# Numerical Study of Hydraulic Flux through a Saturated and Inhomogeneous Porous Medium by means of the FEM

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***Abstract –*** This paper presents the study of water flux in saturated porous medium, homogeneous and inhomogeneous, by the development of particular numerical tool with Finite Element Method (FEM) and formulated in FORTRAN programming language. The finite element model, developed at this article, uses hexahedron finite elements with 8 or 27 nodes. The hydraulic percolation is analyzed by the flux and potential hydraulics determination in soil with sheet piles structures. The study of the massive comprises either clay or clay plus iron mining waste. These numerical applications aims to validate the numerical method implemented using FEM as well as by numerical comparison with similar studies made by other researchers.

***Keywords –*** Flux through a saturated porous medium, anisotropic soil, sheet piles structures, Finite Element Method.

## 1 Introduction

This paper aims the development of a numerical tool, formulated in *Fortran*, in order to study the water flux (under permanent regime) through porous and saturated medium. In this context, it focuses particularly on percolation, in terms of determination of total hydraulic outflow and hydraulic potential. Attention is addressed for soils with contentment, systems that use curtains of *board-stake* spiked in different types of soils. There was a waste chosen in the analysis, waste of Iron Ore. The choice was due to high volumes produced in the activities of extraction in the mining segment. According to [1,2], "waste" as being all the materials resulted from the chemical and physical processes involved in the extraction of minerals. The minerals of economic interest but also processed as waste in mining. Based on the authors, wastes of gross and medium granulometry (such as fine and medium size sands), non-plastics, present high permeability, high shearing strength and low compressive strengths. In the case of fine wastes (such as silts and clays), these present high index of plasticity, high compressive strengths, harder sedimentation and low permeability. Particularly in Brazil the rate between the iron ore produced and wastes generated reaches an order of 2/1. It means, for each two portion of Iron Ore produced is generated one portion of waste. For other minerals, commonly explored in the Country, as coal, phosphate, cupper and the gold, this rate reaches an average of 1/3, 1/5, 1/30 and 1/1000 [1] consequently.

In this work, beyond of Iron Ore wastes, soils based on Clays are treated. Percolation of water in homogeneous or inhomogeneous massive are studied by an unique layer or multilayers of soil through the depth of the massive.

Within the context of containment, courts of *board-stake* of the water flux through porous medium are considered. According to [3], containment has to conform to the construction procedure and material to build it up. It is highlighted, among them, the *molded walls*, *curtains of board-stake*, *stake curtains* and *walls type Berlin*. Among the several systems of containment, the curtains of *board-stake* are plain or curved structures (wooden, concrete or metallic made). These are spiked in the soil in such a way of juxtaposed to each other [4]. Between other characteristics taken into consideration, according to [5], the most important are: - elements of provisory containment or definitive; - low stiffness in its plane; - soils with high groundwater level; - provisory situations are usually anchored or moored at the top and - applicable in any type of land (undependable of the level of the groundwater). This kind of containment is applied in port constructions, underground parking, tunnels, slope protection; etc…In this work is not considered the *analysis of stability* of the curtains of *board-stake*. Once the interest is not the study of mechanical behavior of such structures of containment. The interest is solely the hydraulic behavior of soils on which those are spiked.

The finite element method (FEM) applies in numerical discretization of the percolation equation. It is applicable for saturated porous mediums, homogene-

ous and inhomogeneous, written as a function of hydraulic potential ($\phi$). The FEM applied at this work makes use of tridimensional finite elements type hexaedron, of 8 or 27 nodes, such as those presented by [6]. This author also studied the tridimensional diffusion-reaction among several numerical applications with analytical solutions, those that validated the efficiency of the method.

According to [7], the simpler problems of percolation in saturated porous mediums, usually, like those involve in unidirectional flux. One example is the experimental test to determine the permeability of homogeneous and isotropic soils. Their resolution done by using the Law of Darcy. However, in some practical situations in engineering (like sloping embankments, dams and embankments inhomogeneous, etc…) the water flux is not unidirectional nor neither uniform. In these cases, it needs to resort of differential equations of continuity of Laplace (*DECL*) for tridimensional flux. In this equation, the hydraulic potential is defined for all domains. The solution therefore is unique for each point of the domain and the particular solution of the problem determined by applying particular boundaries conditions. Thus, the *DECL* represents the condition of percolation in permanent regime, in a porous medium for any point into the mass of the soil analyzed. This condition is basically described in the next section. Such section also presents a succinct review about the water outflow (under permanent regime) into porous mediums, homogeneous or inhomogeneous.

