qquad (1)$$

where are the following functions:

$x_i(t)$ - are the PHO characteristic parameters,

N_i - the limit values of *i*-th parameters,

a_{ij} - the coefficients, which may be the function of time too, in a general case,

n- the number of parameters of the PHO,

i=1,2..n.

2.1.2 The limit values of the model

The limit values N_i may vary according to the problem stated: tactical or strategic planning, calculation of the dynamic evolution of the parameters, critical regimes of the PHO, etc.

The non-linear ordinary differential equation array (1) with the corresponding initial data allowed conducting the simulation of PHO determining the peculiarities of evolution in time until the achievement of the planning values or any kind of other regimes, for example saturation, critical points in the system, catastrophes, etc.

2.1.3 The parameters of the mathematical model

The parameters x_i have been stated in [1] as follows:

x_1 - the quantity of the working personnel at the PHO,

x_2 - the quantity of the managers at the PHO,

x_3 - the total amount of product output at PHO,

x_4 - the expenditures for the repairing and restoring at PHO,

x_5 - the expenses required for elimination of a negative influence on the environment,

x_6 - the safety culture level.

The variables may change in the range from 0 to N_i (N_6=1 is chosen just for simplicity). The first step in a development of the PHO model consists in calculation of the constants in the equation array (1) using the statistical data collected. For a lot of diverse PHO this is crucial problem due to absence or uncertainty of the statistical data about its functioning.

Also a serious problem is then identification of the model, which requires computing the coefficients a_{ij} from satisfaction of the model developed with the real object.

2.2 Dimensionless form of the PHO model and analysis of the coefficients

2.2.1 Variables and equations

All variables of the model can be divided on their characteristic scales:

$$y_i = x_i / N_i, b_{ij} = a_{ij} N_j, b_{i0} = a_{i0}, y_6 = x_6 . \qquad (2)$$

Then accounting (2) the equation array (1) is transformed to the following dimensionless form

$$\frac{dy_1}{dt} = [b_{10} + b_{11}(1-y_1) + b_{12}(1-y_2) + b_{13}(1-y_3)](1-y_1),$$

$$\frac{dy_2}{dt} = [b_{20} + b_{21}(1-y_1) + b_{22}(1-y_2) + b_{23}(1-y_3)](1-y_2),$$

$$\frac{dy_3}{dt} = [b_{30} + b_{31}(1-y_1) + b_{32}(1-y_2) + b_{33}(1-y_3) + \\ + b_{34}(1-y_4) + b_{35}(1-y_5) + b_{36}y_6)](1-y_3),$$

$$(3)$$

$$\frac{dy_4}{dt} = [b_{40} + b_{43}(1-y_3) + b_{44}(1-y_4) + \\ b_{45}(1-y_5) + b_{46}y_6)](1-y_4),$$

$$\frac{dy_5}{dt} = [b_{50} + b_{53}(1-y_3) + b_{54}(1-y_4) + \\ + b_{55}(1-y_5) + b_{56}y_6)](1-y_5),$$

$$\frac{dy_6}{dt} = [b_{60} + b_{61}(1-y_1) + b_{62}(1-y_2) + \\ + b_{63}(1-y_3)](1-y_6),$$

Now as one could mention the total amount of the expenses for the repairing and restoring at the PHO together with the expenses required for elimination of negative influence on the surrounding environment cannot be equal or bigger than the total product output at PHO, otherwise such PHO has no interest for its running. Thus, here the restriction $N_4 + N_5 \le N_3$ must be stated. For the dimensionless time t some characteristic interval T was taken.

The equation array (3) has stationary solutions, one of which is the trivial when $y_i = 1$ for all values of the PHO parameters.

2.2.2 Variables and equations

Here we have stated the 6 variables of the mathematical model for the PHO considered. Actually it may be more variable depending on the situation and existing knowledge and statistical data about the object. Investigator must decide upon the number of variables also from the point of view of reasonable simplicity and required rate of fidelity.

2.2.3 Analysis of the coefficients

Coefficients a_{ij} in the equations (1) and coefficients b_{ij} in the equations (3) may be estimated by their possible signs. For example, in a first equation of (1) must be $a_{11} \le 0$, $a_{16} \le 0$, $a_{1j} \ge 0$, *j=2-5.*

Here $a_{11} \le 0$ means that the more people are working at the PHO object, the more the first term in the first equation of (1) decreases the further growing rate of the number of workers (more shortcut of their number is available).

Then $a_{16} \le 0$, because the improvement of the safety culture leads to a shortcut of the employees due to increase of the productivity and safety.

It may be stated that $a_{1j} \ge 0$, *(j=2-5)* due to the fact that the number of managers, quantity of the product produced at PHO, expenses on the repairing, etc. can only lead to increase of the workers needed.

Similarly $a_{21} \ge 0$, $a_{23} \ge 0$, and the rest of the coefficients in the second equation of the system (1) must be negative because only the growing of the workers' number and product may result in a request for growing the number of managers. All the other factors result in decrease of the managers' amount at the PHO.

In the third equation only one coefficient is supposed to be negative: $a_{33} \le 0$. This is because the product amount growing causes the further productivity falling. All the other factors act to its growing.

The forth equation gives $a_{44} \le 0$, $a_{46} \le 0$ because the increase of all expenses tend to their decrease afterwards as far they keep sufficient some time. And the increase of safety culture makes tendency to decrease of all expenses.

In the fifth equation only the last 3 coefficients seem to be negative: $a_{5j} \le 0$, *j=4-6*, where $a_{54} \le 0$ because by growing the expenses for the repairing of different part of PHO there are less money going to ecology and other stuff, $a_{55} \le 0$ because if paid for this need, then less is going later on for this. And $a_{56} \le 0$ because the growing of the safety culture decreases money requested for preventing the pollution of the environment and liquidation of the consequences of the pollutions.

The last equation also must contain the last three coefficients negative: $a_{6j} \le 0$ (*j=4-6*). The reason is simple: the payments for the repairing and other stuff needed are directly connected to a safety culture: the higher is safety culture, the less are those expenditures required. And the safety culture is increasing depending on the current safety culture level.

2.2.4 An indicator of the safety level

It is important to compute the so-called indicator of the safety culture introduced in [1]:

$$V = q\frac{1+\alpha_1 y_1 + \beta_1 y_2}{1+\alpha_2 y_1 + \beta_2 y_2} \cdot \frac{y_3 y_4 y_6}{y_5}, \tag{4}$$

where q is the parameter characterizing the current level of technology in a society, α_i, β_i-coefficients.

The equation (4) shows that by $y_3, y_4, y_6 \to 0$ $V \to 0$, and by $y_5 \to 0$ follows $V \to \infty$. By $y_1, y_2 \to 0$ the indicator of the general safety level is determined not only by the personnel amount but also by the level of individuals' and social interests.

2.3 The time deviations by parameters

2.3.1 Time delaying and time forecasting terms

Some delays in a system might be available, e.g. delays connected with a speed of an information transfer, processing time taken by information processing and analysis before decision making, etc. Then some referring values, for example, achievement of the desired level parameters in a future might be stated too. In such cases, one should introduce time delays or time forecast terms for those parameters.

In general, the time delays τ, in turn, may be functions of time as well, which complicates the problem stated dramatically.

