Excitation Control of the Synchronous Machines with use an Error Function of the Complex Argument.

Oleg N. Agamalov
Electrical Department
Tashlyk Pumped-Storage Power Plant
UKRAINE
olegagamalov@gmail.com, aon@ukrsat.mk.ua

Abstract: - in paper the new structure of the automatic voltage regulator (AVR) and the power system stabilizer (PSS) of the synchronous machines is considered. It is known, that the primary goal, that solved AVR are maintenance of a terminal voltage of the synchronous machine according to the reference. For PSS the goal is mitigation the electromechanical oscillations of the synchronous machine at various disturbances. For these purposes as inputs of AVR and PSS the set of state variables of the synchronous machine is used: terminal voltage, armature current, frequency, speed, power, etc. The key feature of offered excitation control system is use only the phasor of terminal voltage for the decision of all above-stated problems, that considerably reduces influence of noises and disturbances, simplifies its design and operation.

Key-Words: - Hilbert transform, error function of the complex argument (EFCA), Wirtinger calculus, differential of EFCA, small-signal stability, transient stability.

1 Introduction

Now, for excitation control of synchronous machines, the fundamental role played the feedback at terminal voltage [1]. Excitation control is based on the calculation the scalar error between the setpoint (reference) V_{ref} and the measured terminal voltage V_t. The resulting scalar error $\Delta V = V_{ref} - V_t$ used as the input of the automatic voltage regulator (AVR), that is PID controller or its truncated variants (PD, PI) [2], for field current control. The purpose of this regulation is the exact maintenance V_t in accordance with V_{ref} in all possible operating modes. That determined the large values of the proportional gain of the AVR ($K_P \geq 25$). However, as has been determined theoretically and observed in practice [1], in some operation modes of synchronous machines, this leads to a decrease in damping component T_D of the electric moment T_E, proportional to the deviation of the speed $\Delta\omega$. This is causing the rotor oscillations of synchronous machines at disturbances in the grid. Therefore, in addition to the AVR, using feedback to the parameters that reflect the rotor motion – speed, frequency, accelerating power, which used in power system stabilizer (PSS). This greatly complicates the development, subsequent coordination of AVR/PSS and commissioning of the excitation control of the synchronous machines. So, promising is the development of the excitation control, which realizes a new type of the feedback, which reflects the states of the terminal voltage and the rotor motion.

Paper includes sections with the following contents. Section II defines the two-dimensional error function of the complex argument (EFCA) of the terminal voltage phasor and a method of it measuring. This method based on the representation the inputs-outputs of the AVR as analytical signals. Consider the definition of the normal EFCA (NEFKA) for steady state of the synchronous machines. Presented the calculation of the differential EFCA at phasor deviations with use the Wirtinger calculus, so as shown in the paper, EFCA is not analytic function in the whole domain of the terminal voltage phasor. In section III showed the excitation control of the synchronous machine that used EFCA at terminal voltage phasor. Impact this excitation control on the synchronising and the damper components of the electric moment of a synchronous machine for model "synchronous machine-infinite bus" is considered. The results of simulations for study the small-signal and transient stability with proposed excitation control are given in the section IV. The conclusions are summarized the contents of this paper.

2 Error Function of the Complex Argument (EFCA).

The inertia, physical principles of the plant, as well as, the limited speed of the signal propagation is causing the phase delays, observed between the inputs, state variables and outputs of a control system. For error, depending on the phase lag of the plant output, we formulate:

Definition 1. Error function of the complex argument (FFCA) is the difference between the input (set point) $\bar{r}$ and the output $\bar{y}$ of the plant, calculated on the complex plane C, for it the real axis R is determined by the direction of a complex vector $\bar{r}$, and the imaginary axis I shifted an angle $\pi/2$ in the Cartesian coordinate system. That allows to take into account the phase lag deviation of the plant complex output at violation the steady state.