# 2 Flux in Permanent Regime into Tri-dimensional Porous Mediums

The Darcy Law for tridimensional flux ($x$, $y$ and $z$), valid only under conditions of saturations into porous mediums, is written at the following way (Braja, 2012):

$$i = i_x + i_y + i_z = \frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} + \frac{\partial h}{\partial z} = \nabla h \qquad (1)$$

where (i) represents the hydraulic gradient vector, ($\partial h$) represents the variation of total hydraulic load h ($x$, $y$, $z$) within the extremes ($x$, $y$, $z$) of the infinitesimal segment of the hydraulic flux current.

The Darcy vector speed on the other hand can be written under the matricidal manner [9]:

$$q = -K\nabla h = -\begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix}\begin{Bmatrix} \partial h/\partial x \\ \partial h/\partial y \\ \partial h/\partial z \end{Bmatrix} \qquad (2)$$

being: $q = \{q_x \quad q_y \quad q_z\}^T$ the velocity of infiltration, tangent to the flux line to any point to the soil mass; $k_{xx}$, $k_{yy}$ and $k_{zz}$ the coefficient of hydraulic permeability in the Cartesian directions ($x$, $y$, $z$); $\{i_x \quad i_y \quad i_z\} = \{\partial h/\partial x \quad \partial h/\partial y \quad \partial h/\partial z\}$; $K$ is the tensor of hydraulic conductivity (or the tensor of the permeability coefficient) of the saturated soil. The negative sign indicates that the flux occurs in the sense of diminishing the gradient, taken for definition as that for which the potential increase [9].

The outflow passing through the cross section (*A*) into the porous medium, perpendicular to the flux can be calculated by the equation [7]:

$$Q = -KA\nabla h \qquad (3)$$

The hydraulic conductivity indicates the grade of easiness for which the water moves through the soil [7]. According to [10], several factors linked to the water and soil influence the hydraulic permeability of soils. Among them: the voids index of the soil, the temperature of the water, the type of soil, its saturation and the stratification of the land. This last, as being one of the characteristics analyzed in this paper).

The *differential equation of continuity* used for the study of percolation *bi* and *tridimensional* is obtained considering the flux through an *elementary block*. Assuming the Darcy Law at this element, homogeneous soil, non-compressive water and non-saturated medium, the differential equation of flux into non-saturated mediums assumes the following manner [10]:

$$k_{xx}\frac{\partial^2 h}{\partial x^2} + k_{yy}\frac{\partial^2 h}{\partial y^2} + k_{zz}\frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e}\left(e\frac{\partial S}{\partial t} + S\frac{\partial e}{\partial t}\right) \qquad (4)$$

being: (*S*) the grade of the medium saturation, (*e*) the index of voids into the soil and (*t*) the variable time. The grade of saturation is related, on the other hand, directly to the volume of water present into the soil at the time analyzed. In addition, inversely proportional to the volume of voids in it, in other words, $S = v_w/v_v$, and varies within 0 (dry soil) to 100% (saturated soil). For the voids index, it is directly related to the volume of voids into the soil and inversely related to the volume of solid grains into the soil. It means, $e = v_v/v_s$, and varies within 1 to 13 (extremes' case, according to [11], observed in the clays in México).

Considering the studied soil as homogeneous, anisotropic and saturated ($\partial S/\partial t = 0$) and where compression and expansion do not occur in porous medium ($\partial e/\partial t = 0$) there are:

$$k_{xx}\frac{\partial^2 h}{\partial x^2} + k_{yy}\frac{\partial^2 h}{\partial y^2} + k_{zz}\frac{\partial^2 h}{\partial z^2} = 0 \qquad (5)$$

The Eq. (5) is known as *differential equation of continuity of Laplace* for permanent flux into porous mediums, homogeneous and saturated. This equation is obtained assuming a Cartesian system *x*, *y* and *z* (where *z* is in vertical direction – positive upward, Figure 1). Those as coincident with the main directions of the tensor of the hydraulic permeability, thus, being non-permanent elements to this main diagonal nulls [12], [as shown in Eq. (2)]. In the particular case of Eq. (5), is considered the studied domain as plain and bidirectional (*x,z*) being the soil homogeneous and isotropic ($k_{xx} = k_{yy} = k_{zz} = k$). Usually, when the flux is treated as a plain problem (bidirectional) is considered one typical section of the soil. This section located within two vertical planes and parallel with an unity of thickness. This fact is due to the fact the hydraulic load, upstream and downstream, does not change with the longitudinal dimension of the massive soil [10].