2.3.2 The equation arrays with deviating arguments

The time delays τ, in a first approach, can be assumed constants at least for a short time period. With account of the above-mentioned, the above-described PHO model (3) is presented in the form:

$$\begin{aligned}
&\frac{dz_1}{dt} = [b_{10} + b_{11}z_1(t-\tau_{11}) + b_{12}z_2(t-\tau_{12}) + \\
&b_{13}z_3(t-\tau_{13})]z_1(t-\tau_{10}), \\
&\frac{dz_2}{dt} = [b_{20} + b_{21}z_1(t-\tau_{21}) + b_{22}z_2(t-\tau_{22}) + \\
&b_{23}z_3(t-\tau_{23})]z_2(t-\tau_{20}), \\
&\frac{dz_3}{dt} = [b_{30} + b_{31}z_1(t-\tau_{31}) + b_{32}z_2(t-\tau_{32}) + \\
&b_{33}z_3(t-\tau_{33}) + b_{34}z_4(t-\tau_{34}) + b_{35}z_5(t-\tau_{35}) + \\
&b_{36}z_6(t-\tau_{36})]z_3(t-\tau_{30}), \\
&\frac{dz_4}{dt} = [b_{40} + b_{43}z_3(t-\tau_{43}) + b_{44}z_4(t-\tau_{44}) + \\
&b_{45}z_5(t-\tau_{45}) + b_{46}z_6(t-\tau_{46})]z_4(t-\tau_{40}), \\
&\frac{dz_5}{dt} = [b_{50} + b_{53}z_3(t-\tau_{53}) + b_{54}z_4(t-\tau_{54}) + \\
&b_{55}z_5(t-\tau_{55}) + b_{56}z_6(t-\tau_{56})]z_5(t-\tau_{50}), \\
&\frac{dz_6}{dt} = [b_{60} + b_{61}z_1(t-\tau_{61}) + b_{62}z_2(t-\tau_{62}) + \\
&b_{63}z_3(t-\tau_{63}))]z_6(t-\tau_{60}),
\end{aligned} \tag{5}$$

where the deviations τ_{ij} =const. In general τ_{ij} are functions of the time themselves, z_i.=1-y_i.

The mathematical model (5) for the PHO with deviated arguments τ_{ij} thus obtained describes a PHO evolution in time influenced upon its history. In a similar way the positive time shifts can be introduced to the system (5) with respect to the processes of some planning levels for the future parameters' levels (e.g. managers orient in their activity to the desired level of some parameters).

2.3.3 Peculiarities of the equation array

In general the equation array (5) is difficult to solve because so many time deviations cause big numerical problems due to necessity of the calculating algorithm to account all time moments corresponding to the respective time delays while the numerical solution of the differential equations is going according to the accuracy stated and may go outside of some time moments connected to τ_{ij}.

The other situation is with time forecasting terms (when corresponding value τ_{ij} in the equations (5) is negative). Then the right hand of the system (5) contains some terms depending on the time moments in the future. Such terms may be computed only iteratively step-by-step approaching to the solution in a number of iterations.

Obviously, τ_{ij}=0 for all $t \le \tau_{ij}$ because otherwise the know prehistory of the PHO is required before starting the solution process. Thus, if the initial conditions are stated for the equation array (5) as

$$t = 0, \quad z_i = z_{i0}, \tag{6}$$

where z_{i0} are known values, then each of the time delay arguments τ_{ij} starts to work only after the moment of time $t = \tau_{ij}$. These moments may be called switching moments of time.

2.4 Analysis of the equation array

The equation array (5) could be treated under some special conditions and restrictions due to real peculiarities of the PHO. Therefore further consideration of PHO modelling could be representing as the problem for concrete case.

2.4.1 The Elsgoltz's theorem

The system (5) can be simplified in a vicinity of the current moment of time t using the Taylor time series for the functions $z_i(t-\tau_{ij})$. According to the Elsgoltz theorem [10], the best approximations for the $z_i(t-\tau_{ij})$ are the linear ones:

$$z_i(t-\tau_{ij}) \approx z_i(t) - \tau_{ij}\dot{z}_i, \tag{7}$$

where $\dot{z}_i = dz/dt$.

Unfortunately this theorem does not satisfy many real cases. The deviating arguments can be reason for serious non-linearity even in the simplest equations. For example, J.D. Murray [11] considered one non-linear model with time delay from biology

$$\frac{dx}{dt} = -\frac{\pi}{2\tau}x(t-\tau),$$

which has solution

$$x(\mathrm{t}) = \mathrm{A}{*}\cos(\pi\, t/(2\,\tau)),$$

where instead of π might be $\pi(4n+1)$.
Here n is any natural number. The Elsgoltz's theorem does not allow obtaining this solution.

2.4.2 Transformation of the equation array with the Elsgoltz's theorem

With the theorem (7), the equation array (5) is transformed to a much simpler form

$$\begin{aligned}
\dot{z}_1 &= [b_{10} + b_{11}(z_1(t) - \tau_{11}\dot{z}_1) + b_{12}(z_2(t) - \tau_{12}\dot{z}_2) + \\
&\quad + b_{13}(z_3(t) - \tau_{13}\dot{z}_3)](z_1(t) - \tau_{10}\dot{z}_1), \\
\dot{z}_2 &= [b_{20} + b_{21}(z_1(t) - \tau_{21}\dot{z}_1) + b_{22}(z_2(t) - \tau_{22}\dot{z}_2) + \\
&\quad + b_{23}(z_3(t) - \tau_{23}\dot{z}_3)](z_2(t) - \tau_{20}\dot{z}_2), \\
\dot{z}_3 &= [b_{30} + b_{31}(z_1(t) - \tau_{31}\dot{z}_1) + b_{32}(z_2(t) - \tau_{32}\dot{z}_2) + \\
&\quad + b_{33}(z_3(t) - \tau_{33}\dot{z}_3) + b_{34}(z_4(t) - \tau_{34}\dot{z}_4) + \\
&\quad + b_{35}(z_5(t) - \tau_{35}\dot{z}_5) + b_{36}(z_6(t) - \tau_{36}\dot{z}_6)](z_3(t) - \tau_{30}\dot{z}_3), \\
\dot{z}_4 &= [b_{40} + b_{43}(z_3(t) - \tau_{43}\dot{z}_3) + b_{44}(z_4(t) - \tau_{44}\dot{z}_4) + \\
&\quad + b_{45}(z_5(t) - \tau_{45}\dot{z}_5) + b_{46}(z_6(t) - \tau_{46}\dot{z}_6)](z_4(t) - \tau_{40}\dot{z}_4), \\
\dot{z}_5 &= [b_{50} + b_{53}(z_3(t) - \tau_{53}\dot{z}_3) + b_{54}(z_4(t) - \tau_{54}\dot{z}_4) + \\
&\quad + b_{55}(z_5(t) - \tau_{55}\dot{z}_5) + b_{56}(z_6(t) - \tau_{56}\dot{z}_6)](z_5(t) - \tau_{50}\dot{z}_5), \\
\dot{z}_6 &= [b_{60} + b_{61}(z_1(t) - \tau_{61}\dot{z}_1) + b_{62}(z_2(t) - \tau_{62}\dot{z}_2) + \\
&\quad + b_{63}(z_3(t) - \tau_{63}\dot{z}_3)](z_6(t) - \tau_{60}\dot{z}_6).
\end{aligned} \tag{8}$$

The equation array (8) thus obtained evidently shows that deviating arguments even in simplest case of the Elsgoltz's theorem cause dramatically changes of the type of differential equations. They cause substantial additional and more serious non-linearity of the equations.

Now the mathematical model described by system (8) contains the products of the derivatives, so that solution of the equation array (8) may absolutely differ from the one for the equation array (3). The same big difference is supposed for the behaviors of the PHO, respectively.

