Bidding Strategies for Generation Companies in a Day-ahead Market using Fuzzy Adaptive Particle Swarm Optimization

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Abstract: - This paper presents a methodology based on Fuzzy Adaptive Particle Swarm Optimization (FAPSO) for the preparation of optimal bidding strategies corresponding unit commitment by Generation companies (Gencos) in order to gain maximum profits in a day-ahead electricity market. In a competitive electricity market with limited number of suppliers, Gencos are facing an oligopoly market rather than a perfect competition. Under oligopoly market environment, each Genco may increase its own profit through a favorable bidding strategy. In FAPSO the inertia weight is tuned using fuzzy IF/THEN rules. The fuzzy rule-based systems are natural candidates to design inertia weight, because they provide a way to develop decision mechanism based on specific nature of search regions, transitions between their boundaries and completely dependent on the problem. The proposed method is tested with a numerical example and results are compared with Genetic Algorithm (GA) and different versions of PSO. The results show that fuzzying the inertia weight improve the search behavior, solution quality and reduced computational time compared to GA and different versions of PSO.

Key-Words: - Bidding Strategy, Electricity Market, Fuzzy Inference, Market Clearing Price (MCP), Particle Swarm Optimization (PSO).

1 Introduction

A recent change in regulatory policies in electricity industries has created competitive environments and markets for power suppliers. Therefore, economical operation and profit have become the primary objectives of market participants and preparing an optimal bidding strategy has significant impact all around the world have introduced competition in many industry sectors as electricity, gas and telecommunications. This paper focuses on market systems based on sealed-bid auctions. In this market, participants submit their offers to sell and to buy to the market operator, who determines the Market Clearing Price (MCP). In this environment, participants build their offers maximizing their expected profits. This process is known as strategic bidding. [1]. However, the electricity markets are oligopolistic in practice, and power suppliers may seek to increase their profit by bidding a price higher than marginal production cost. Knowing their own costs, technical constraints and their anticipation of rival and market behavior, suppliers face the problem of constructing the best optimal bid.

In general, there are three basic approaches to model the strategic bidding problem viz. i) based on the estimation of market clearing price ii) estimation of rival's bidding behavior and iii) on game theory. David [2] developed a conceptual optimal bidding model for the first time in which a Dynamic Programming (DP) based approach has been used. Gross and Finaly adopted a Lagrangian relaxation-based approach for strategic bidding in England-Wales pool type electricity market [3]. Jainhui [4] used evolutionary game approach to analyzing bidding strategies by considering elastic demand. Ebrahim and Galiana developed Nash equilibrium based bidding strategy in electricity markets [5]. David and Wen [6] proposed to develop an overall bidding strategy using two different bidding schemes for a day-ahead market using Genetic Algorithm (GA). The same methodology has been extended for spinning reserve market coordinated with energy market by David and Wen [7]. Ugedo developed a stochastic-optimization approach for submitting the block bids in sequential energy and ancillary services markets and uncertainty in demand and rival's bidding behavior is estimated by stochastic residual demand curves based on decision trees [8]. To construct linear bid curves in the Nord-pool market stochastic programming model has been used by Fleten [9]. The opponents' bidding behaviors are represented as a discrete probability distribution function solved using Monte Carlo method by David and Wen [10].

The deterministic approach based optimal bidding problem was solved by Hobbs [11], but it is difficult to obtain the global solution of bi-level optimization problem because of non-convex objective functions and non-linear complementary conditions to represent market clearing. These difficulties are avoided by representing the residual demand function by Mixed Integer Linear Programming (MILP) model [12] in which unit commitment and uncertainties are also taken into account. The generators associated to the competitors' firms have been explicitly modeled as an alternative MILP formulation based on a binary expansion of the decision variables (price and quantity bids) by Pereira [13]. Javier developed a stochastic mixed-integer quadratic programming model for optimal bidding strategies of thermal and generic programming units in a day-ahead electricity market [14]. Azadeh formed optimal bidding problem for day-ahead market as multi objective problem and solved using GA [15]. Jain and Srivastava [16] considered risk constraint, for single-sided and double- sided bidding and solved using GA. Ahmet used PSO to determine bid prices and quantities under the rules of a competitive power market [17]. Kanakasabhapathy and Swarup [18] developed strategic bidding for pumped-storage hydroelectric plant using evolutionary tristate PSO. Bajpai developed lineal and blocked bid model bidding strategy in a competitive electricity market using PSO and FAPSO [19, 20]. Recently the combination of PSO and Simulated Annealing (SA) is used to predict the bidding strategy of generation companies [21].