Adopting a new variable, designated as *hydraulic potential function*, and considering the porous medium homogeneous and anisotropic, the Eq. (5) assumes the following way:

$$k_{xx}\frac{\partial^2 \phi}{\partial x^2} + k_{yy}\frac{\partial^2 \phi}{\partial y^2} + k_{zz}\frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (6)$$

being: $\phi(x,y,z) = h(x,y,z) + C$ and *C* constant determined by the imposed boundary conditions of the problem.

Based on [13], the boundary condition can be: *essentials* (of *Dirichlet*) where the values written to the function $\phi$ are imposed in certain path of the domain boundary, it means: $\phi = \bar{\phi}$, and/or *naturals* (*Neumann*), where the derivative of the function $\phi$ has pre-established values, it means: $\phi_n = -(k_{xx}\partial\phi/\partial x)n_x - (k_{yy}\partial\phi/\partial y)n_y - (k_{zz}\partial\phi/\partial z)n_z$, being normal to the boundary, to ($n_x$, $n_y$, $n_z$) components of the vector (*n*).

The Figure 1 shows the curtain *board-stake* (*GHJIG'H'J'I'*) used in several numerical simulations along this paper. This is considered a medium semi-infinite (*ABCDA'B'C'D'*) of soil by the boundary conditions utilized.

In Figure 1, $h_m$ is the hydraulic load to the upstream and $h_j$ the hydraulic load at downstream, applied respectively on the planes *AFF'A'* and, *DD'EE'*, $\phi_n = 0$ on planes *ABB'A'*, *BCC'B'*, *CDD'C'*, *GII'G'*, *GHG'H'* e *JHH'J'*.
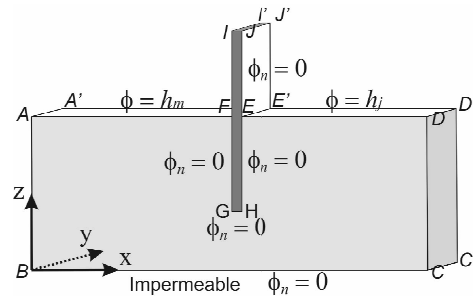


**Fig. 1** Boundary conditions applied to the medium of semi-infinite soil composed by a containment system board-stake

## 3 Formulation by FEM for the Study of Permeability in Porous Mediums

The introduction of this approximation needs to define the variational formulation of the problem, as follow;

First we should determine $v_i^e \in V^e$ with $V^e \in H^1(\Omega)$, in a way that;

$$\int_{\Omega^e} R \cdot \bar{\phi}^e d\Omega = 0, \ \forall \bar{\phi}^e \in V^e, \ i = 1, 2, ..., \qquad (7)$$

once

$$\phi \approx \bar{\phi}^e = \sum_{j=1}^{N_{nodes}} N_j \bar{\phi}_j^e \qquad (8)$$

where $N_{Nodes}$ is the number of nodes in each finite element.

The Eq. (7) is valid for $v_i^e = N_i$, $i = 1, 2, ..., N_{nodes}$, in this context, the function weigh is equal of the function of interpolation. The formulation by the Method of Galerkin is symmetric and positive, as defined for the problem handled at this paper;

Taking the residual equation as the equation below;

$$R \cong k_{xx}\frac{\partial^2 \phi}{\partial x^2} + k_{yy}\frac{\partial^2 \phi}{\partial y^2} + k_{zz}\frac{\partial^2 \phi}{\partial z^2} \qquad (9)$$

and substituting (9) into Eq. (7) it is obtained the following integral on the element,

$$\int_{\Omega^e}\left[k_{xx}\frac{\partial^2 \phi}{\partial x^2} + k_{yy}\frac{\partial^2 \phi}{\partial y^2} + k_{zz}\frac{\partial^2 \phi}{\partial z^2}\right]N_j d\Omega = 0 \qquad (10)$$