Thus, the time deviating arguments may cause cardinal changes in the system, which may go to the critical regimes with the loosing of the system stability and other critical states of the system. Also it might be concluded that time delays and deviating arguments in general are the source of the strongest changes in the system solutions, which lead to a mistuning of the system parameters and as a consequence of this – to available catastrophes.

2.4.3 Transformation of the equation array with the Elsgoltz's theorem

The above-mentioned features of the model predetermine the difference in numerical methods applied for solution of the equation arrays (3), (5) and (8).

One of the approaches to solve the equation array (8) can be the following. First the time delays only for the main variable in each of the equations are taken into account as the most important (influential ones): first equation in touch with the first variable x_1, the second – in touch with the second variable x_2, etc. Then the more general iterative algorithm may be built to continue accounting the other delays in the equation array.

For the first step in the above algorithm the equations may be simplified as following

$$\begin{aligned}&\left[1+\tau_{10}(b_{10}+b_{11}z_1(t)+b_{12}z_2(t)+b_{13}z_3(t))\right]\dot{z}_1 =\\ &=[b_{10}+b_{11}z_1(t)+b_{12}z_2(t)+b_{13}z_3(t)]z_1(t),\end{aligned}$$

$$\begin{aligned}&\left[1+\tau_{20}(b_{20}+b_{21}z_1(t)+b_{22}z_2(t)+b_{23}z_3(t))\right]\dot{z}_2 =\\ &=[b_{20}+b_{21}z_1(t)+b_{22}z_2(t)+b_{23}z_3(t)]z_2(t),\end{aligned}$$

$$\begin{bmatrix}1+\tau_{30}(b_{30}+b_{31}z_1(t)+b_{32}z_2(t)+b_{33}z_3(t)+\\ +b_{34}z_4(t)+b_{35}z_5(t)+b_{36}z_6(t))\end{bmatrix}\dot{z}_3 =$$

$$=\begin{bmatrix}b_{30}+b_{31}z_1(t)+b_{32}z_2(t)+b_{33}z_3(t)+\\ +b_{34}z_4(t)+b_{35}z_5(t)+b_{36}z_6(t)\end{bmatrix}z_3(t),$$

$$\begin{bmatrix}1+\tau_{40}(b_{40}+b_{43}z_3(t)+b_{44}z_4(t)+\\ +b_{45}z_5(t)+b_{46}z_6(t))\end{bmatrix}\dot{z}_4 = \tag{9}$$

$$=\begin{bmatrix}b_{40}+b_{43}z_3(t)+b_{44}z_4(t)+\\ +b_{45}z_5(t)+b_{46}z_6(t)\end{bmatrix}z_4(t),$$

$$\begin{bmatrix}1+\tau_{50}(b_{50}+b_{53}z_3(t)+b_{54}z_4(t)+\\ +b_{55}z_5(t)+b_{56}z_6(t))\end{bmatrix}\dot{z}_5 =$$

$$=\begin{bmatrix}b_{50}+b_{53}z_3(t)+b_{54}z_4(t)+\\ +b_{55}z_5(t)+b_{56}z_6(t)\end{bmatrix}z_5(t),$$

$$\begin{bmatrix}1+\tau_{60}(b_{60}+b_{61}z_1(t)+\\ +b_{62}z_2(t)+b_{63}z_3(t))\end{bmatrix}\dot{z}_6 =$$

$$=[b_{60}+b_{61}z_1(t)+b_{62}z_2(t)+b_{63}z_3(t)]z_6(t).$$

After solution of the equation array (9) the next iteration with the other deviating terms in (8) may be performed similarly.

3 Stationary solution of the Problem

3.1 The stationary state of the non-linear PHO dynamical system

3.1.1 Solution for the stationary state

According to the equation array (3) the stationary state of the aggregate mathematical model is defined as follows

$$\sum_{j=1}^{n} a_{ij}x_j = a_{io} + \sum_{j=1}^{n} a_{ij}N_j. \tag{10}$$

Solution of the algebraic equation array (10) is obtained in the form

$$x_j = \det\left\{a^*_{ij}\right\}/\det\left\{a_{ij}\right\},$$

where $\det\left\{a_{ij}\right\}$, $\det\left\{a^*_{ij}\right\}$ are the determinant and adjacent determinant of the linear algebraic equation array (10). The adjacent determinant is obtained by replacing the j-th column with the right hand $a_{io}+\sum_{j=1}^{n}a_{ij}N_j$ of the (10).

The matrix of the equations (10) is

$$\left\{a_{ij}\right\}=\begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0\\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36}\\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46}\\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56}\\ a_{61} & a_{62} & a_{63} & 0 & 0 & 0\end{pmatrix}. \tag{11}$$

To compute $\det\{a_{ij}\}$ the third and sixth raws of the matrix are exchanged:

$$\{a_{ij}\} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{pmatrix}, \quad (12)$$

and then determinant is computed as a product of two determinants of the third order:

$$\det\{a_{ij}\} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{61} & a_{62} & a_{63} \end{vmatrix} \cdot \begin{vmatrix} a_{44} & a_{45} & a_{46} \\ a_{54} & a_{55} & a_{56} \\ a_{34} & a_{35} & a_{36} \end{vmatrix} = \quad (13)$$

$$= A_{126} \cdot A_{453} .$$

3.1.2 Analysis of the solution for stationary state

Here and later on the determinants are assigned according to the lines of the matrix (12). For the adjacent determinants the expressions are different for the first three variables (x_1, x_2, x_6) and the second three variables (x_4, x_5, x_3), respectively:

$$\det\{a_{ij}^*\} = A_j^* \cdot A_{453}, \quad j=1, 2, 6;$$

$$\det\{a_{i4}^*\} =$$
$$= A_{126}\left[a_{36}(a_{55}a_4^* - a_{45}a_5^*) + a_{35}(a_{46}a_5^* - a_{56}a_4^*)\right] +$$
$$+ A_3^*\left[a_{36}(a_{45}a_{53} - a_{55}a_{43}) + a_{35}(a_{56}a_{43} - a_{53}a_{46})\right] + ,$$
$$+ A_4^*(a_{45}a_{56} - a_{55}a_{46})$$

$$\det\{a_{i5}^*\} =$$
$$= A_{126}\left[a_{35}(a_{44}a_5^* - a_{54}a_4^*) + a_{34}(a_{56}a_4^* - a_{46}a_5^*)\right] +$$
$$+ A_3^*\left[a_{36}(a_{54}a_{43} - a_{53}a_{44}) + a_{34}(a_{53}a_{46} - a_{56}a_{43})\right] + ,$$
$$+ A_4^*(a_{46}a_{54} - a_{56}a_{44}) \quad (14)$$

$$\det\{a_{i6}^*\} =$$
$$= A_{126}\left[a_{34}(a_{45}a_5^* - a_{55}a_4^*) + a_{35}(a_{54}a_4^* - a_{44}a_5^*)\right] +$$
$$+ A_3^*\left[a_{34}(a_{55}a_{43} - a_{53}a_{45}) + a_{35}(a_{53}a_{44} - a_{54}a_{43})\right] + ,$$
$$+ A_4^*(a_{44}a_{55} - a_{54}a_{45})$$

where are

$$A_1^* = \begin{vmatrix} a_1^* & a_{12} & a_{13} \\ a_2^* & a_{22} & a_{23} \\ a_6^* & a_{62} & a_{63} \end{vmatrix}, \quad A_2^* = \begin{vmatrix} a_{11} & a_1^* & a_{13} \\ a_{21} & a_2^* & a_{23} \\ a_{61} & a_6^* & a_{63} \end{vmatrix},$$

$$A_3^* = \begin{vmatrix} a_{11} & a_{12} & a_1^* \\ a_{21} & a_{22} & a_2^* \\ a_{61} & a_{62} & a_6^* \end{vmatrix}, \quad A_4^* = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1^* \\ a_{21} & a_{22} & a_{23} & a_2^* \\ a_{61} & a_{62} & a_{63} & a_6^* \\ a_{31} & a_{32} & a_{33} & a_3^* \end{vmatrix}.$$