In general, strategic bidding is an optimization problem that can be solved by various conventional and non-conventional (heuristic) methods. Depending on the bidding models, objective function and constraints may not be differentiable; in that case conventional methods cannot be applied. Whereas, heuristic methods such as GA, Simulated Annealing (SA) and Evolutionary Programming (EP), Particle Swarm Optimization (PSO) etc., have main limitations of their sensitivity to the choice of parameters, such as the crossover and mutation probabilities in GA, temperature in SA, scaling factor in EP and inertia weight, learning factors in PSO.

Therefore, Fuzzy Adaptive Particle Swarm Optimization (FAPSO) is proposed to overcome the shortcomings of PSO and GA. In FAPSO the inertia weight (w) is adjusted using fuzzy *IF/THEN* rules. The fuzzy rule-based systems are natural candidates to design inertia weight, because they provide a way to develop decision mechanism based on specific nature of search regions, transitions between their boundaries and completely dependent on the problem.

The main contribution of this paper is, the optimal biding problem is formulated as a blocked bid model in which suppliers and rivals bidding coefficients are determined with the help of probability density function (*pdf*) using FAPSO instead of Monte Carlo method [20]. Then an optimal bid price for each block has to be determined. Based on the bid prices, run the unit commitment using FAPSO. The result shows that the proposed algorithm can generate better quality solution within shorter computation time and stable convergence characteristics compared to GA and different versions of PSO.

The paper is organized as follows. Section 2 presents the mathematical formulation of optimal bidding problem. Section 3 contains a brief over view of the proposed FAPSO method and the application of FAPSO for solving the optimal bidding problem. Section 4 reports the results and discussions compared with GA and different versions of PSO. Section 5 summed up the final outcome of the paper as Conclusion.

2 Problem Formulation

The following notations are used in this paper:

M: Number of units;
T: Scheduling Period;
$I_i(t)$: i^{th} unit status at time t (1/0 for on/off);
$P_i(t)$: Output power of i^{th} unit at time t;
$P_i^{max}(t)/P_i^{min}(t)$: Maximum/minimum output power of unit i at time t considering ramp Rate;
$D(t)$: Demand power at time t;
MU_i/MD_i: Minimum up/down time of unit i;
$X_i^{on}(t)/X_i^{off}(t)$: Duration of continuously on/off of unit i at time t;
c_i^u: Start-up cost of unit i
c_i^d: Constant shut down cost;
h: Hot start-up cost ($), considered when unit has been shut down for a short time.
τ : Cooling time constant (h).
T_{off}: number of hours of a generator shutdown.
δ: Cold start-up cost ($), considered when unit has been shut down for a long time;
$a1, b1, c1$: cost coefficients;
$c2, c3$: constants of the valve point loading effect;

Consider a system consist of '$M+1$' Generators or suppliers, an inter-connected network controlled by an Independent System Operator (ISO) and a Power

Exchange (PX), an aggregated consumer (load) which does not participate in demand-side bidding. Generator-G, for which an optimal bidding strategy has to be developed, is having *M* rivals in the market. Each generator bids for every one hour trading period under stepwise protocol and uniform Market Clearing Price (MCP) system. Generators submit their bids in terms of quantity (MW) and price ($) for each hour in 24-h horizon to compete in a day-ahead market. Generator-G and each rival generator can bid maximum *I* blocks of output for each trading period.

