Structural Dynamic Finite Element Model Updating using Derringer's Function: A Novel Technique

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Abstract: - Aim of this research paper is to develop a new structural dynamic finite element model updating (FEMU) technique using Derringer's desirability function. Proposed FEMU technique allows formation of sub-objectives of model updating problem in a detailed and flexible but in a simplified and user friendly manner. Beauty of the proposed technique is that user can set target value, lower limit, upper limit, weightage on lower limit, weightage on upper limit and relative importance of each sub-objective of model updating problem. Two updating parameters (Elastic modulus and moment of inertia) of a beam structure are varied to develop experimental design matrix. This design matrix is further used to generate response surfaces for first five natural frequencies using response surface methodology. Derringer's function approach is used for formulating individual desirability function for each sub-objective by considering corresponding natural frequency predicted by response surface model, simulated experimental natural frequency (target value), desired upper and lower limits on predicted frequency, weights on upper and lower limits, and, importance of the related mode (sub-objective). Individual desirability functions are then combined to produce a single overall desirability function, thereby reducing the complex multi-objective FEMU problem to single objective FEMU problem. Optimum (maximum) value of overall desirability function is then used to find out the value of updating parameters. Updating parameters are then used to produce an updated FE model. Dynamic results of updated FE model are then compared with their simulated experimental counterparts and it is found that absolute average error between FE and simulated experimental results is reduced from 14.6% (before updating) to just 0.02% (after updating), thereby suggesting successful implementation of proposed FEMU technique.

Key-Words: - Structural dynamics; Finite Element; Model updating; Response surface methodology; Derringer's function.

Abbreviations

ANOVA	analysis of variance
FE	finite element
FEMU	finite element model updating
PRESS	predicted residual error sum of squares
RS	response surface
RSM	response surface methodology

Nomenclature

A	coded parameter related to elastic modulus
B	coded parameter related to moment of inertia
C_i	coefficient of i^{th} linear term of polynomial model
D	overall desirability function
D_i	coefficient of i^{th} quadratic term of polynomial model
d_i	i^{th} Individual desirability function
E	elastic modulus
F	test statistic of F-test
f	response function
I	moment of inertia
i	integer
m	number of independent parameters
n	number of individual desirability functions / natural frequencies / modes
R^2	coefficient of determination
r_i	relative importance of i^{th} individual desirability function
V	variance-covariance matrix
w_1	weight on lower limit of individual desirability function
w_2	weight on upper limit of individual desirability function
X	design matrix as being a set of value combinations of coded parameters
X_i	i^{th} independent parameter
Y	RS predicted response

ε experimental error
σ standard deviation
ω_i experimental / FE predicted i^{th} natural frequency
$\widehat{\omega}_i$ RS predicted i^{th} natural frequency
$\widehat{\omega}_{iLL}$ lower limit for RS predicted i^{th} natural frequency
$\widehat{\omega}_{iT}$ target value for RS predicted i^{th} natural frequency
$\widehat{\omega}_{iUL}$ upper limit for RS predicted i^{th} natural frequency

1 Introduction

Use of lighter materials in latest machines and structures is increasing day by day. For example, in automotive, aircraft, and spaceship engines, there is an ever existing demand of attaining better fuel economy; which can be met to a good extent by using thin products and low density materials such as aluminium and plastics composites instead of the conventionally used high density materials such as steels. Particularly, in satellites, some parts are so thin that they can get collapsed just due to their own weight if tested under the effect of gravity. Thin and light weight products have lot more tendencies to vibrate than their thick and heavy weight counterparts. Excessive vibrations may even cause pre-mature failure of products. Prediction of accurate dynamic behavior constitutes one of the most important processes in design of rotors of turbines and many other machines [1]. Thus it becomes imperative for engineers to understand the dynamic behavior of structures through their dynamic analysis. Therefore better dynamic testing and analysis tools are becoming an urgent need of hour. Dynamic analysis aims at understanding, evaluating, analyzing and modifying (if required) the structural dynamic behavior. Structural dynamic behavior can be represented by many terms such as natural frequencies, eigenvalues, mode-shapes, damping ratios, frequency response functions etc. Further, to analyze the dynamic behavior of structures, either experimental route or theoretical approach [2–3] can be followed.

Theoretical route involves the formation of an analytical model of the system either using a classical method [4] or through finite element (FE) method [5]. Application of classical method is generally limited to simple systems only, while FE method is preferred for real life complex systems. However, FE method is not able to predict dynamic responses of structures with complete accuracy due to presence of certain errors such as incorrect values of material (such as modulus of elasticity, density etc.) and structural (thickness, moment of inertia etc.) properties. Thus there is a need to correct (update) an FE model so that its vibration behavior matches with experimental dynamic response. The procedure used to update the FE model is called finite element model updating (FEMU) [6].

Most of the early contributions in the field of FEMU have been reviewed by Imregun and Visser [7], and, Mottershead and Friswell [8]. FEMU methods can be broadly classified into direct and iterative methods. Direct methods are non-iterative in nature and are essentially one step methods [9–12]. Updated FE models produced by such methods may not be symmetric and positive definite, hence such methods are not much useful in industry. Industrial applications generally rely upon the use of iterative methods [13–32]. In present research work, FEMU has been considered as a multi-objective optimization problem. In such problems, experimental responses are considered as targets, while parameters of FE model are identified (corrected or updated) in such a way that FE responses match with corresponding experimental response values. By considering FEMU as an optimization problem, its solution can be found by first defining the multi-objective functions and constraints; then certain optimization algorithm is applied to find out the optimal solution (here updating parameters). FEMU being a multi-objective optimization problem; success and efficiency of any FEMU technique will depend a lot upon the formulation of objective functions and constraints. Therefore, purpose of present research work is to present a more detailed, flexible, simplified, and user friendly formulation of objective functions of FEMU. Another aim is to explore the application of Derringer's function approach [33] in FEMU.

In this paper, one dedicated sub-objective is defined for each frequency predicted by response surface method (RSM) in such a way that the user can set the target value (experimental response), desired upper and lower limits on predicted frequency, weights on upper and lower limits, and, relative importance for each sub-objective separately. Then, by employing the concept of Derringer's function [33], all sub-objectives are converted into scale free values called individual desirability functions. The Individual desirability function of any particular mode will attain a unit value when the frequency predicted by response surface (RS) model of that particular mode matches exactly with the corresponding experimental (target) frequency. Value of the individual desirability function will be zero if the RS predicted frequency falls short of the desired lower limit or if it exceeds the set upper limit. If RS predicted frequency is away from target value but within the set limits then value of individual desirability will be less than unity and will be governed by shape of desirability function. Further, individual desirability functions are then multiplied together to form an overall desirability function. Thus multi-objective FEMU problem is transformed into a single objective optimization problem, where the main objective is that overall desirability function should approach unity. It is to be mentioned here that overall desirability function can approach unity only if all individual desirability functions also approach the unit value separately.