3.2 Peculiarities of the stationary solution

3.2.1 Stationary values of the parameters of PHO x_1, x_2, x_6 and x_4, x_5, x_3

The solution of the system of algebraic equations (10) is presented as

$$x_j = A_j^* / A_{126}, \quad j=1, 2, 6;$$

$$x_5 = \Big\{\left[a_{35}(a_{44}a_5^* - a_{54}a_4^*) + a_{34}(a_{56}a_4^* - a_{46}a_5^*)\right] +$$
$$+ \frac{A_3^*}{A_{126}}\left[a_{36}(a_{54}a_{43} - a_{53}a_{44}) + a_{34}(a_{53}a_{46} - a_{56}a_{43})\right] +$$
$$+ A_4^*(a_{46}a_{54} - a_{56}a_{44}) / A_{126}\Big\} / A_{453}, \quad (15)$$

$$x_3 = \Big\{\left[a_{34}(a_{45}a_5^* - a_{55}a_4^*) + a_{35}(a_{54}a_4^* - a_{44}a_5^*)\right] +$$
$$+ \frac{A_3^*}{A_{126}}\left[a_{34}(a_{55}a_{43} - a_{53}a_{45}) + a_{35}(a_{53}a_{44} - a_{54}a_{43})\right] +$$
$$+ A_4^*(a_{44}a_{55} - a_{54}a_{45}) / A_{126} + \Big\} / A_{453}.$$

3.2.2 Specific relations between the values x_1, x_2, x_6 and x_4, x_5, x_3

The solution obtained (15) shows that the variables x_1, x_2, x_6 (the amount of workers and managers and the safety culture level) do not depend on the variables x_4, x_5, x_3 (ecology, repairing and amount of the product produced), while x_4, x_5, x_3 depend on all variables though according to (1) x_4, x_5 do not depend on x_1, x_2.

Thus, dependence of x_4, x_5 on x_1, x_2 is indirect, through the other variables, which depend on x_1, x_2. Now using the solution (15) one can get the feature of the stationary states available studied in detail. The special, critical, catastrophic and other specific regimes may be learnt. Also using the solution (15) thus obtained investigator can perform optimization of the stationary state of PHO according to the goal stated.

4 The numerical modeling and computer simulation of the PHO

4.1 Statement of the Cauchy problem for the PHO equation array

4.1.1 The initial values of the PHO parameters

Now for the mathematical models in any of the forms (5) or (8), (9) with deviating in time arguments the Cauchy problem may be stated:

$$t = 0, \qquad z_i = z_i^0, \tag{16}$$

where $z_i = z_i^0$ are the values of the functions at the initial moment, τ_{ij} are constants in the first approach and they determine the time shifts by corresponding parameters.

By $\tau_{ij} > 0$ the delays are in the system by corresponding PHO parameters while by $\tau_{ij} < 0$ there are available some outstrips by parameters.

4.1.2 Prehistory of the PHO

Instead of the conventional initial data in the form (16) the Cauchy problem for the model in the form (5) or (8), (9) may be stated as prehistory of the PHO object. In such case the PHO initial state is computer from the known prehistory. And then the equation array (5) is solved with the initial data (16) accounting the time shifts. Otherwise as stated before the time shifts should be turned on at the moment when they are achieved in time.

This may cause substantial difficulties in the numerical solution of the Cauchy problem. Therefore specific numerical methods are applied here.

4.1.3 Pre-stochastic behavior of the PHO

Modeling of the non-linear dynamical systems reveals many interesting effects, for example «pre-stochastic» behavior leading to appearing the strange attractor [12]:

$$dN_i/dt = N_i(-m_i + k_i V_{i-1}(N_{i-1}) - W_i(N_i)N_{i+1}),$$
$$V_i(N_i) = VN_i/(K+N_i), \;\; W_i(N_i) = V/(K+N_i). \tag{17}$$

where N_{i-1}, N_i, N_{i+1} are the values of function in the three consecutive points of the region, and a model is cellular automata, which means that only the neighboring points are interacting.

4.2 Numerical modeling of the PHO

4.2.1 Development of the algorithm for numerical modeling

The algorithm for numerical solution of the Cauchy problem (5), (16) is built as follows. The main computer program from the code SGDemo called from command line:

```
        PROGRAM     SGDemo
!
        USE DFLIB
        USE SCIGRAPH        !   SciGraph    --
Scientific Graphs for Compaq Visual Fortran
!
        integer I
!
        call     EquSys()
!
        I = setexitqq(QWIN$EXITPERSIST)
!
        stop     '\n finish\n'C
        end
```

Then connects the libraries DFLIB and SCIGRAPH, where from later on the standard FORTRAN-subroutines DFLIB and SCIGRAPH (graph building) and called.

Then the computer program EquSys for solution of the equation array is called:

```
        subroutine        EquSys
!
        character*(*),  parameter ::    filename =
'Input.dat'C
!
        integer           nodep, n, iswitch

        real              t_ini, t_fin
!
```

```
        integer, externalN_Data
!
        n = N_Data(filename, nodep, t_ini, t_fin, iswitch)
!
        if (0 .LT. n)      call      SOLVSHOW(nodep, t_ini, t_fin, iswitch, n)
!
        return
        end      subroutine
```

With the parameter filename the name of the input file is done as 'Input.dat', which contains the description of task stated for solution. Function N_Data, in turn, gets from the function Get_Path the total access pass to the file and reads its first and second lines. Function Get_Path is searching the file 'Input.dat' in the directory where from the exe-file of the code is called. If the file is absent there, it displays the standard frame of the Windows selection file, where the user must pick-up the file place.

The first line of the file 'Input.dat' containes the four numbers:

Nodep- integer number, equal to a number of the numerical grid nodes in numerical algorithm;

t_ini- number with flowing point (the initial moment of time);

t_fin- number with flowing point (the final moment of time for numerical solution);

iswitch- integer number (switcher), equal zero.

The second line of the file 'Input.dat' must contain one integer number:

n- dimension of the solving equation array (here it is n = 6).

Then the program EquSys calls subroutine SOLVSHOW, which performs the numerical solution of the equation array and printing the result on display.

4.2.2 Peculiarity of the computer program

The above consequence of the algorithmic procedures is due to the mechanism of the automatic distribution of the computer memory in FORTRAN.

At the moment of start the main subroutine SOLVSHOW the number n is already known (read from the file). This allow FORTRAN executive system selecting the memory size for the stated value n.

Subroutine SOLVSHOW calls the subroutines M_Data and T_Data, which continue reading the input file 'Input.dat' and fill the structures of the automatically reserved memory depending on n and determined dynamically at the moment of entering to the subroutine SOLVSHOW.

The references to the structures dataV and dataI are described as common blocks and are accessible to all the rest subroutines of the task:

```
        common        /dataV/ dataV
        common        /dataI/   dataI
```

This allow decreasing the amount of the parameters to transfer for further calls of the subroutines and making their text more readable.