To integrate the left side of the Eq. (10) is utilized the integration per parts, as defined by [14], at the way as follow;

$$\int_{\Omega^e}\left[k_{xx}\frac{\partial^2 \phi}{\partial x^2} + k_{yy}\frac{\partial^2 \phi}{\partial y^2} + k_{zz}\frac{\partial^2 \phi}{\partial z^2}\right]N_j d\Omega =$$

$$-\int_{\Omega^e}\left[k_{xx}\frac{\partial N_i}{\partial x}\frac{\partial \phi}{\partial x} + k_{yy}\frac{\partial N_i}{\partial y}\frac{\partial \phi}{\partial y} + k_{zz}\frac{\partial N_i}{\partial z}\frac{\partial \phi}{\partial z}\right]d\Omega$$

$$+\int_{\Gamma_q} N_i k_{xx}\frac{\partial \phi}{\partial x}l d\Gamma_q + \int_{\Gamma_q} N_i k_{yy}\frac{\partial \phi}{\partial y}m d\Gamma_q$$

$$+\int_{\Gamma_q} N_i k_{zz} \frac{\partial \phi}{\partial z} n d\Gamma_q \qquad (11)$$

This work considers the boundary conditions of first and second type, written mathematically as follow:

$$\phi = \phi_b \quad \text{in } \Gamma_b \qquad (12a)$$

and

$$k_{xx}\frac{\partial \phi}{\partial x}l + k_{yy}\frac{\partial \phi}{\partial y}m + k_{zz}\frac{\partial \phi}{\partial z}n = 0 \qquad \text{in } \Gamma_q \qquad (12b)$$

where $\Gamma_b \cup \Gamma_q = \Gamma$ e $\Gamma_b \cap \Gamma_q = 0$, $\Gamma$ represents the boundary, and $m$ and $n$ represent the cosines directors. Then, applying (12b) in (11), there is:

$$\int_{\Omega^e}\left[k_{xx}\frac{\partial^2\phi}{\partial x^2} + k_{yy}\frac{\partial^2\phi}{\partial y^2} + k_{zz}\frac{\partial^2\phi}{\partial z^2}\right]N_j d\Omega =$$

$$-\int_{\Omega^e}\left[k_{xx}\frac{\partial N_i}{\partial x}\frac{\partial \phi}{\partial x} + k_{yy}\frac{\partial N_i}{\partial y}\frac{\partial \phi}{\partial y} + k_{zz}\frac{\partial N_i}{\partial z}\frac{\partial \phi}{\partial z}\right]d\Omega$$

$$(13)$$

Substituting (9) and (13) into Eq. (10) and after some calculations by algebra is obtained a linear system,

$$[K]\{\phi_i^e\} = \{F\} \qquad (14)$$

in which

$$K_{ij} = -\int_{\Omega^e} k_{xx}\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}d\Omega$$

$$-\int_{\Omega^e} k_{yy}\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}d\Omega - \int_{\Omega^e} k_{zz}\frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}d\Omega \qquad (15a)$$

$$F_i = 0 \qquad (15b)$$

In this paper is used the transformation of global coordinates to local coordinates ($x\to\xi$, $y\to\eta$, $z\to\zeta$). For that, ahead, towards the computation of integrals into the matrixes via method of *Quadrature of Gauss-Legendre* [15]. More details about the numerical integration can be met in [16-17].

# 4 Numerical Applications

Two different numerical application are proposed in this paper. These aim to validate the implemented numerical method via *FEM* and to serve as a reference of comparison to similar studies proposed by other researchers. The first numerical application studies the permanent flux in porous and homogeneous mediums and isotropic. The second in mediums porous, inhomogeneous, and anisotropic.

## 4.1 Evaluation of FEM in the study of percolation of water in homogeneous and isotropic soils

The first numerical application is based in the illustration shown in the Figure 2 of a curtain of *board-stake*. This is utilized as a system of containment in a porous and homogeneous medium, with the coefficient of hydraulic permeability equal of $k = k_{xx} = k_{yy} = k_{zz} = 5\times10^{-7}$ m/s [7].