Basic theory of D-optimal design, RSM and desirability function in brief has been discussed in section 2 of this research paper. Section 3 discusses simulated experimental set-up of the beam structure. Development of response surfaces, formulation of objective functions of FEMU, optimization of overall desirability along with its application to FEMU of a beam structure have been presented in section 4. Success of the proposed technique is measured by performing confirmation experiments as outlined in section 4.5. Section 5 discusses the conclusions drawn out of present research work.

2 Theory

The novel technique of model updating proposed in this paper is based upon the use of D-optimal design, RSM and desirability function; basic theory of which is presented in subsections 2.1, 2.2 and 2.3 respectively.

2.1 D-optimal Design

There are several design optimality criterion available in literature such as D-optimality, A-optimality, G-optimality. Among all, D-optimality is the most popular one [34]. In general, modeling accuracy, namely, goodness-of-fit, can be measured by a variance-covariance matrix $\mathbf{V}$ given by (1).

$$\mathbf{V} = \sigma^2(\mathbf{X}'\mathbf{X})^{-1} \tag{1}$$

where, σ is the standard deviation. Naturally, it is expected to minimize $(\mathbf{X}'\mathbf{X})^{-1}$ in order to obtain an RS model. In statistics, minimizing $(\mathbf{X}'\mathbf{X})^{-1}$ is equivalent to maximizing the determinant of $\mathbf{X}'\mathbf{X}$. Therefore, the criteria for constructing the design matrix with a maximized $|\mathbf{X}'\mathbf{X}|$ from a set of candidate samples can be defined as D-optimality. Initial ‘D’ stands for ‘determinant’. By using D-optimal designs, generalized variance of a predefined model is minimized, which means the ‘optimality’ of a specific D-optimal design is model dependent. Unlike standard designs, D-optimal designs are straight optimization and their matrices are generally not orthogonal with the effect estimates correlated.

2.2 Response Surface Methodology

Response surface methodology is a collection of mathematical and statistical techniques that are useful for modeling and analysis of problems in which a response of interest is influenced by several input variables and the objective is to optimize this response [35, 36]. It is a sequential experimentation strategy for empirical model building and optimization. By conducting experiments and applying regression analysis, a model of the response to some independent input variables can be obtained. Based on the model of the response, a near optimal point can then be deduced. RSM is often applied in the characterization and optimization of processes. Though, in this paper, RSM has been applied to FEMU problem. In RSM, it is possible to represent independent process parameters in quantitative form as written in (2).

$$Y = f(X_1, X_2, X_3, \dots X_m) \pm \varepsilon \tag{2}$$

where, Y is the response predicted by RSM, f is the response function, ε is the experimental error, and $X_1, X_2, X_3, \dots X_m$ are independent parameters.

By plotting the expected response of Y, a surface, known as the response surface is obtained. The form of f is unknown and may be very complicated. Thus, RSM aims at approximating f by a suitable lower ordered polynomial in some region of the independent design variables. If the response can be well modeled by a linear function of the m independent variables, the function Y can be written as:

$$Y = C_0 + C_1X_1 + C_2X_2 + \cdots + C_mX_m \pm \varepsilon \tag{3}$$

However, if a curvature appears in the system, then a higher order polynomial such as the quadratic model as shown in (4) may be used.

$$Y = C_0 + \sum_{i=1}^{m} C_i X_i + \sum_{i=1}^{m} D_i X_i^2 \pm \varepsilon \quad (4)$$

The objective of using RSM is not only to investigate the response over the entire factor space, but also to locate the region of interest where the response reaches its optimum or near optimal value. By studying carefully the RS model, the combination of factors, which gives the best response, can then be established. RSM is a sequential process and its procedure can be summarized as shown in Fig. 1.

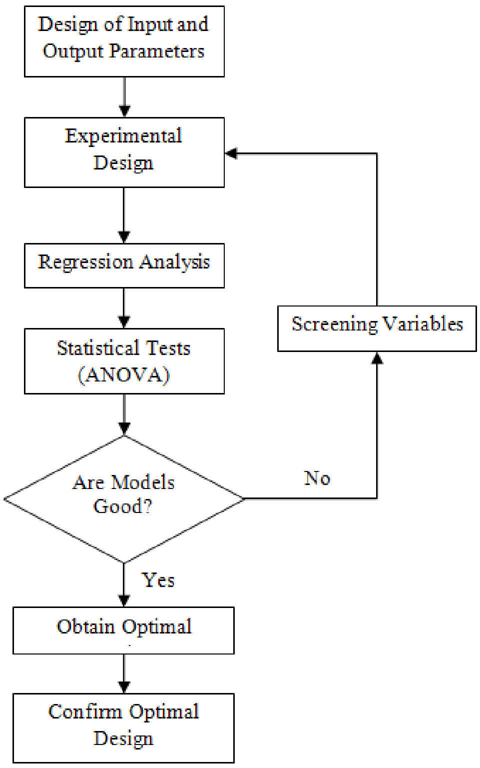

Fig. 1 Procedure of response surface methodology [37].

2.3 Desirability Function

Derringer and Suich [33] describe a multiple response method called desirability. It is an attractive method for industry for optimization of multiple quality characteristic problems. The method makes use of an objective function, D, called the desirability function and transforms an estimated response into a scale free value (d_i) called desirability. The desirable ranges are from zero to one (least to most desirable, respectively). The factor settings with maximum total desirability are considered to be the optimal parameter conditions. The simultaneous objective function is a geometric mean of all transformed responses:

$$D = (d_1 \times d_2 \times d_3 \times ... \times d_n)^{1/n} = \left(\prod_i^n d_i\right)^{1/n} \quad (5)$$

where n is the number of responses in the measure. If any of the responses falls outside the desirability range, the overall function becomes zero. It can be extended to

$$D = \left(d_1^{r_1} \times d_2^{r_2} \times d_3^{r_3} \times d_4^{r_4} \times d_5^{r_5}\right)^{1/\sum r_i} (6)$$

where r_i represents the relative importance of the i^{th} individual desirability function. This is a better representation than (5), because it can reflect the possible difference in importance of a number of individual desirability functions [38].