For large thermal generators, input-output characteristics are not always smooth due to sequential opening of multi-number of valves to obtain ever-increasing output of the unit [22]. Typically, as each steam admission valve in a turbine starts to open, it produces a rippling effect on the unit curve. This rippling effect of valve point loading has been modeled as a recurring rectified sinusoidal function, which confirms the importance of precise production cost function application in strategic bidding. Considering non-differentiable, non-convex production cost function $c_i^p(t)$, the operating cost function $c_{i(t)}$ for the i^{th} block of generator-G can be written as

$$c_i(t) = c_i^p(t) + c_i^u(t) + c_i^d(t) \quad (1)$$

Where

$$c_i^p(t) = a1(p_i(t))^2 + b1(p_i(t)) + c1 + \left|c2\sin(c3(p_i^{\min} - p_i(t)))\right| \quad (2)$$

$$c_i^u = h + \delta\left(1 - \exp\left(\frac{-T_{off}}{\tau}\right)\right)$$

The optimal bidding strategy of generator-G can be formulated as profit maximization problem in terms of dispatched power output $p_i(t)$ and Market Clearing Price *M(t)*. The product of dispatched power and MCP gives the revenue obtained. The cumulative profit for *I* blocks of the generator over time period "*T*" is expressed as:

$$\text{Maximize } F(M(t), p_i(t)) = \sum_{t=1}^{T}\sum_{i=1}^{I}(M(t) * p_i(t) - c_i(t)) \quad (3)$$

Subject to

1) System power balance

   The generated power from all the committed units must satisfy the load demand which is defined as

$$D(t) = \sum_{i=1}^{M} p_i(t) \quad (4)$$

2) Generation limits

$$p_i^{\min} \le p_i(t) \le p_i^{\max}, \forall t \in T \quad (5)$$

3) Minimum up/down time

   Once a unit is committed/de-committed, there is a predefined minimum time after it can be de-committed/committed again.

$$(1 - I_i(t+1)MUT_i \le X_i^{on}(t), ifI_i(t) = 1 \quad (6)$$

$$I_i(t+1)MD_i \le X_i^{off}(t), ifI_i(t) = 0$$

4) Limitations on bid price

$$c_i(t) \le p_i(t) \le p_{\max} \quad (7)$$

It is clear that, market participants can set MCP at the level that returns the maximum profit to them if they know bidding strategy of other firms. But in sealed bid auction based electricity market, information for the next bidding period is confidential in which suppliers cannot solve optimization problem formed in Eq. (3) directly. However, bidding information of previous round will be disclosed after ISO decide MCP and everyone can make use of this information to strategically bid for the next round of transaction between suppliers and consumers. An immediate problem for each supplier is how to estimate the bidding coefficients of rivals.

Let, from the i^{th} supplier's point of view, rival's (*j*) bidding coefficients (*j≠i*) obey a joint normal distribution with the following probability density function (*pdf*):

$$pdf_i(a_j, b_j) = \frac{1}{2\Pi\sigma_j^{(a)}\sigma_j^{(b)}\sqrt{1-\rho_j^2}} \times \exp\left\{-\frac{1}{2(1-\rho_j^2)}\left[\left(\frac{a_j - \mu_j^{(a)}}{\sigma_j^{(a)}}\right)^2 - \frac{2\rho_j(a_j - \mu_j^{(a)})(b_j - \mu_j^{(b)})}{\sigma_j^{(a)}\sigma_j^{(b)}} + \left(\frac{b_j - \mu_j^{(b)}}{\sigma_j^{(b)}}\right)^2\right]\right\} \quad (8)$$

Here ρ_j is the correlation coefficient between a_j and b_j. $\mu_j^{(a)}, \mu_j^{(b)}$, $\sigma_j^{(a)}$ and $\sigma_j^{(b)}$ are the parameter of the joint distribution. The marginal distributions of a_j and b_j are both normal with mean values $\mu_j^{(a)}$ and $\mu_j^{(b)}$, and standard deviations $\sigma_j^{(a)}$ and $\sigma_j^{(b)}$ respectively. Using probability density function of Eq. (8) for Generator-G and rivals, the joint distribution between a_j and b_j, the optimal bidding problem with the objective function of Eq. (3) and constraints Eq. (4)-(7) becomes a stochastic optimization problem and FAPSO algorithm is very efficient to solve the stochastic optimization problem, presented in the following section.