In the Figure 2a, the upstream water height ($h_m$) of the curtain *board-stake* is adopted as being 5.6m and the downstream (hj) 2.0 m. The depth of natural soil ($h_n$) is equal of 11 m, with length ($l$) of the massive analyzed 26 m. The parcel of *board-stake* in contact with the soil has a height of 7.0 m, this means $h_p = 7$ m and the thickness of those *board-stake* assumed as 0.50 m. The results, in terms of total hydraulic out-flow (Eq. 3) and total hydraulic potential for the points $A(5,0,4)$ e $B(16.5,0,6.5)$ of the domain $(x,y,z)$ are presented in Fig. 2a. These data obtained via FEM, and compared with the analytical calculations presented by Braja (2012).

The calculus of percolation in the porous medium realized by [7] made use of the equation $Q = 2.38k\left((h_m - h_j)/n_d\right) = 2.38k(H/n_d)$, obtained when *applying the method of the flow nets*. It assumes the equation of bidirectional continuity as an isotropic medium (particular case of Eq. (5)). It represents two families of orthogonal curves, worked between themselves and called *equipotential lines and flow*. In this equation, $(n_d)$, is the number of equipotential channels of the flux, given by the number of the equipotential lines less than 1, which is, for this application equal to 6 according to [7]. Now the total hydraulic potential at the points $A$ and $B$ of the porous medium (Fig. 2a) are represented by *the method of the flux nets,* given when calculating $H$ subtracted by the loss of total load $(n_d^p \Delta h)$. Being $n_d^p$ the number of load losses until $P$ considered and $\Delta h$ the loss of load for each decrease of the potential.
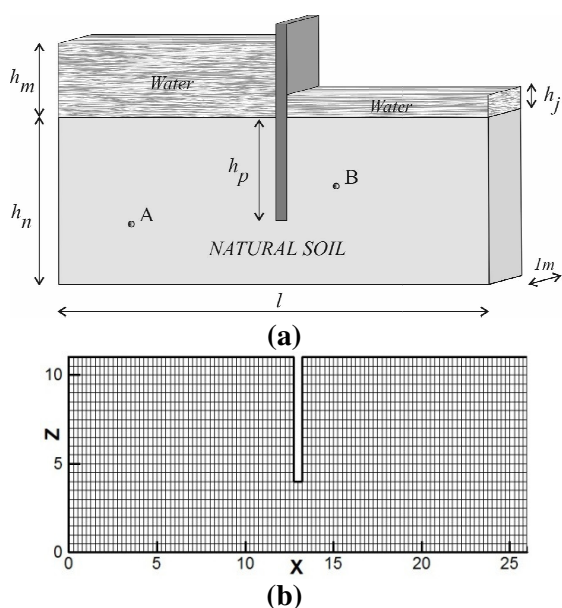
**Fig. 2** (a) Geometry of curtain board-stake and porous medium analyzed, and (b) computational mesh with $\Delta x = 0.25$, $\Delta y = 0.20$, $\Delta z = 0.50$, $N_{nost} = 14406$, $N_{elem} = 11300$.

The mesh of the finite element (Fig. 2b) applied in this numerical simulation for the calculus of the distribution of the total hydraulic potential in the porous medium uses quadratic finite elements with 8 nodes. Being $N_{elem}$ and $N_{nost}$ respectively the number of finite elements and nodes in the domain discretized via FEM.

The Figure 3 illustes the distribution of the total hydraulic potential in the porous medium and isotropic, studied and obtained by FEM. It is observed that the total hydraulic potential varies between 5.6 m upstream and 2.0 m downstream from the curtain board-stake. In that, at the points $A$ and $B$ the total hydraulic potentials resulted by FEM are respectively 5.120 m and 2.705 m. It is known that the hydraulic flow occurs normally from the higher potential, 5.6 m, in the direction of the lower, 2.0 m and the flux lines (which indicate geometrically the path done by a particle into the water). These last perpendiculars to the lines of hydraulic potentials presented in Fig. 3.
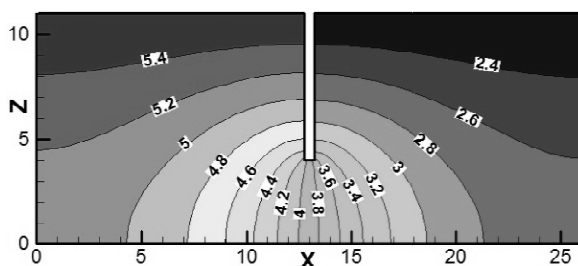