Desirability is an objective function that ranges from zero outside of the limits to one at the goal. The numerical optimization finds a point that maximizes the desirability function. Adjusting the weight or importance may alter the characteristics of a goal. For several responses, all goals get combined into one desirability function. For simultaneous optimization, each response must have a lower and upper limit assigned to each goal. The "Goal" field for responses must be one of five choices: "none", "maximum", "minimum", "target", or "in range". Factors will always be included in the optimization at their design range by default, or as a maximum, minimum of target goal. The meanings of the goal parameters are:

- Maximum:
  - o $d_i = 0$ if response < lower limit
  - o $0 \leq d_i \leq 1$ as response varies from lower to upper limit
  - o $d_i = 1$ if response > upper limit

- Minimum:
  - o $d_i = 1$ if response < lower limit
  - o $1 \leq d_i \leq 0$ as response varies from lower to upper limit
  - o $d_i = 0$ if response > upper limit
- Target:
  - o $d_i = 0$ if response < lower limit
  - o $0 \leq d_i \leq 1$ as response varies from lower limit to target
  - o $1 \geq d_i \geq 0$ as response varies from target to upper limit

- o $d_i = 0$ if response > upper limit

- Range:
  - o $d_i = 0$ if response < lower limit
  - o $d_i = 1$ as response varies from lower to upper limit
  - o $d_i = 0$ if response > upper limit

The d_i for "in range" are included in the product of the desirability function "D", but are not counted in determining "n": $D = (\prod d_i)^{1/n}$. If the goal is none, the response will not be used for the optimization.

Derringer's desirability function has been investigated by many researchers in a number of fields like chromatography [39, 40], manufacturing [38, 41, 42], and renewable energy [43]. Although Derringer's desirability function has been successfully used in many other multi-objective optimization problems, but its benefits have not yet been explored in the field of FEMU. This research paper develops a model updating technique by utilizing Derringer's desirability function for detailed, flexible and user-friendly formulation of objective functions of FEMU problem.

In this paper, Derringer's desirability function was used with the purpose of efficient and flexible formulation of objectives of FEMU based multi-objective optimization problem. While formulating the objectives, experimentally (simulated experiments) obtained natural frequencies for first five modes were considered as target values $(\widehat{\omega}_{iT})$ for RS model. The lower limit $(\widehat{\omega}_{iLL})$ and upper limit $(\widehat{\omega}_{iUL})$ were set separately for each RS predicted natural frequency. Here, $\widehat{\omega}_{iUL}$ and $\widehat{\omega}_{iLL}$ were taken as ±5% of the corresponding $\widehat{\omega}_{iT}$ value. Weight on lower and upper limit were taken as w_1 and w_2 respectively. The weight w_1 was taken as unity, so that the individual desirability function could vary linearly between its target value $(\widehat{\omega}_{iT})$ and lower limit $(\widehat{\omega}_{iLL})$. Similarly a unit value of w_2 was decided to allow linear variation of the individual desirability function between its target value $(\widehat{\omega}_{iT})$ and upper limit $(\widehat{\omega}_{iUL})$.The natural frequencies predicted by RS model were treated as response variables $(\widehat{\omega}_i)$ to be optimized. Then, five individual desirability functions were developed (one for each response variable) using the method of Derringer and Suich [33] as represented in (7).

$$d_i = \begin{cases} \left[\frac{\widehat{\omega}_i - \widehat{\omega}_{iLL}}{\widehat{\omega}_{iT} - \widehat{\omega}_{iLL}}\right]^{w_1}, & \widehat{\omega}_{iLL} \le \widehat{\omega}_i \le \widehat{\omega}_{iT} \\ \left[\frac{\widehat{\omega}_{iUL} - \widehat{\omega}_i}{\widehat{\omega}_{iUL} - \widehat{\omega}_{iT}}\right]^{w_2}, & \widehat{\omega}_{iT} < \widehat{\omega}_i \le \widehat{\omega}_{iUL} \\ 0, & \widehat{\omega}_i < \widehat{\omega}_{iLL}\ or\ \widehat{\omega}_i > \widehat{\omega}_{iUL} \end{cases} \quad (7)$$

It is clear from (7) that, if $\widehat{\omega}_i$ and $\widehat{\omega}_{iT}$ match perfectly, it will lead to a unit value of i^{th} individual desirability function. A unit value is also the maximum value that an individual desirability function can attain. More the mismatch between $\widehat{\omega}_i$ and $\widehat{\omega}_{iT}$, lower will be the value of i^{th} individual desirability function. Moreover, if $\widehat{\omega}_i$ is not within the limits defined by $\widehat{\omega}_{iUL}$ and $\widehat{\omega}_{iLL}$, then the value of i^{th} individual desirability function falls to zero.

Further, the individual desirability functions for all five modes were utilized to produce the overall desirability function using (6). In the present research work, all five modes were considered equally important and hence the value of r_i was set as unity for all individual desirability functions. It is to be mentioned here that the overall desirability function will achieve a unit value only if all the individual desirability functions are each equal to unity. If any of the individual desirability function value falls down, the value of overall desirability function will also get reduced accordingly. Thus, the complex FEMU problem was simplified to maximization of scale-free and single overall desirability function. After locating the maxima of overall desirability function, corresponding values of the updating parameters were found. Updating parameters were then used to produce an updated FE model of beam structure.

3 Experimental: A Beam Structure

An undamped cantilever beam structure as drawn in Fig. 2 was considered in the present case study. This particular structure was taken because of its resemblance with many real life products such as wing of an airplane, blade of the rotor of a turbine; wing of a ceiling fan, an integrated chip of a mechatronic product etc. The beam is of mild steel material having the dimensions 910 x 49 x 7 mm^3, density of 6728 kg/m^3 and Young' modulus of elasticity as 200 GPa.

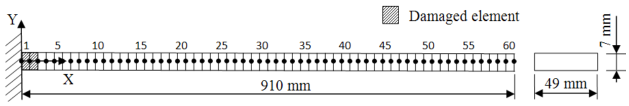

Fig. 2 FE model of cantilever beam structure.

4 Procedure

Procedure adopted for structural dynamic model updating of the cantilever beam structure is explained in following subsections.

4.1 Calculation of FE and Simulated Experimental Natural Frequencies

FE model of the beam was developed in Matlab [44] by using 60 beam elements each having two nodes. FE model was then used for producing the FE natural frequencies for first five modes. After that a perturbation was introduced into the FE model by reducing the elastic modulus (E) and moment of inertia (I) of first two elements near fixed end by 60%. The so-called perturbed FE model was processed in Matlab to produce the simulated experimental results, now onwards to be called as experimental results only. It is to be noted here that simulated experimental results have been used earlier also by many researchers for model updating related research work [45–48]. FE and experimental results for first five natural frequencies are presented in Table 1. From Table 1, it is quite clear that there existed a mismatch between FE and experimental results. Purpose of FEMU was to reduce this mismatch by updating the physical parameters of FE model. So, the experimental results were taken as targets; while the input parameters of the FE model were updated by using the newly developed technique of FEMU that used the concepts of RSM and Derringer's function. Before applying RSM to current FEMU problem, experimental design was performed as described in sub-section 4.2.

Table 1 Comparison of experimental and FE results.