3 Proposed FAPSO Algorithm

PSO is similar to the other evolutionary algorithms in that the system is initialized with a population of random solutions. Each potential solution, call particles, flies in the *D*-dimensional space with a velocity which is dynamically adjusted according to the flying experiences of its own and its colleagues [23]. The location of the i^{th} particle is represented as $X_i = (x_{i1,} x_{i2...} x_{iD})$. The best previous position of the i^{th} particle is recorded as P_{besti}. The index of the best, P_{best} among all the particles is represented by the symbol *g*. The location P_{bestg} is also called G_{best}. The rate of velocity for the i^{th} particle is represented as $V_i = (v_{i1}, v_{i2} \ldots v_{iD})$. The modified velocity and position of each particle are calculated using current velocity and the distance from P_{best} to G_{best} as Eq. (9) and (10).

$$V_r^{k+1} = w^k V_r^k + a_1 rand_1 * (P_{best}^k - X_r^k) + a_2 rand_2 * (G_{best}^k - X_r^k) \quad (9)$$

$$X_r^{k+1} = X_r^k + V_r^{k+1} \quad (10)$$

Where, k is the iteration counter and k_{max} is the maximum iteration number.

The PSO search process is a nonlinear and complicated process and a linear decreasing Inertia Weight Approach (IWA) PSO or linearly decreasing learning factors in Velocity Updated Relaxed (VUR) PSO has a linear transition of search ability from global to local search, which does not truly reflect the actual search process required to find the optimum [24]. This especially is true for dynamic optimization problems. Therefore, for better performance, the inertia weight should be nonlinearly, dynamically changed to have better dynamics of balance between global and local search abilities. Therefore, the FAPSO is proposed, to design fuzzy adaptive inertia weight (*w*) using fuzzy "*IF/ THEN*" rules for solving the optimal bidding problem.

The fuzzy system consists of four principle components: fuzzification, fuzzy rules, fuzzy reasoning and defuzzification which are described as follows [25].

(*i*) *Fuzzification*: To obtain a better gravitational constant value under the fuzzy environment, two inputs are considered, a) Normalized Fitness Value (NFV). b) Current inertia weight (*w*) and output is the correction of the inertia weight (Δw). The triangular membership functions are considered for the fuzzification of the input variables are presented in three linguistic values, S (Small), M (Medium) and L (Large), where as the output variable (Δw) is presented in three fuzzy sets of linguistic values; NE (negative), ZE (zero) and PE (positive) with associated triangular membership functions, as shown in Figure 1.

(*ii*) *Fuzzy rules*: The Mamdani-type fuzzy rules are used to formulate the conditional statements that comprise fuzzy logic. For example:

IF (NFV is *S*) AND (*w* is *M*) *THEN* change in inertia weight (Δw) is *NE.*

The fuzzy rules are designed to determine the change in inertia weight (Δw). As three linguistic variables are considered for the NFV, *w* and Δw, total nine rules are designed as shown in Table 1. Each rule represents a mapping from the input space to the output space.

(*iii*) *Fuzzy reasoning*: The fuzzy control strategy is used to map the inputs to the output. The AND operator is typically used to combine the membership values for each fired rule to generate the membership values for the fuzzy sets of output variables in the consequent part of the rule. Since there may be other rules fired in the rule sets, for some fuzzy sets of the output variables there may be different membership values obtained from different fired rules.