**Fig. 3** Profile of total hydraulic potential in the section XZ ($y = 1$) of the computational mesh with $\Delta x = 0.25$, $\Delta y = 0.20$, $\Delta z = 0.50$, $N_{nost} = 14406$, $N_{elem} = 11300$.

One new mesh ($\Delta x = \Delta y = \Delta z = 0.125$) applying hexahedron finite elements with 27 nodes ($N_{nost} = 165897$ and $N_{elem} = 144640$) is used to calculate the total hydraulic outflow in the massive soil. The mesh utilized at this paper are limited due to computational processing capacity restrictions (Hardware 2.1 GHz and 8 RAM).

With this mesh, the total hydraulic outflow obtained in the section $x = 13$ m, $0 \le y \le 1$ e $0 \le z \le 4$ is equal to Q = $6.828 \times 10^{-5}$ m$^3$/s per unit of length (m) of the curtain of board-stake.

**Table 1** Hydraulic outflow and total hydraulic potential in a homogeneous and isotropic soil, obtained by the Net of Flux and by FEM.

| Parameters analysed | Braja (2012) ($I$) | MEF proposed ($II$) | Percentage $100 \times \left| (I - II)/(I) \right|$ |
|---|---|---|---|
| Total hydraulic outflow (m$^3$/s/m) | $6.740 \times 10^{-5}$ | $6.828 \times 10^{-5}$ | 1.3 |
| Total hydraulic potential in **A** (m) | 5.033 | 5.120 | 1.7 |
| Total hydraulic potential in **B** (m) | 2.765 | 2.705 | 2.2 |

Table 1 presents a summary of the total hydraulic outflow through porous medium and the total hydraulic potential in the points $A$ and $B$ of massive, supplied by [7], making use of flux net, via FEM (proposed at this paper).

It is observed in Table 1 the small variation between the values of total hydraulic outflow (per linear meter of curtain) and between the values of total hydraulic potentials, (for both points $A$ and $B$), in the massive analyzed via method of net of flux and by MEF.

## 4.2 Study of water flow in inhomogeneous soil, isotropic or anisotropic

The next numerical application is based on the same geometric domain of last one (Fig. 2a). At this way the hydraulic load upstream and the downstream of containment by board-stake, and porous medium, are changed according to the Figure 4.
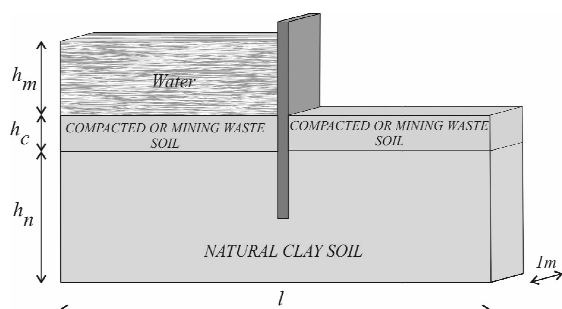
**Fig. 4** Geometry of the spiked containment in the inhomogeneous soil.

The massive of soil at this application consists of two types of clay soils, distinguished by its hydraulic permeability in the natural conditions (in situ), and consequently compacted. During the phase of construction of the system of containment, the natural soil could be compacted by mechanical processing, resulting in the dismissing of its permeability and increase of its mechanical strength. The hydraulic loads upstream and downstream were kept constants by mean of an hydraulic pumping; which maintains the level of water unchanged, it means $h_m = 5.6$ m and $h_j = 0$ m.