Response variable	Experimental results (Hz)	FE results (Hz)
ω_1	5.75	7.45
ω_2	39.90	46.66
ω_3	117.11	130.64
ω_4	236.13	256.01
ω_5	397.34	423.20

4.2 Generation of Response Surface Models for First Five Natural Frequencies

Statistically significant RS models of first five natural frequencies were developed by using the concepts of D-optimal design, RSM and analysis of variance (ANOVA). RS models were preferred over original FE model because of their computational advantages. While implementing the D-optimal design, using Design-Expert software [49], firstly the range of each updating parameter (Elastic modulus E, and moment of inertia I) was decided. Lower and upper limits for both updating parameters were taken as -50% and +50% of their actual values respectively. Lower limit for elastic modulus (E) was taken as 40 GPa (i.e., 0.5 times 80 GPa, where 80 GPa is the actual value of elastic modulus) and upper limit of this parameter was taken as 120 GPa (i.e., 1.5 times 80 GPa). For moment of inertia (I) parameter, lower and upper limits were considered as 280.12 x 10^{-12} m^4 (i.e., 0.5 times 560.23 x 10^{-12} m^4, where 560.23 x 10^{-12} m^4 is the actual value of moment of inertia) and 840.35 x 10^{-12} m^4 (i.e., 1.5 times 560.23 x 10^{-12} m^4) respectively. Relationship between updating parameters (E and I) and response variables (ω_1, ω_2, ω_3, ω_4 and ω_5) was assumed to be quadratic. A quadratic fit was assumed because it was giving better results than a linear or a cubic model. A coded parameter A was defined in such a way that it varied linearly between -1 and +1 over the complete range of E. Similarly, another coded parameter B was defined and related to I. Coordinate exchange method [50] was used for candidate selection, because it does not require a candidate list, which if unchecked grows exponentially as the size of the problem increases [51]. D-optimality criterion was used to develop experimental design matrix of physical updating parameters. The design matrix consisted of a total of 15 test runs. The experimental design matrix contained the information about various combinations of different levels of input variables at which different experimental runs were to be performed. Experimental design matrix was then imported in Matlab and used in conjunction with unperturbed FE model of beam in order to find out corresponding natural frequencies of the beam structure as tabulated in Table 2. Results presented in Table 2 were then used to develop the RS models for first five natural frequencies.

Before using any RS model, checking of goodness of fit of each RS the model is very much required. In order to check the adequacy of the model, ANOVA was performed [52]. F-test method was used to carry out the hypothesis testing to check the significance of different parameters. The results of ANOVA using F-test for first natural frequency are presented in Table 3.

Table 2 Design of experimental matrix and results for first five natural frequencies.

Run No.	Updating parameters		Response variables (Natural frequencies in Hz)				
	Elastic modulus E, A	Moment of inertia I, B	ω_1	ω_2	ω_3	ω_4	ω_5
1	61.6	417.67	4.9	37.9	114.2	232.5	392.7
2	40.0	840.35	5.3	38.8	115.5	234.1	394.8
3	120.0	840.35	6.7	43.2	122.9	243.8	406.7
4	40.0	530.31	4.6	37.4	113.4	231.4	391.2
5	80.0	560.24	5.7	39.9	117.1	236.1	397.3
6	120.0	280.12	5.3	38.8	115.5	234.1	394.8
7	80.0	840.35	6.3	41.5	119.8	239.6	401.6
8	120.0	840.35	6.7	43.2	122.9	243.8	406.7
9	98.0	420.18	5.6	39.6	116.6	235.5	396.5
10	102.1	684.54	6.3	41.7	120.1	240.0	402.0
11	120.0	529.42	6.2	41.3	119.4	239.1	400.9
12	40.0	280.12	3.6	35.8	111.3	228.5	386.3
13	59.8	698.95	5.6	39.6	116.7	235.6	396.7
14	80.0	280.12	4.7	37.5	113.6	231.7	391.7
15	40.0	840.35	5.3	38.8	115.5	234.0	394.8

Table 3 ANOVA for $\hat{\omega}_1$.

Source	Sum of squares	Degrees of freedom	Mean square	F-Value	Prob > F
Model	9.93	5	1.99	2766.49	< 0.0001*
A	4.89	1	4.89	6806.66	< 0.0001*
B	4.85	1	4.85	6759.53	< 0.0001*
A × B	0.03	1	0.03	43.00	0.0001*
A^2	0.15	1	0.15	210.81	< 0.0001*
B^2	0.16	1	0.16	216.06	< 0.0001*
Residual	0.01	9	0.00		
Lack of Fit	0.01	7	0.00		
Pure Error	0.00	2	0.00		
Cor Total	9.94	14			
Standard deviation = 0.03				R^2 = 0.9993	
Mean = 5.55				R^2 Adjusted = 0.9990	
Coefficient of variation (%) = 0.48				Predicted R^2 = 0.9987	
PRESS = 0.01				Adequate precision = 179.67	

*: Significant

Table 3 suggests that RS model for $\hat{\omega}_1$ is statistically significant. Lack of fit term is not significant as its mean square value is zero. Pure error is zero, which is expected because there are no measurement errors, since the experimental runs were performed by running the perturbed FE model. Further, factor *A* (Elastic modulus), factor *B* (Moment of inertia), second order terms of factors *A* as well as *B* and also the interaction of factor *A* with factor *B* all are significant. Value of R^2 and adjusted R^2 is over 99%. This means that the RS model gives a sufficiently accurate relationship between updating parameters and response variables. Moreover, the "Predicted R^2" value is 0.9987, which is in good agreement with the "Adjusted R^2" value of 0.9990. The predicted residual error sum of squares (PRESS), which is a measure of discrepancy between experimental response values and RS predicted response values, is just 0.01; which shows that the quadratic model well fits each point in the design. Further, in order to check for normal distribution of residuals, normal probability plot of the residuals for $\hat{\omega}_1$ was drawn in Fig. 3.

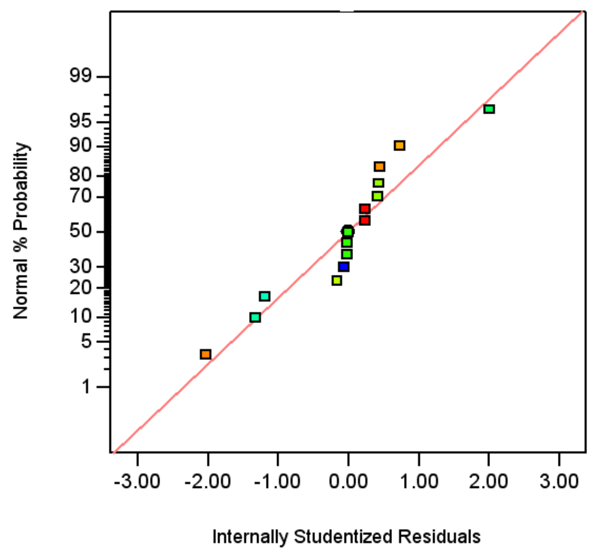

Fig. 3 Normal probability plot of residuals for $\hat{\omega}_1$.