Table 1
Fuzzy rules for the variation of inertia weight (*w*)

Rule No	Antecedent		Consequent
	NFV	*w*	Δw
1	*S*	*S*	*ZE*
2	*S*	*M*	*NE*
3	*S*	*L*	*NE*
4	*M*	*S*	*PE*
5	*M*	*M*	*ZE*
6	*M*	*L*	*NE*
7	*L*	*S*	*PE*
8	*L*	*M*	*ZE*
9	*L*	*L*	*NE*

To obtain a better inertia weight under the fuzzy environment, the variables selected as input to the fuzzy system are the current best performance evaluation (*NFV*) and current inertia weight (*w*); whereas the output variable is the change in inertia weight (Δw). The NFV is defined as;

$$NFV = \frac{FV - FV_{\min}}{FV_{\max} - FV_{\min}} \quad (11)$$

The Fitness Value (*FV*) calculated from Eq. (3) at the first iteration may be used as FV_{min} for the next iterations. Whereas FV_{max} is a very large value and is greater than any acceptable feasible solution. Typical inertia weight is $0.4 \le w \le 1.0$. Both positive and negative corrections limits are required for the inertia weight. Therefore, a correction range of -0.1 to +0.1 has been chosen for the inertia weight correction.

$$w^{t+1} = w^{t} + \Delta w \tag{12}$$

(*iv*) *Defuzzification*: defuzzification of every input and output, the method of centroid (center-of-sums) is used for the membership functions shown in Figure 1.

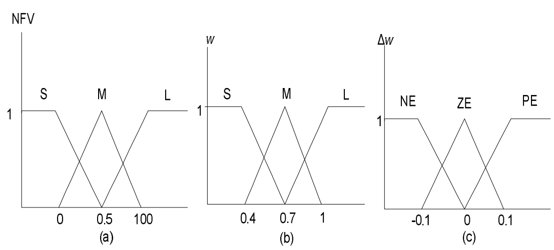

Fig.1 Membership functions of input variables (a) Normalized Fitness Value (NFV) (b) Inertia weight (*w*) output variable (c) change in inertia weight (Δw)

3.1FAPSO algorithm for bidding problem

In the optimal bidding problem, each particle is composed of the strategic variable. For the considered supplier in linear bid model using probability density function the position of *r* represents the optimum value of b_j. For each generated particle, profit maximization objective function of Eq. (3) is taken as a fitness function. $P_{best\ r}$ represents the best position of the particle *r* and the best position reached by the swarm G_{best} in the final iteration gives optimal value of strategic variable. The computational steps for searching bidding coefficients using FAPSO algorithm are described below.

Step1. Read input data μ σ , ρ , *b* and maximum iterations. // where μ = mean, σ = standard deviation, ρ = correlation coefficient of Eq. (8), b = cost coefficient//

Step2.The initial population and initial velocity for each particle should be generated randomly.

Step3. The objective function is to be evaluated for each individual using Eq. (8).

Step4. The individual that has the minimum objective function should be selected as the global position.

Step5. The r^{th} individual is selected.

Step6. The best local position (P_{best}) is selected for the r^{th} individual.

Step7. Update the FAPSO parameter (*w*) using fuzzy *IF/THEN* rules.

Step8. Calculate the next position for each individual based on the inertia weight (*w*) of Eq. (9) and then checked with its limit.

Step9. If all individuals are selected, go to the next step, otherwise $k=k+1$ and go to step5.

Step10. If the current iteration number reaches the predetermined maximum iteration number, the search procedure is stopped, otherwise go to step 2.

Step11.The last G_{best} (i.e. b_j) is the solution of the problem

Step12. Calculate the optimal bid prices of each block and run the unit commitment for different loads.

Step13. Calculate profit of each supplier using Eq. (3).