The subroutine M_Data reads the matrix of the coefficients of the equation array and the right column of the equation array from the input file, as well as the initial value of the unknown function, matrix of the time shifts and the column of the time shifts according to the problem stated.

4.2.3 The work of the algorithm by operations

Let consider the column of the unknown functions as z_i(t), then b_{ij} – square matrix of coefficients the equation array, b_{i0}– column of the right hand of the equation array, τ_{ij}– square matrix of the time shifts; τ_{i0}– column of the time shifts; n– dimension of the system.

Let us assume that by asymptotic $t \gg \tau_{ij}$ the system has form:

$$\sum_{j=1}^{n} b_{ij} \cdot \bar{z}_j + b_i = 0. \qquad (18)$$

At the initial moment of time $t=0$ the system has form (5). Subroutine M_Data computes the stationary solutions $\bar{z}_j$ of the equation array (5) and goes to «normalized» variables with the initial values:

$$w_i(t) = \frac{z_i(t)}{\bar{z}_i}. \quad w_i(t_0) = \frac{z_i(t_0)}{\bar{z}_i}.$$

Building the solution $z_i(t)$ for $t > 0$ requires the values of $z_i(t_0-\tau_{ij})$. They are stated in the input file 'Input.dat' by pairs ($z_i(t_0-\tau_{ij})$, τ_{ij}), the same way as the complex numbers in the FORTRAN.

Subroutine T_Data proves the consistence of the introduced by subroutine M_Data values ($z_i(t_0-\tau_{ij})$, τ_{ij}) and “non-degeneration” of the matrix b_{ij}, $\det|b_{ij}| \neq 0$. Non-contradiction of the results $z_i(t_0-\tau_{ij})$ obtained means:

$$\max|z_i(t_0-\tau_{ij})-z_i(t_0-\tau_{il})|<\varepsilon \text{ by } |\tau_{ij}-\tau_{kl}|<\delta,$$

where from follows that the variables cannot do abrupt changes in the infinitely close moments of time. The values ε and δ are computed by calling the standard function of the FORTRAN epsilon (REAL), which returns a positive model number that is almost negligible compared to unity in the model representing real numbers.

After execution of the above proof the program stage2 is called, which first builds the spline-interpolation of the initial data stated at the moments of time τ_{ij}, preceded the initial moment of time stated in the task, where from the solution of the Cauchy problem (5), (16) is built.

The data obtained are plotted in the new window of the Windows system by calling the function XYDemo. Then the program for solution of the differential equation array with the Runge-Kutta-Merson using the automatic time step PrkM is called. First the stationary solution is found and plotted for the beginning and estimation.

Then the sixth-step Runge-Kutta-Merson method is called for solution of the differential equations. The plot of solution is done in the new window by calling the function XYDemo. For comparison the solution of the problem was done also by the five-step Runge-Kutta-Merson method, which did not reveal substantial difference with the sixth-step method.

4.3 Computer simulation of the PHO

4.3.1 Analysis of the results by the regimes of the PHO without time delays

The computer simulation of the PHO using the algorithm and computer program developed has been performed. It allowed revealing interesting features of the systems studied.

The computations performed for the grid number 999, time interval from 0 to 9.99, the parameter -1 in the program was used for testing the program. For the case without time delays the results of computation are given in the Fig. 1 by parameters presented in the Table 1. As an initial data the stationary solution was chosen in the computer simulations. The functions $z_i(t)$ (for the values i from 6 до 1) are assigned as Y(6)-Y(1), respectively, and in Fig. 1 they are shown from the left to the right.

Table 1. The values of the coefficients and right hands of the equations (5)

coefficients of the 1st equation	1,1 1,2 1,3 1,4 1,5 1,6	-1,7
coefficients of the 2nd equation	2,1 2,2 2,3 2,4 2,5 2,6	-2,7
coefficients of the 3rd equation	3,1 3,2 3,3 3,4 3,5 3,6	-3,7
coefficients of the 4th equation	4,1 4,2 4,3 4,4 4,5 4,6	-4,7
coefficients of the 5th equation	5,1 5,2 5,3 5,4 5,5 5,6	-5,7
coefficients of the 6th equation	6,1 6,2 6,3 6,4 6,5 6,6	-6,7

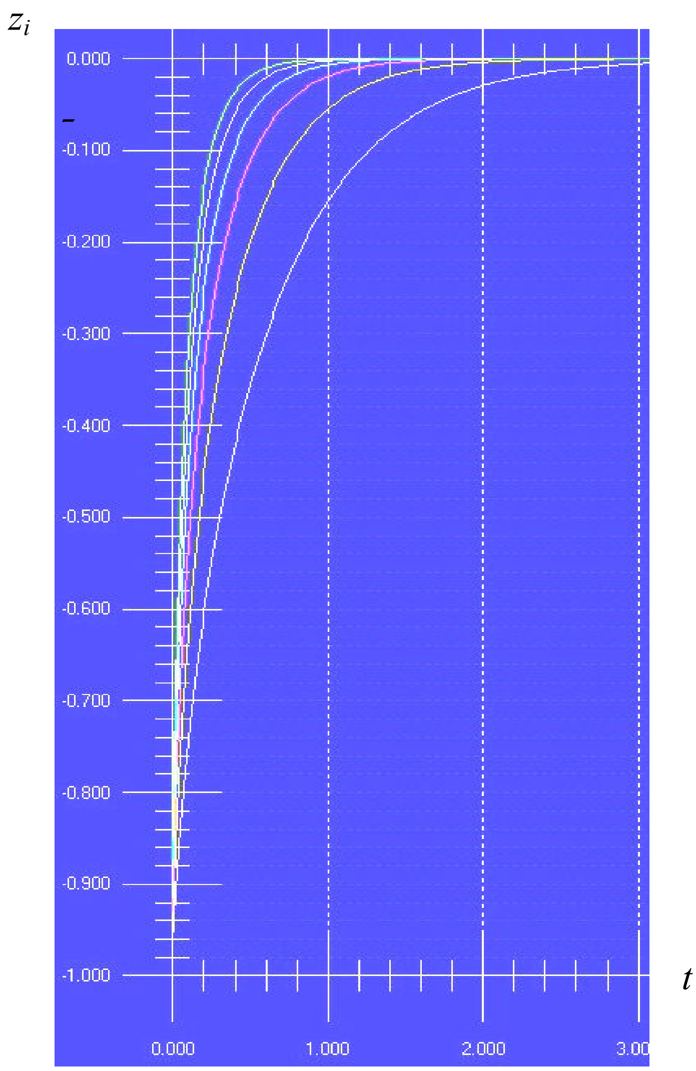

Fig. 1 Parameters of PHO without time delays depending on time

4.3.2 Regimes of the PHO without time delays but with account of the coefficients signs

Similar results presented in Fig. 2 for the data from Table 2 were got with account of the above-mentioned analysis of the coefficients in the equation array (5) based on the information about the system performance and basic features.

Comparison of the data in Fig. 1 and Fig. 2 shows that calculation with the accounts of the PHO features makes the regimes different. The amount of workers on the PHO goes more smoothly while the other parameters go even more close and fast to their stationary state (asymptotic curves).

It was revealed that by some parameters the model of PHO is practically not sensitive even to big variation of the coefficients. For example, for the values from the Table 2, the computer simulations with the coefficient b_{62} does not reveal any changes even for the difference in the value of coefficient up to thousand times. Thus, the number of the managers at PHO (even abnormal grow of managers' quantity) does not influence the level of the safety culture. It reaches the high level and afterwards the safety culture does not depend anymore on the managers. What is interesting, the other parameters of the system do not feel it too.