In Fig. 3, the residuals are falling along a straight line, which shows that the residuals are normally distributed. Fig. 4 shows the values of the first natural frequency predicted by the RS model versus the values actually observed. Fig. 4 proves that the RS model is fairly well fitted with the observed values. Using regression analysis method [52] in conjunction with the data points provided in

Table 2, the polynomial equation for first natural frequency predicted by the RS model is written as:-

(in coded terms)

$$\widehat{\omega}_1 = 5.76 + 0.76A + 0.76B - 0.072AB - 0.24A^2 - 0.25B^2 \quad (8)$$

(in actual factors)

$$\widehat{\omega}_1 = 0.46 + 0.05E - 6.77 \times 10^{-3}I - 6.47 \times 10^{-6}EI - 1.52 \times 10^{-4}E^2 - 3.15 \times 10^{-6}I^2 \quad (9)$$

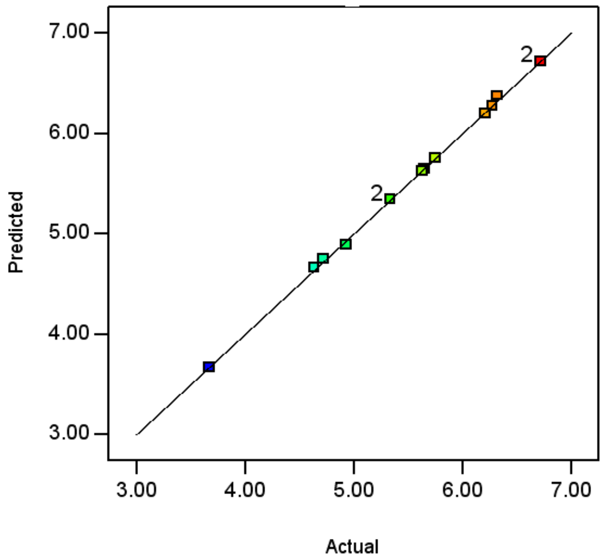

Fig. 4 Predicted versus actual values of $\widehat{\omega}_1$.

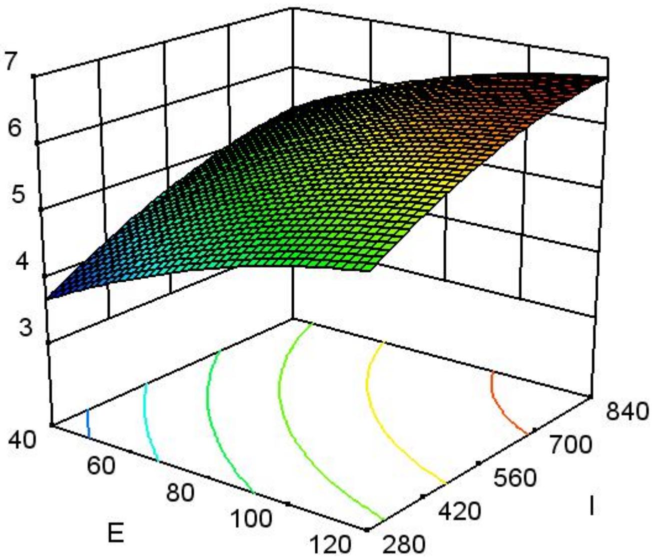

Fig. 5 Response surface for $\widehat{\omega}_1$.

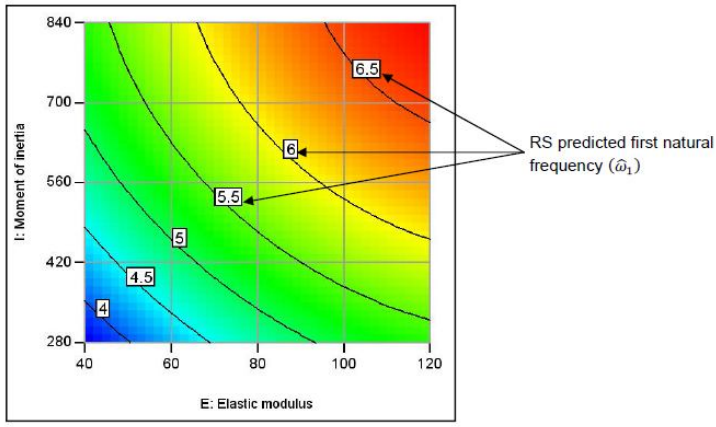

Fig. 6 Contour plot for $\widehat{\omega}_1$.

Three-dimensional distribution of RS of $\widehat{\omega}_1$ with respect to updating parameters E and I is drawn in Fig. 5. Fig. 6 shows the contour plot of $\widehat{\omega}_1$ in relation to the updating parameters. It can be seen from Fig. 6 that as the elastic modulus (E) or the moment of inertia (I) increase the value of $\widehat{\omega}_1$ also increases. Similar analysis was also performed for next four natural frequencies. ANOVA tables for $\widehat{\omega}_2$, $\widehat{\omega}_3$, $\widehat{\omega}_4$ and $\widehat{\omega}_5$ are shown in Table 4 to 7 respectively.

Table 4 ANOVA for $\widehat{\omega}_2$.

Source	Sum of squares	Degrees of freedom	Mean square	F-Value	Prob > F
Model	64.85	5	12.97	1286.23	< 0.0001*
A	30.15	1	30.15	2990.30	< 0.0001*
B	29.90	1	29.90	2965.07	< 0.0001*
$A \times B$	00.59	1	00.59	58.48	< 0.0001*
A^2	00.35	1	00.35	34.28	0.0002
B^2	00.37	1	00.37	36.76	0.0002
Residual	00.09	9	00.01		
Lack of Fit	00.09	7	00.01		
Pure Error	00.00	2	00.00		
Cor Total	64.94	14			
Standard deviation = 0.10			R^2 = 0.9986		
Mean = 39.67			R^2 Adjusted = 0.9978		
Coefficient of variation (%) = 0.25			Predicted R^2 = 0.9927		
PRESS = 0.47			Adequate precision = 119.05		

*: Significant

Table 5 ANOVA for $\hat{\omega}_3$.

Source	Sum of squares	Degrees of freedom	Mean square	F-Value	Prob > F
Model	166.14	5	33.23	2092.43	< 0.0001*
A	74.61	1	74.61	4698.01	< 0.0001*
B	73.97	1	73.97	4658.24	< 0.0001*
$A \times B$	03.58	1	03.58	225.43	< 0.0001*
A^2	00.46	1	00.46	28.90	0.0004
B^2	00.50	1	00.50	31.18	0.0003
Residual	00.14	9	00.02		
Lack of Fit	00.14	7	00.02		
Pure Error	00.00	2	00.00		
Cor Total	166.28	14			
Standard deviation = 0.13				R^2 = 0.9991	
Mean = 116.96				R^2 Adjusted = 0.9987	
Coefficient of variation (%) = 0.11				Predicted R^2 = 0.9956	
PRESS = 0.72				Adequate precision = 149.21	

*: Significant

Table 6 ANOVA for $\hat{\omega}_4$.