**Table 2.** Outflow in region $x = 13$ m, $0 \leq y \leq 1$ m and $0 \leq z \leq 4$ m according to mesh refinement by hexahedron elements with 27 nodes.

| Mesh | | | | | Outflow for each thickness of compacting $[m^3/s/m] \times 10^{-8}$ | | |
|---|---|---|---|---|---|---|---|
| $\Delta x$ | $\Delta y$ | $\Delta z$ | $N_{Nost}$ | $N_{elem}$ | 0.20 m | 0.40 m | 0.60 m |
| 0.250 | 0.250 | 0.100 | 57925 | 45200 | 7.836 | 7.339 | 6.905 |
| 0.250 | 0.125 | 0.100 | 104265 | 90400 | 7.926 | 7.423 | 6.984 |
| 0.250 | 0.250 | 0.005 | 115325 | 90400 | 7.811 | 7.316 | 6.882 |
| 0.125 | 0.125 | 0.100 | 206901 | 180800 | 7.911 | 7.411 | 6.972 |
| 0.125 | 0.100 | 0.100 | 252879 | 226000 | 7.964 | 7.460 | 7.018 |

Three different thicknesses of compacted clayed soil ($h_c$) are evaluated: 0.20; 0.40; 0.60 m, being the thickness of natural soil ($h_n$) beneath of the compacted soil, as a function ($h_c$) under the way: $h_n(h_c) = 11 - h_c$ [m].

The coefficients of hydraulic permeability of the clayed soils, compacted and natural, are equal respectively to $5.31 \times 10^{-9}$ and $3.78 \times 10^{-8}$ m/s and kept experimentally by [18] at the temperature of 20 ℃.

Table 2 presents the total hydraulic outflows of the massive inhomogeneous, per unit of length of the curtain of *board-stake*, obtained numerically via FEM, with hexagonal elements with 27 nodes. In this simulation, as the previous case, the domain is discretized for different meshes and the total hydraulic outflows through the massive obtained.

It notes by the Table 2 that the increase of refinement of the mesh results in a small variation of hydraulic pressure. Showing that, and for higher refinements, compared with the previous done, low influence in the outflow results is found.

The profiles of distribution of the total hydraulic potential through the stratified massive are shown in the following figures. For each different thickness of superficial and compacted soil analyzed the values are obtained by using hexagonal finite element with 8 nodes, with mesh of $\Delta x = 0.125$, $\Delta y = 0.125$, $\Delta z = 0.100$ e $N_{nost} = 206901$, $N_{elem} = 180800$.



**(a)**



**(b)**



**(c)**
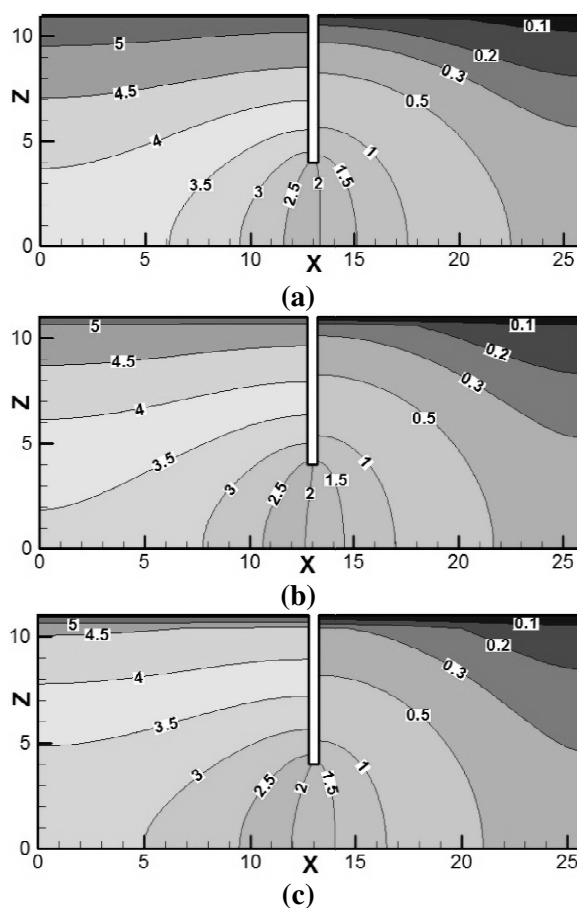
**Fig. 5** Profile of total hydraulic potential in the section XZ, with $y = 1$, in the stratified massive with compacted clayed soils under thicknesses: 0.20 m (a), 0.40 m (b) e 0.60 m (c).