The flowchart for the proposed algorithm shown in Figure 2

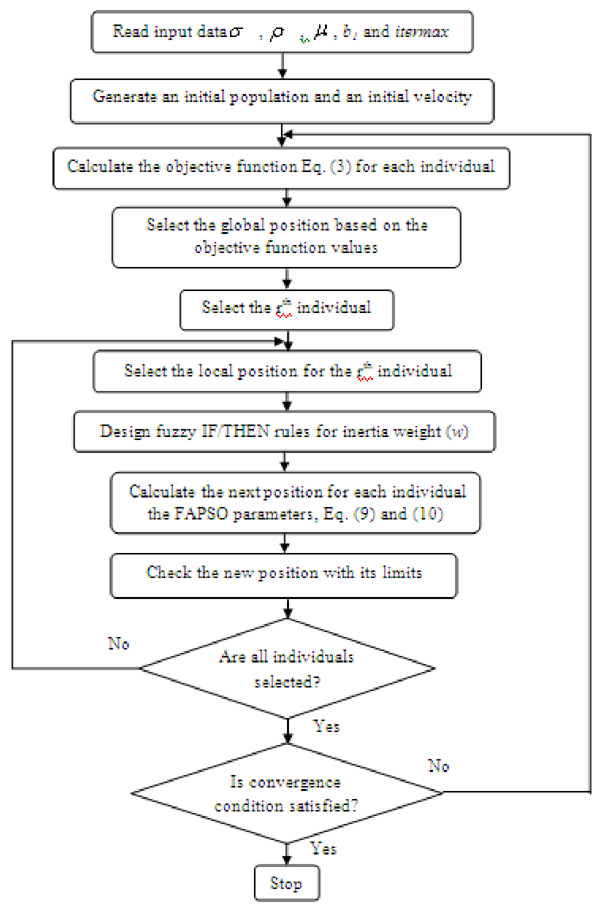

Fig. 2 Flowchart for the proposed FAPSO algorithm

4 Results and discussions

The effectiveness of the proposed FAPSO has been demonstrated considering a numerical example based on problem formulated in the previous section. The problem is formulated in a dynamically changing environment and bidding strategies are developed for multi-hourly trading in a day-ahead market. In this work, the parameters used for FAPSO, IWAPSO, VURPSO and GA are shown in Table 2. Simulations are carried on 2.66GHz, PIV Processor, 3GB RAM and MATLAB 7.8 version is used. *c1*, *c2* learning factors; *w* inertia weight; *mf* momentum factor for PSO; P_e elitism probability; P_c crossover probability; P_m mutation probability; *l* chromosome length for GA.

Normal probability distribution parameters of rivals' block bid prices in Table 3 are taken in such a manner that each block has a unique *pdf*. The parameters of all three power blocks of generator-G are shown in Table 4. At the start of first hour, blocks 1, 2, and 3 of generator-G are assumed to be ON, ON, and OFF, respectively. The bid price for each hour and each block of generator-G is obtained using FAPSO algorithm.

Table 2
Parameter values used for different approaches

FAPSO	IWAPSO	VURPSO	GA
No. of particles= 40; Max. iterations= 100; *c1*=3.5, *c2*=2.5; *w*= 0.4 to 1.0	No. of particles= 40; Max. iterations= 100; *c1*=3.5,*c2* =2.5; *w*=1.0 to 0.5	No. of particles= 40; Max. iterations= 100; *c1*, *c2* vary from 6.5 to1.5; *mf*=0.3	Population size = 40; Generations=100 ; P_e =0.1; P_c=0.8; P_m=0.001, *l*=16

Table 5 shows the optimal bid price for all generators for each block, the average of all bid prices for each block is known as ISO (Independent System Operator) MCP. Run the unit commitment for all the generators based on this MCP and then units are committed/de-committed accordingly. Table 6 shows the dispatched and non-dispatched powers of generator-G and rivals during each trading period. The following observations can be made from the results shown in Table 6.

- Block 3 of generator-G is not committed in the hours of negative benefit (from 1 to 8 h) because of its high production cost and low system demand.
- Cold start-up cost is accounted in the production cost of block 3, when it is committed at 9 h, because it has been shut down for a long time (8 h).
- At the end of 12 h, block 3 is decommitted due to low system demand, and minimum down time constraint is active on 13 h.
- Block 3 is recommitted at 14 h, and hot start-up cost is accounted in the production cost of this hour, because it has been shut down for a short time (1h).
- Block 3 is again decommitted from 20 to 24 h due to low system demand.