Source	Sum of squares	Degrees of freedom	Mean square	F-Value	Prob > F
Model	279.83	5	55.97	11013.91	< 0.0001*
A	125.40	1	125.40	24677.74	< 0.0001*
B	124.40	1	124.40	24481.33	< 0.0001*
$A \times B$	06.17	1	06.17	1213.65	< 0.0001*
A^2	00.69	1	00.69	134.98	< 0.0001*
B^2	00.72	1	00.72	141.84	< 0.0001*
Residual	00.05	9	00.01		
Lack of Fit	00.05	7	00.01		
Pure Error	00.00	2	00.00		
Cor Total	279.88	14			
Standard deviation = 0.071				R^2 = 0.9998	
Mean = 235.99				R^2 Adjusted = 0.9997	
Coefficient of variation (%) = 0.030				Predicted R^2 = 0.9994	
PRESS = 0.17				Adequate precision = 342.03	

*: Significant

Table 7 ANOVA for $\hat{\omega}_5$.

Source	Sum of squares	Degrees of freedom	Mean square	F-Value	Prob > F
Model	456.28	5	91.26	2846.28	< 0.0001*
A	211.21	1	211.21	6587.63	< 0.0001*
B	209.70	1	209.70	6540.62	< 0.0001*
$A \times B$	4.61	1	4.61	143.73	< 0.0001*
A^2	1.90	1	1.90	59.36	< 0.0001*
B^2	1.93	1	1.93	60.09	< 0.0001*
Residual	0.29	9	0.03		
Lack of Fit	0.29	7	0.04		
Pure Error	0.00	2	0.00		
Cor Total	456.57	14			
Standard deviation = 0.18				R^2 = 0.9994	
Mean = 397.00				R^2 Adjusted = 0.9990	
Coefficient of variation (%) = 0.045				Predicted R^2 = 0.9973	
PRESS = 1.24				Adequate precision = 176.75	

*: Significant

After fitting the RS model to the experimental results, the RS models for next four natural frequencies are given by the regression equations (10) to (17). Corresponding contour plots have been presented in Figs. 7 to 10.

(in coded terms)

$$\hat{\omega}_2 = 39.90 + 1.89A + 1.89B + 0.32AB - 0.37A^2 - 0.38B^2 \tag{10}$$

$$\hat{\omega}_3 = 117.13 + 2.97A + 2.97B + 0.78AB - 0.42A^2 - 0.44B^2 \tag{11}$$

$$\hat{\omega}_4 = 236.17 + 3.86A + 3.85B + 1.02AB - 0.52A^2 - 0.53B^2 \tag{12}$$

$$\hat{\omega}_5 = 397.45 + 5A + 5B + 0.89AB - 0.86A^2 - 0.87B^2 \tag{13}$$

(in actual factors)

$$\hat{\omega}_2 = 30.6 + 0.07E + 9.94 \times 10^{-3}I + 2.83 \times 10^{-5}EI - 2.3 \times 10^{-4}E^2 - 4.87 \times 10^{-6}I^2 \tag{14}$$

$$\widehat{\omega}_3 = 104.90 + 0.08E + 0.01I + 6.97 \times 10^{-5}EI - 2.64 \times 10^{-4}E^2 - 5.63 \times 10^{-6}I^2 \quad (15)$$

$$\widehat{\omega}_4 = 220.65 + 0.10E + 0.01I + 9.14 \times 10^{-5}EI - 3.23 \times 10^{-4}E^2 - 6.80 \times 10^{-6}I^2 \quad (16)$$

$$\widehat{\omega}_5 = 374.05 + 0.17E + 0.02I + 7.90 \times 10^{-5}EI - 5.38 \times 10^{-4}E^2 - 1.11 \times 10^{-5}I^2 \quad (17)$$

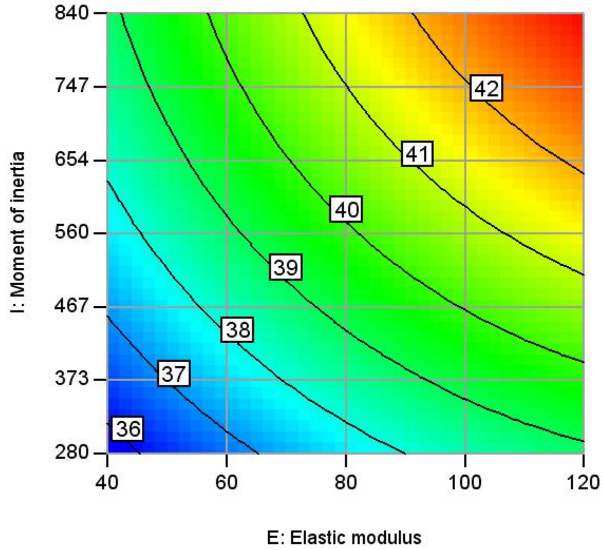

Fig. 7 Contour plot for $\widehat{\omega}_2$.

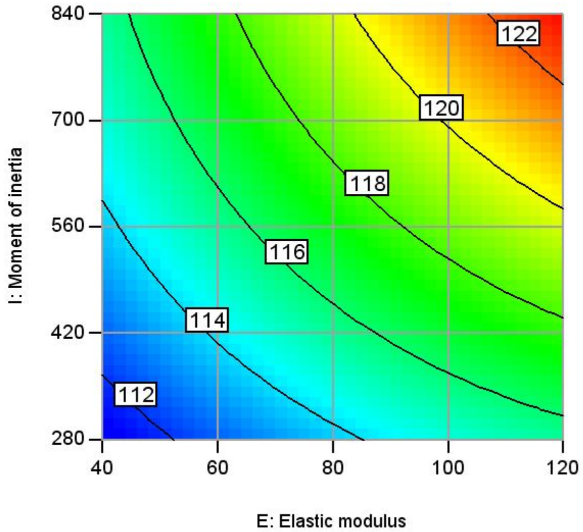

Fig. 8 Contour plot for $\widehat{\omega}_3$.