This phenomenon may be formulated as the property of autonomous influence of the coefficients of the system on its parameters, which means that any changes of the coefficients of the system do not lead to the substantial variations of the parameters of the PHO. They only affect locally in the prompt growing of the parameters to their asymptotic curves.

Table 2. The values of the coefficients and right hands of the equations (5)

coefficients of the 1st equation	-1,1 1,2 1,3 1,4 1,5 -1,6	-1,7
coefficients of the 2nd equation	2,1 -2,2 2,3 -2,4 - 2,5 - 2,6	-2,7
coefficients of the 3rd equation	3,1 3,2 -3,3 3,4 3,5 3,6	-3,7
coefficients of the 4th equation	4,1 4,2 4,3 -4,4 4,5 -4,6	-4,7
coefficients of the 5th equation	5,1 5,2 5,3 -5,4 -5,5 -5,6	-5,7
coefficients of the 6th equation	6,1 6,2 6,3 -6,4 -6,5 -6,6	-6,7

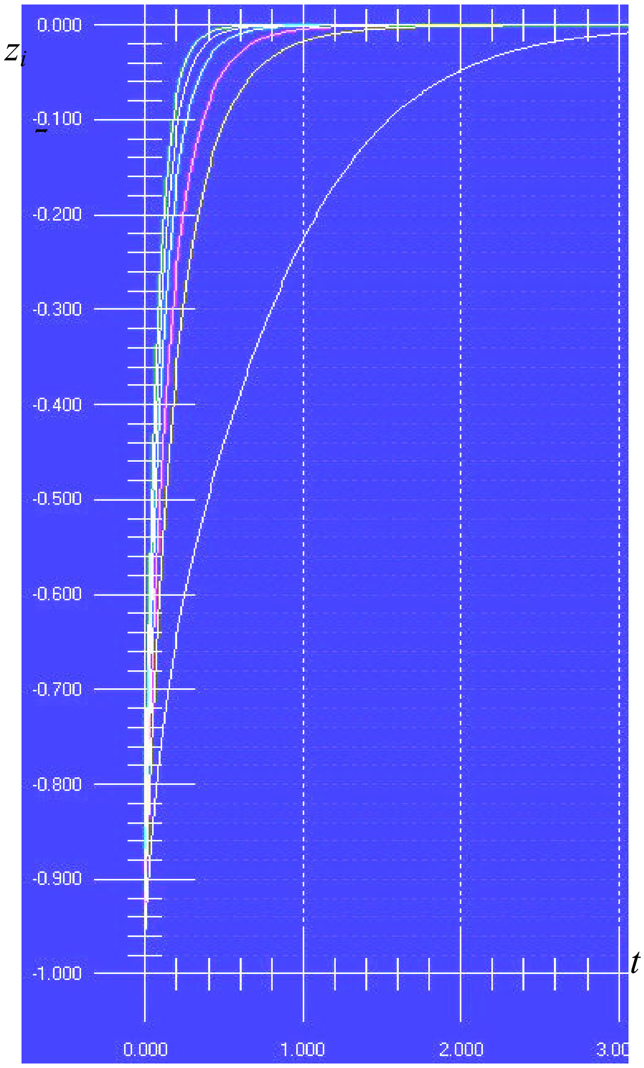

Fig. 2 Parameters of PHO without time delays, with account of coefficients, depending on time

Despite the non-linear character of the system, in an absence of the perturbations (oscillations and strong impulse changes) of the parameters of PHO and deviating arguments, the system is stable and is going smoothly to the asymptotic solution.

The deviating arguments play the role of serious non-linearity even in case of the simplest equations, as shown in the literature too [11].

4.3.3 Modeling of some special regimes

Let us consider some special regimes, e.g. in case of the anomaly intensive growing of the amount of managers (crisis of the managers' pressure).

The results of computer simulations are presented in Fig. 3 for the data given in the Table 3.

Comparison of the data in Fig. 2 and Fig. 3 evidently shows that with increase of the coefficient b_{20} the curves 3, 5, 6 are going to their asymptotes very fast, and the curves 1, 2, 4 and separate into the group (the growing rate of the workers, managers and expenditures for the repairment and maintaining of the PHO is falling down).

Then by b_{20}=-18 the situation becomes even more dramatic as clearly observed from the Fig. 4.

This analysis has shown only influence of the two coefficients on the system, by keeping all the other of them constant. In reality all coefficients may vary, so that the regimes may be more complex.

The modeling and simulation according to the methodology developed is helpful in investigation of the wide range of diverse situations and regimes. Such peculiarities revealed are of interest for the optimal control of the PHO.

Table 3. The values of the coefficients and right hands of the equations (5)

coefficients of the 1st equation	1,1 1,2 1,3 0 0 0	-1,7
coefficients of the 2nd equation	2,1 2,2 2,3 0 0 0	- 10,7
coefficients of the 3rd equation	3,1 3,2 3,3 0 0 0	-3,7
coefficients of the 4th equation	4,1 4,2 4,3 0 4,5 4,6	-4,7
coefficients of the 5th equation	0 0 5,3 5,4 5,5 5,6	-5,7
coefficients of the 6th equation	0 0 6,3 6,4 6,5 6,6	-6,7

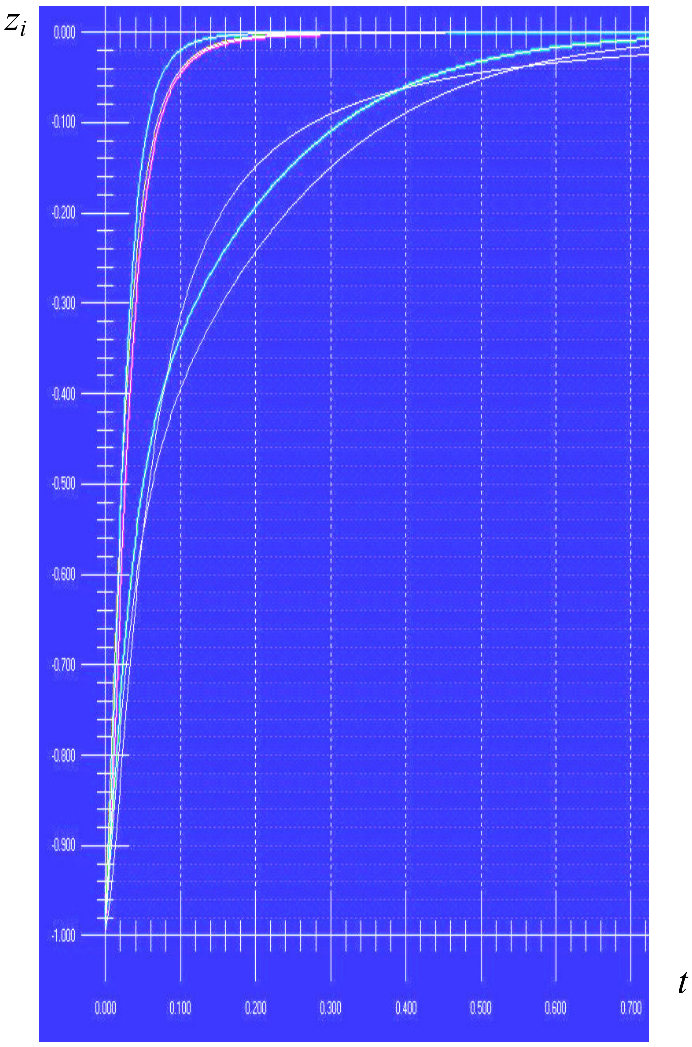

Fig. 3 Functions z_i (for i from 6 до 1)