In replacement of the compacted clayed soil, a reinforcement utilizing waste of iron ore is applied on the same thicknesses of those compacted soils, analyzed previously. The purpose is evaluate the influence of waste iron ore on the values of total hydraulic outflow in the massive. The value of hydraulic permeability (anisotropic) of the iron ore waste is assumed as $k_{xx}/k_{zz} = 4$ com $k_{zz} = 10^{-7}$ m/s, being presented by [2] in his study of the behavior of containments of iron ore waste. The resultant values are localized 14.5 m far

from the beach where the waste is launched at the containment.

The total hydraulic outflow though the massive in the anisotropic soil and inhomogeneous, calculated in the region $x = 13$ m, $0 \leq y \leq 1$ e $0 \leq z \leq 4$, is obtained and shown in the Table 3. This is for different thicknesses of iron ore waste utilized. Those values are achieved using a mesh with $\Delta x = 0.125$ and $\Delta y = \Delta z = 0.100$ and hexagonal elements with 27 nodes ($N_{nost} = 252879$ e $N_{elem} = 226000$). The Table 4 presents also the total hydraulic outflow obtained through an homogeneous and isotropic massive, composed by an unique natural clayed soil with permeability $k = k_{xx} = k_{yy} = k_{zz} = 3.78 \times 10^{-8}$ m/s, for comparison reasons with the values of the massive inhomogeneous and anisotropic.

**Table 3** Total hydraulic outflow in the region $x = 13$ m, $0 \leq y \leq 1$ m e $0 \leq z \leq 4$ m for massive composed by different types of soil.

| TYPE OF SOIL | Hydraulic outflow for each thickness of compaction [m³/s/m] × 10⁻⁸ | | |
|---|---|---|---|
| | 0.20m | 0.40m | 0.60m |
| Compacted clayed + natural clay | 7.964 | 7.460 | 7.018 |
| Waste of iron ore + natural clay | 8.652 | 8.756 | 8.864 |
| Natural clay | | 8.550 | |

It is verified that the total hydraulic outflow through the massive inhomogeneous, composed by a compacted layer of clayed soil, decreases with the increase of the compacted clayed layer. This is not observed with the substitution of that compacted layer by the waste of iron ore (once the permeability of this soil is higher than the compacted clayed soil). Beyond of this, the total hydraulic outflow throughout the homogeneous massive (natural clay) is lower than those two inhomogeneous soils analyzed (due to the lower hydraulic permeability). Also, based on the values presented in this table it is realized that the replacement of the compacted clayed soil by the waste of iron ore does not contribute for the decrease of the total hydraulic outflow along the massive, composed by natural clayed soil.

# 5 Conclusions

It is not easily found in the scientific literature numerical papers associated with the study of the hydraulic outflow in the containment systems. Mainly treating *stake-board* spiked in homogeneous soils or not, or those which handle wastes of mining. In this context, this paper presents a numerical formulation via FEM that may be extended to the study of flux. Being in permanent regime, on several systems of containment and porous mediums (homogeneous or inhomogeneous) and isotropic or anisotropic soils.

The numerical simulation via FEM presented in this paper to study the hydraulic flux in massive of soils is much relevant, mainly when the porous medium is not homogeneous. It means, in stratified mediums and when the soils have anisotropic nature in terms of hydraulic permeability. Nevertheless it is verified the absence of analytical or numerical studies on this theme.

The numerical simulations presented on this paper demonstrate that the tridimensional finite elements of hexahedron type, with 8 and 27 nodes, can be applied in the modelling. In the study of the hydraulic flux on massive homogeneous as well as in isotropic ones. Beyond of this, this type of finite element also can used in modelling and in the study of hydraulic flux in massive stratified, composed of anisotropic soils. Mainly for those where the hydraulic behavior cannot be evaluated directly by the classical model of net flux. It is important to reinforce that the results of finite element are influenced by the refinement grade of the mesh and by the type of element utilized. On the other hand, it is not dependable of such variables when the interest is on obtaining the distribution of hydraulic potential along the massive of soil analyzed. Once the calculation of hydraulic flow requires the knowledge of derivatives of $\phi$, an expressive refinement is necessary when using finite element type hexahedron with 8 nodes. An alternative is applying instead of hexahedron with 8 nodes other ones with 27 nodes, resulting in (for the same domain analyzed) a mesh less refined.

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