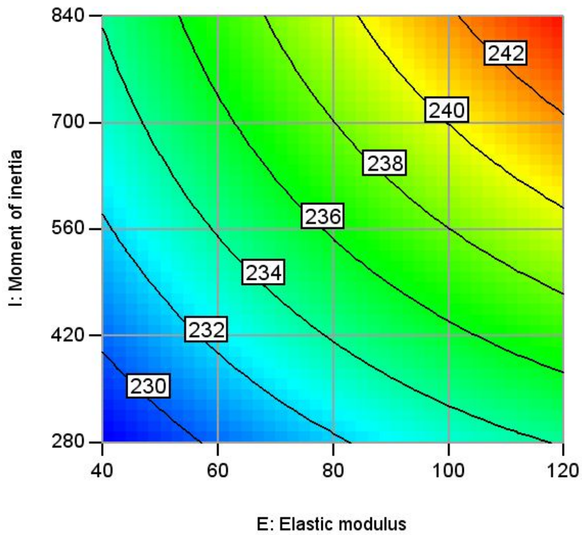

Fig. 9 Contour plot for $\widehat{\omega}_4$.

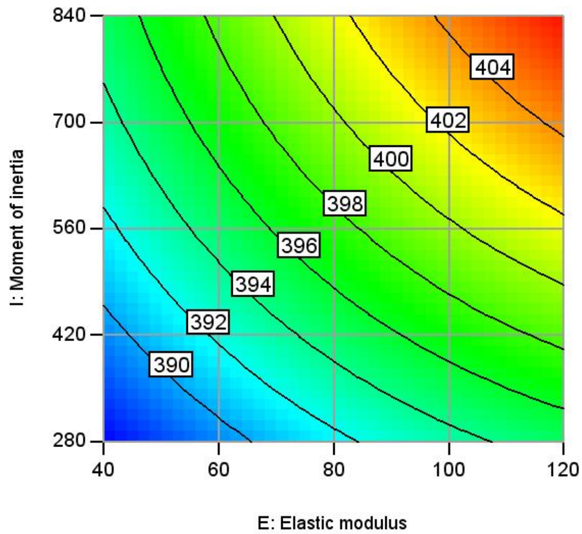

Fig. 10 Contour plot for $\widehat{\omega}_5$.

After having generated the RS models for all response variables, next step was to define the sub-objectives of the FEMU based multi-objective optimization problem, which is explained in next section.

4.3 Formulation of Individual Desirability Functions

Individual desirability functions were formulated by using the data provided in Table 8. The target values shown in Table 8 are the actual experimental values of corresponding natural frequencies. Weighting on lower as well as upper limit of each sub-objective were set to unity in order to give equal weightage to the corresponding lower and upper limits. All relative importance coefficients were also set to unity so as to give equal importance to all the sub-objectives. If any particular mode is more important than others, then its importance can be reflected by increasing the value of its corresponding relative importance coefficient. The values given in Table 8 were combined with the mathematical equation of individual desirability function shown in (7) to formulate the individual desirability functions for first five RS predicted natural frequencies.

Table 8 Details of the formulation of sub-objectives of FEMU.

RS Predicted response variable	$\widehat{\omega}_{iT}$ (Hz)	$\widehat{\omega}_{iLL}$ (Hz)	$\widehat{\omega}_{iUL}$ (Hz)	w_1	w_2	r_i
$\widehat{\omega}_1$	5.75	5.69	5.81	1	1	1
$\widehat{\omega}_2$	39.90	39.50	40.30	1	1	1
$\widehat{\omega}_3$	117.11	115.94	118.28	1	1	1
$\widehat{\omega}_4$	236.13	233.77	238.49	1	1	1
$\widehat{\omega}_5$	397.37	393.37	401.32	1	1	1

Graphically, the individual desirability functions of different modes have been drawn in Figs. 11 to 15. Value of any individual desirability function for any particular mode will be unity, only if RS predicted value of natural frequency of that particular mode will be equal to corresponding experimental value. Thus FEMU problem was converted to an optimization problem, where the five sub-objectives were to maximize the scale-free individual desirability functions.

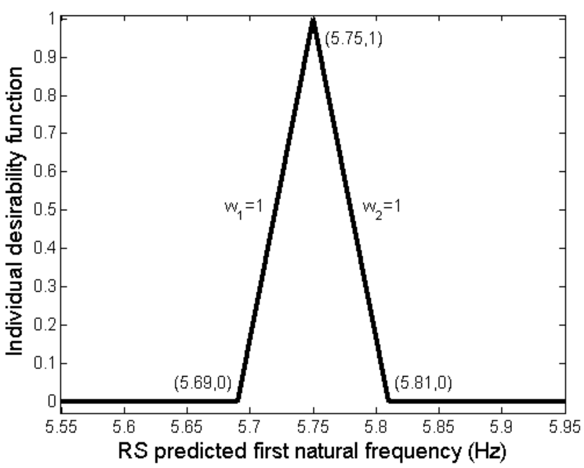

Fig. 11 Individual desirability function for $\widehat{\omega}_1$.

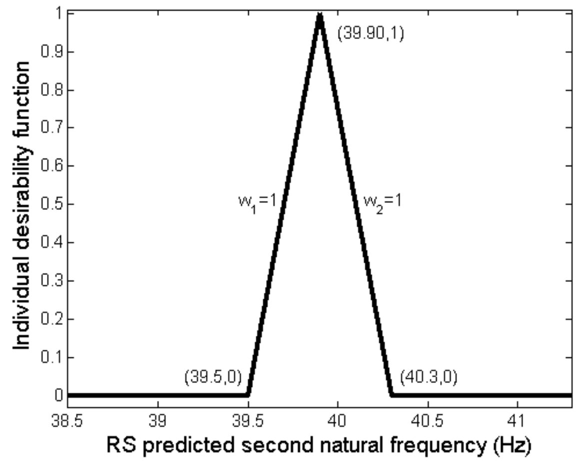

Fig. 12 Individual desirability function for $\widehat{\omega}_2$.

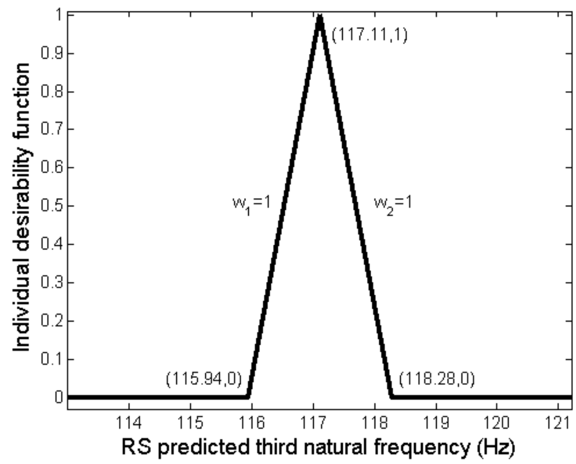

Fig. 13 Individual desirability function for $\widehat{\omega}_3$.