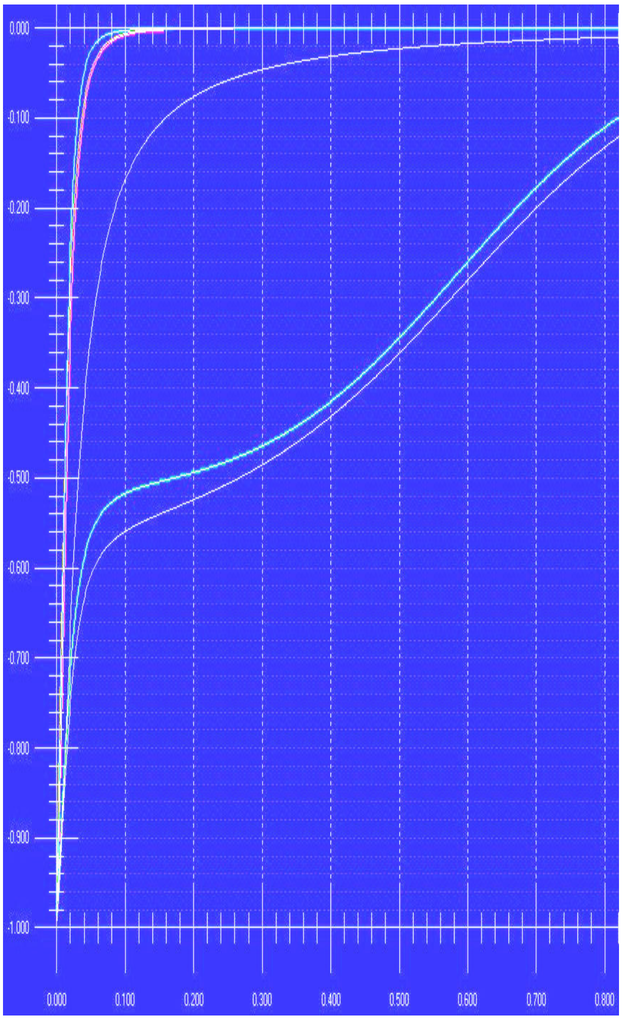

Fig. 4 Functions z_i (for i from 6 до 1) for the same data as in Fig. 3, only b_{20}=-18

Even without deviating arguments the model has shown available critical regimes by some specific combination of the parameters. There are available different features including the catastrophic situations.

4.4 Computer simulation of the PHO with deviating arguments

The deviating arguments may cause dramatic changes in behaviors of the PHO. It was mentioned about such phenomena in the literature [11, 12].

The results of computer simulations for the initial data given in the Table 4, are presented in Figs 5, 6. It is clearly observed that the time delays and prehistory of the system may cause oscillations, («memory of the system»).

Fig. 5 shows that the amount of the workers and the amount of product produced on PHO are both nearly stable and are not prone to any oscillations despite complex behaviors of other parameters.

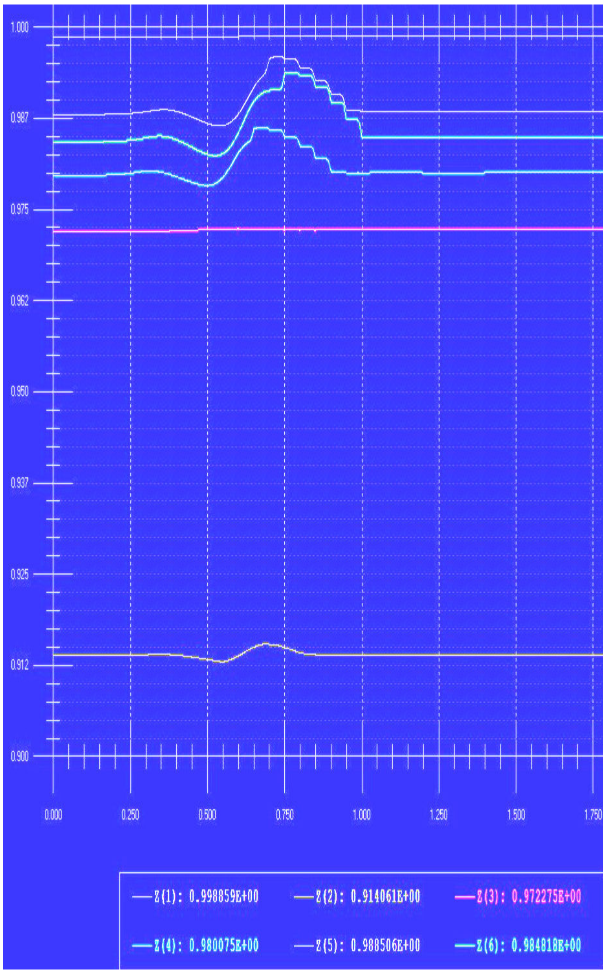

Fig. 5 Functions z_i (for i from 6 to 1, from the top to the bottom of picture) for time delays in Table 4

Table 4. Matrix of the time delays in the system (5) and values of functions in the prehistory points

(-0.380,1.1), (-0.31,1.2), (-0.22,1.3), (-0.14,1.4), (-0.057,1.5), (0.027,1.6), (0.11,1.7)	0,17
(0.087,2.1), (0.11,2.2), (0.13,2.3), (0.15,2.4), (0.170,2.5), (0.190,2.6), (0.21,2.7)	0,27
(0.250,3.1), (0.26,3.2), (0.27,3.3), (0.28,3.4), (0.290,3.5), (0.300,3.6), (0.31,3.7)	0,37
(0.320,4.1), (0.34,4.2), (0.35,4.3), (0.37,4.4), (0.380,4.5), (0.390,4.6), (0.41,4.7)	0,47
(0.370,5.1), (0.40,5.2), (0.42,5.3), (0.44,5.4), (0.460,5.5), (0.490,5.6), (0.51,5.7)	0,57
(0.40,5.60), (0.44,5.65), (0.47,5.7), (0.51,5.75), (0.54,5.8), (0.58,5.85), (0.61,5.9)	0,67

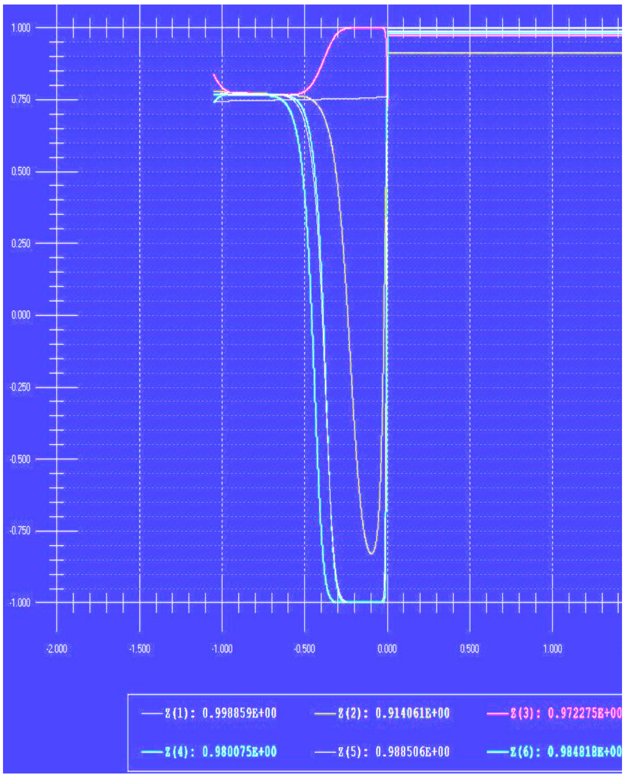

Fig. 6 Functions z_i (for i from 6 to 1) with interpolated prehistory

As one could see the number of managers was oscillating a little. But the safety culture, expenditures for the repairment and maintenance, ecology could oscillate a lot in the prehistory time. Nevertheless by the parameters stated these oscillations are very fast decreasing and the PHO becomes stable.