Figure 3 and 4 shows hourly and cumulative profit curves of generator-G, where cumulative profit is the aggregation of hourly profit. At hour 8 and 13, a sharp increase in hourly profit 22,450.853 and 48,118.080 is directly related to sudden rise in bid price to 13.06 to 21.535 $/MWh, respectively. This sharp increase is followed by a sudden decrease in hourly profit to $ 20855.565 and 42,564.080 at hours 9 and 14, respectively because of the high start-up cost of block 3, which is committed at these hours. In spite of marginal difference in bid price at 17 and 18 h, a huge increase in profit from $6340.166 to $74163.55 is obtained by decreasing dispatched power output of block 3 of generator-G from 200 to 50 MW and, therefore, the production cost. The cumulative profit of generator-G over 24-h period using FAPSO approach is $94,178.

Due to the randomness of the evolutionary algorithms, their performance cannot be judged by the result of a single run. Many trails with different initializations should be made to reach a valid conclusion about the performance of the algorithms. An algorithm is robust, if it can guarantee an acceptable performance level under different conditions. In this paper, 10 different runs have been carried out. The best, worst, average values, total profit and PD over a period found by all the methods are shown in Table 7.

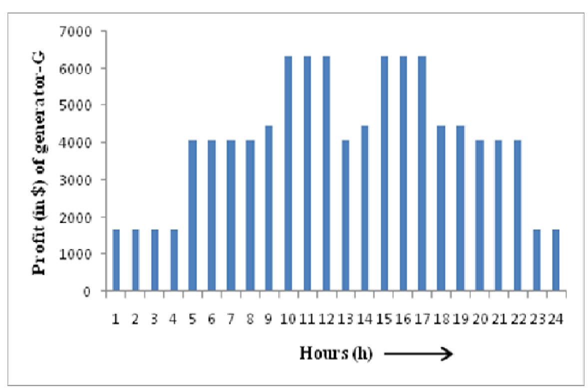

Fig.3 Hourly profit of generator-G

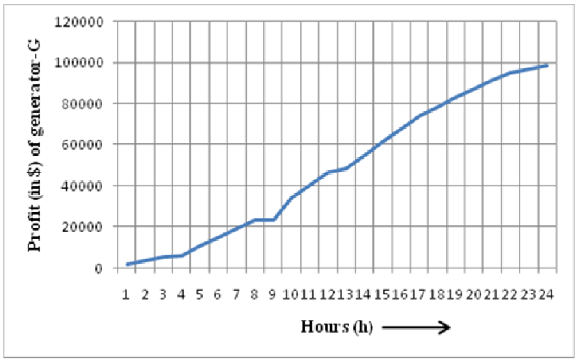

Fig.4 Cumulative profit of generator-G

The Percentage Deviation (PD) is computed as follows.

$$PD = \frac{(Best - Worst)}{Best} \times 100\% \cdot$$

Table 7 shows that the PD is minimum for the proposed FAPSO method compared to GA and different versions of PSO, for a given data and it is clearly observed that the optimal bidding strategies obtained by FAPSO producing higher profits compared to GA and different versions of PSO. In addition to that, FAPSO shows good consistency by keeping small variation between the best and worst solution. In other words, the simulation results show that, the FAPSO algorithm converges to global solution has a shorter c.p.u. time and small percentage deviation because, it can easily follow the frequently changing demand in each trading period through dynamically changing inertia weight using fuzzy *IF/THEN* rules. As a result, the final solution lands at global optimum, which avoids premature convergence and permits a faster convergence.

5 Conclusion

In this paper, the application of FAPSO was proposed to obtain bidding strategies for Gencos in a day-ahead electricity market with an objective of maximizing total profit considering unit commitment and valve point effects in the cost function. PSO is an efficient tool for solving complex optimization problems. The results of the PSO are greatly dependent on the inertia weight and the method often suffers from the problem of being trapped in local optima. To overcome these drawbacks of PSO, the inertia weight has been dynamically adjusted in FAPSO using fuzzy "*IF/THEN*" rules. The numerical results reveal the superiority of the proposed FAPSO compared to GA and different versions of PSO with respect to total profit and convergence. The result shows that, the proposed algorithm produces more profit and rapid convergence. Hence it can be used for real-time applications.