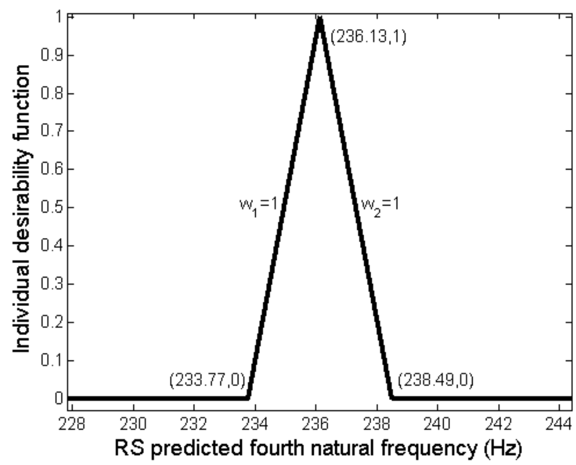

Fig. 14 Individual desirability function for $\widehat{\omega}_4$.

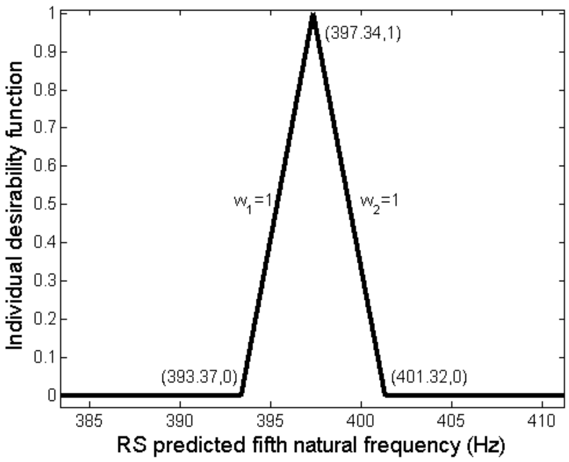

Fig. 15 Individual desirability function for $\hat{\omega}_5$.

4.4 Formulation and Optimization of Overall Desirability

Individual desirability functions were then combined to produce a single overall desirability function by using (6). Now the multi-objective optimization problem is transformed to a single objective one; which was later optimized using the RSM. It is found that the optimum value of the overall desirability function is 0.999928653, which is very near to unity. This optimum value of the overall desirability function was achieved when the elastic modulus (E) and moment of inertia (I) were set at 79.90 GPa and 559.63 x 10^{-12} m^4 respectively. The updated values of the elastic modulus (E) and moment of inertia (I) were then used to update the FE model of the beam structure. The response surface of the overall desirability function is also drawn in Fig. 16. It can be seen from Fig. 16 that high value of overall desirability function is achieved at only a few locations; while in most part of the design space the overall desirability function approaches zero. If elastic modulus takes a value lower than 52 GPa, the overall desirability function falls to zero; thereby restricting the design space. Moreover, if the moment of inertia parameter falls below 373 x 10^{-12} m^4, the overall desirability function again falls to zero irrespective of the value of elastic modulus. This information is very important in the dynamic design applications such as eigenvalues optimization of beams [53]. In order to have some desired dynamic characteristics of a mechanical component (such as beam), one can explore a variety of solutions readily available from the solution space provided by the overall desirability function value.

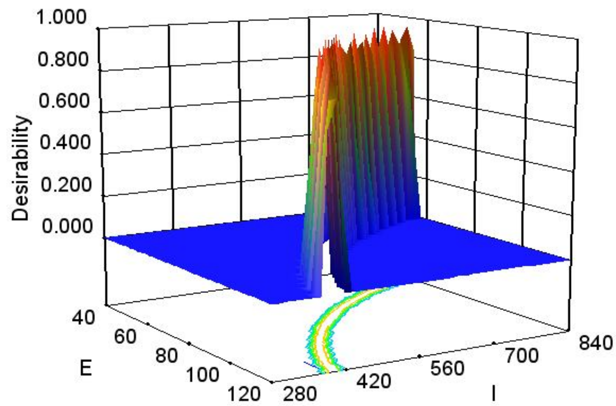

Fig. 16 Response surface of overall desirability function.

4.5 Confirmation Experiments and Calculation of Errors in FEMU Results

Confirmation experiments were performed to check the actual errors by implementing the optimal solution obtained through Derringer's function approach. The FE model was updated by taking Young' modulus of elasticity (E) as 79.90 GPa and moment of inertia (I) as 559.63 x 10^{-12} m^4. Corresponding FE results were then compared with their experimental counterparts as shown in Table 9. Before FEMU, there was an absolute average error of 14.6% considering all five modes, which gets reduced to just 0.02% after FEMU. Further, it is seen that the updated values of all five FE natural frequencies are now matching with their target experimental counterparts; thereby proving the success of the current FEMU technique. Finally the experimental results were compared with their FE counterparts before and after FEMU by drawing Fig. 17. It is quite clear from Fig. 17 that the present FEMU technique has efficiently handled and reduced the errors in FE predicted results.

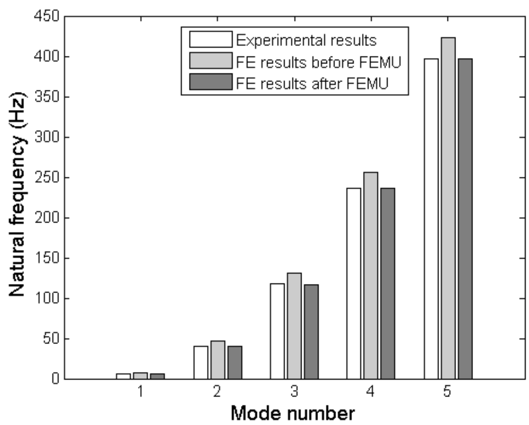

Fig. 17 Comparison of experimental results with FE results before and after FEMU.

Table 9 Updated results.

Response variable	Experimental results (Hz)	FE results before FEMU (Hz)	Initial error (%)	FE results after FEMU (Hz)	Final error (%)
ω_1	5.75	7.45	29.44	5.75	-0.06
ω_2	39.90	46.66	16.94	39.89	-0.02
ω_3	117.11	130.64	11.55	117.10	-0.01
ω_4	236.13	256.01	8.42	236.11	-0.01
ω_5	397.34	423.20	6.51	397.32	-0.01

5 Conclusions

The paper presents an application of Derringer's function in solving a multi-objective FEMU problem. It is shown that by using the proposed technique, objective functions of FEMU can be formulated in a very detailed, flexible, simplified and user friendly manner. Depending upon the preferences of the user, the target value, lower limit, upper limit, weight on lower limit, weight on upper limit and relative importance of each response variable can be set using this technique. The proposed technique of FEMU is especially useful where different vibration modes are not equally important but have different relative importance. Success of the proposed technique is validated by its application on FEMU of a simulated cantilever beam structure. Updating results show that by using the proposed technique, absolute average error of 14.6% considering all five natural frequencies has been reduced to just 0.02%. This shows an improvement of 99.86% in error. Further, the method can be easily extended to all cases involving large number of updating parameters.

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