All the date in the Table 4 are presented in the form: (Amp, T) I,j, where T– value of of the time delay of the (*i,j*) variable; Amp– the initial value (*i*) variable at the moment T_j.

The other multiple numerical simulations have shown that despite dramatic prehistory for many PHO with available exceeding of the accepted ranges of the parameters, big oscillations, etc. the PHO is stabilizing in all cases during some time interval, which may be long under certain conditions.

5 Mathematical model of the corium melt penetrating pool of coolant

The other problem of the severe accidents at nuclear power plants (NPP) considered here is more studied but still not with account of the delayed parameters. We try to fill this gap in the rest of this paper.

5.1 Modeling of the corium melt cooling during severe accidents at NPP

5.1.1 Equation for penetration of the melted corium into the volatile coolant

Development of the mathematical models for different conditions of the melted corium penetration into a pool of volatile coolant has been considered in many papers, e.g. [5-9, 13-16]. But the time delays were not considered for this problem yet. This problem is considered here at first.

The non-linear differential equation for the jet of corium penetrating pool of volatile coolant (mostly it is considered water for this purpose at NPPs), with account of the time delays, is proposed as follows

$$\rho_1 \frac{d(hv_1)}{dt} = (\rho_1 - \rho_2)gh(t-\tau_1) - \beta_c \rho_2 v_1^2 (t-\tau_2), \tag{19}$$

where h is the length of a jet penetration into the pool, ρ_1, ρ_2 are the densities of the jet and fluid in the pool, respectively, t is time, v_1 is the jet velocity. Obviously here is $v_1 = dh/dt$. And τ_1, τ_2 are the corresponding time delays for the buoyancy and vapor drag forces.

Here g is the acceleration due to gravity β_c is a coefficient depending on the velocity of flow and form-factor of the jet head. For the simplicity, in a first approach to estimate the action of a drag force, it may be taken β_c=0.5.

5.1.2 The time delays of the buoyancy and vapor drag forces

The buoyancy force may have the time delay due to delay of the reaction of the coolant pool on the new position of the penetrating jet at each current moment of time. This is because jet is intensively radiating due to high temperature of the corium melt (2000-3000 Celsius degrees) and therefore it is penetrating the volatile coolant in some vapor “sack” indeed.

Water or other coolant starts evaporate prior to the contact of the high-temperature melt with volatile coolant. This is the reason why corium jet is actually going into a pool of coolant having all the time intensive vapor going around the jet.

Thus, buoyancy force is applied to the jet through this vapor layer. Therefore there are some time delays on action of the buoyancy force, which depend on the features of the vapor flow around the penetrating jet.

Similarly with the time delay of the vapor draft force, this is acting at the current moment with delay equal to a time of evaporation of the coolant in front of the jet head.

In general, this problem requires numerical solution of the two-phase two-component Navier-Stokes equations, which are not closed and need separate investigations for development of the closing relations.

5.1.3 Dimensionless form of the equation array

The equation array (19) may be transformed to a dimensionless form:

$$\frac{d(hv_1)}{dt^2} + \beta_c \rho_{2/1} \left(\frac{dh(t-\tau_2)}{dt} \right)^2 + \frac{\rho_{2/1} - 1}{Fr} h(t-\tau_1) = 0, \quad (4.1) \tag{20}$$

where for the dimensionless variables the same assignments as previously taken were preserved.

The following scales were used for the velocity and for the time: u_0 and r_0/u_0, respectively. ere $\rho_{21} = \rho_2/\rho_1$, $Fr=u_0^2/(gr_0)$ is the Froude number, which characterizes the ratio of the inertia and buoyancy forces, r_0 is a jet radius at the initial moment of a jet penetration into a pool.

Having no data about the values of the time delays τ_1, τ_2, the differential equations of a jet penetrating pool of coolant with account of the time delays, the equations in the dimension form (19) or in the dimensionless form (20), are considered first qualitatively. After estimation of the values τ_1, τ_2 the equations (19) (20) may be used for quantitative calculations too.

5.2 Specific features of the jet penetration equations with the time delays

5.2.1 Character of the jet penetration into a pool

Taking analysis of a jet penetration equation (20) one can mention that the time delays may change this equation substantially, both quantitatively and qualitatively.

By numerical solution of the non-linear differential equation (20) the time delays τ_1, τ_2 are switched on after the time moment equal to the maximal value of τ_1, τ_2: numerical solution of the equation is going with the algorithm of automatic time step, therefore these time moments, which correspond to the time delays are introduced in a special way, for example, with specific approximation method.

According to the Elsgoltz's theorem [10], function with the time delay may be presented in a linear approach as the following series by the small values of the delays:

$$h(t-\tau_i) \approx h(t) - \tau_i dh/dt\,. \qquad (21)$$

Then with account of (21), the equation (20) is transformed to the following

$$(h - 2\beta_c \rho_{2/1} \tau_2 \frac{dh}{dt})\frac{d^2h}{dt^2} + (1+\beta_c\rho_{2/1})(\frac{dh}{dt})^2 + \qquad (22)$$
$$+\tau_1 \frac{1-\rho_{2/1}}{Fr}\frac{dh}{dt} + \frac{\rho_{2/1}-1}{Fr}h = 0\,,$$

The equation (22) thus obtained has changed the equation type comparing to the equation (20). Non-linearity of this equation is stronger than the previous one, which may cause more complex different solutions.

5.2.2 Influence of the time delays on chaotic character of a jet penetration into a pool

By the character the most influential in the phenomena of a jet penetration into a pool is the time delay τ_2.

This time delay in the action of the vapor drag force predetermines the chaotic character of the coolant vaporization and influence of the vapor flow on the corium jet penetration into a pool.

The time delays are in general functions of time themselves; therefore it may serve as one of the possible mechanisms for chaotization of the jet penetration phenomena.

It is well know from the literature [17-25] that the behaviors of the real complex systems are determined by the effect of past action, which means that system depends not only on the current its state but also on the previous its states. Such systems are described by the differential equations with the time delays.

The values of the time delays for specific tasks can be estimated from solution of the Navier-Stokes equations for the jet penetration into a pool with account of the coolant evaporation and its flow.

Conclusion

The new models for the complex real system with the time delays and time outstrips developed may serve for modelling and computer simulations of such systems. This allow revealing the new phenomena of the complex systems' dynamics including possibilities for forecasting of the specific regimes, both beneficial as well as critical including the catastrophic ones.

The examples of considered models in the paper were general type potentially hazardous objects and concrete type of the severe accidents at the nuclear power plant for development of the passive protection systems against severe accidents at the NPP.

The results obtained may be useful for further development of the complex models with deviating arguments and for practical applications.

The methodology and computer programs developed allow computer simulation of the diverse phenomena from nuclear power safety field and they are applicable in a more bright sense too.

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