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Table 3
Data of rivals' bidding parameters

	Block 1 (i=1)			Block 2 (i=2)			Block 3(i=3)		
	Q_i^n (MW)	μ_i^n ($/MWh)	σ_i^n ($/MWh)	Q_i^n (MW)	μ_i^n ($/MWh)	σ_i^n ($/MWh)	Q_i^n (MW)	μ_i^n ($/MWh)	σ_i^n ($/MWh)
Rival 1 (n=1)	200	10	2.5	300	20	3	400	30	3
Rival 2 (n=2)	300	15	3	400	40	2	500	50	4
Rival 3(n=3)	250	10	2	300	15	2.5	300	20	2.5
Rival 4(n=4)	300	20	4	350	25	5	450	40	5

Table 4
Data of generator-G power blocks

	a1 ($/MW^2/h)	b1 ($/MWh)	c1 ($/h)	c2 ($/h)	c3 (rad/MW)	q^{max} (MW)	q^{min} (MW)	MUT (h)	MDT (h)	h ($)	δ ($)	τ (h)	c_i^d ($)
Block 1	0.00482	7.97	78	150	0.063	200	50	1	1	1000	1500	1	100
Block 2	0.00194	15.85	310	200	0.042	400	100	2	1	1500	2500	1	200
Block 3	0.00156	32.92	561	300	0.0315	600	100	4	2	2000	4000	8	400

Table 5
Optimal bid prices (in $/MWh) of generators for each block

	Rival 1	Rival 2	Rival 3	Rival 4	Generator-G	MCP (ISO)
Block 1	9.9838	15.00	10.00	20.044	10.291	13.06
Block 2	19.991	40.20	14.99	24.99	17.65	21.535
Block 3	29.978	50.00	20.00	39.99	34.059	34.808

Table 6
Dispatched power output of generator-G and rivals

Load (MW)	Hour	Generator-G			Rival-1			Rival-2			Rival-3			Rival-4		
		Block 1	Block 2	Block 3	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3	Block 1	Block 2	Block 3
1500	1	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND
1500	2	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND
1500	3	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND
1500	4	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND
2000	5	200	400	ND	200	50	ND	300	ND	ND	250	300	ND	300	ND	ND
2000	6	200	400	ND	200	50	ND	300	ND	ND	250	300	ND	300	ND	ND
2000	7	200	400	ND	200	50	ND	300	ND	ND	250	300	ND	300	ND	ND
2500	**8**	200	400	**ND**	200	300	ND	300	ND	ND	250	300	250	300	ND	ND
3000	9	200	400	50	200	300	400	300	ND	ND	250	300	300	300	ND	ND
3500	10	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
3500	11	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
3500	12	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
2500	**13**	200	400	**ND**	200	300	ND	300	ND	ND	250	300	250	300	ND	ND
3000	14	200	400	50	200	300	400	300	ND	ND	250	300	300	300	ND	ND
3500	15	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
3500	16	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
3500	17	200	400	200	200	300	400	300	ND	ND	250	300	300	300	350	ND
3000	18	200	400	50	200	300	400	300	ND	ND	250	300	300	300	ND	ND
3000	19	200	400	50	200	300	400	300	ND	ND	250	300	300	300	ND	ND
2500	**20**	200	400	**ND**	200	300	ND	300	ND	ND	250	300	250	300	ND	ND
2000	21	200	400	ND	200	50	ND	300	ND	ND	250	300	ND	300	ND	ND
2000	22	200	400	ND	200	50	ND	300	ND	ND	250	300	ND	300	ND	ND
1500	23	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND
1500	24	200	400	ND	200	150	ND	ND	ND	ND	250	300	ND	ND	ND	ND

Table 7
Performance comparison of different approaches

		FAPSO	IWAPSO	VURPSO	GA
Total profit ($)	Best($)	94178	92086	91672	91468
	Worst($)	93265	90869	89347	88339
	Ave.($)	93721	91477	90509	89903
	PD (%)	0.009	0.013	0.025	0.034
Average c.p.u. time (sec)		0.270	0.446	0.648	2.33