EFCA can be represented in vector or complex form: $\bar{e} = \bar{r} - \bar{y} = e_R + je_I$, where $j = \sqrt{-1}$ - imaginary unit, e_R - real (in-phase with the setpoint), and e_I - imaginary (quadrature setpoint) components, fig. 1:

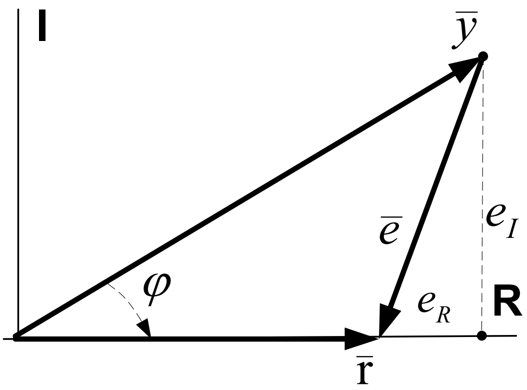


Fig.1. Error Function of the Complex Argument (EFCA)

$$\begin{aligned} &\bar{e} = \bar{r} - \bar{y} = e_R + je_I = r - \left(y\cos(\varphi) + jy\sin(\varphi)\right), \\ &e_R = r - (y \cdot \cos(\varphi)); \\ &e_I = -y \cdot \sin(\varphi). \end{aligned} \quad (1)$$

In the steady state, between the input and output of the plant, there is some constant phase delay, defining steady states of the inertial elements within its structure. That state corresponds to the normal error function of the complex argument (NEFCA), fig. 2:

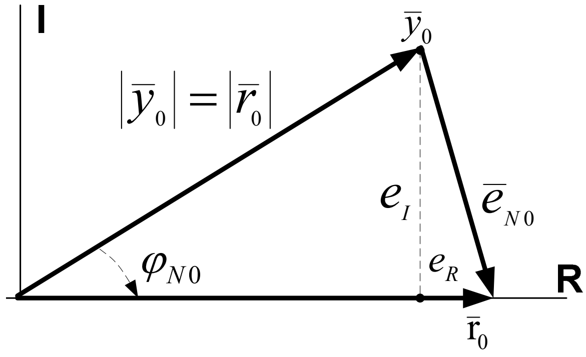


Fig.2. Normal Error Function of the Complex Argument (NEFCA)

Definition 2. Normal EFCA (NEFCA) exist in the steady state (equilibrium point) of the plant and characterized by equality the magnitudes $|\bar{y}_0| = |\bar{r}_0|$ and the normal phase lag φ_{N0}, defined the steady states of the inertial elements within its structure.

At violation the steady state, occurs the deviations the magnitude and phase lag of the plant output. The deviation of the phase lag $\Delta\varphi$ is calculated, using the Hilbert transform (*HT*) [3, 4], defining the input and output as analytical signals, representing a complex sum of the two orthogonal signals. The imaginary part of the analytical signal $\operatorname{Im} Z_s(t) = \tilde{s}(t)$ or the Hilbert transform *HT* of the real signal $s(t)$ is determined by its convolution with the function $1/\pi t$:

$$\tilde{s}(t) = HT[s(t)] = s(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau \quad (2)$$

Then, the analytical signal:

$$\begin{aligned} &Z_s(t) = s(t) + j \cdot \tilde{s}(t) = S(t)e^{ja(t)}, \\ &S(t) = \sqrt{s^2(t) + \tilde{s}^2(t)}, a(t) = arctg\frac{\tilde{s}(t)}{s(t)} \end{aligned} \quad (3)$$

and the difference between instantaneous phases of the two arbitrary signals [5, 6] is:

$$\Delta\varphi_{yr} = \varphi_y(t) - \varphi_r(t) = arctg\frac{\tilde{y}(t)\cdot r(t) - y(t)\cdot\tilde{r}(t)}{y(t)\cdot r(t) + \tilde{y}(t)\cdot\tilde{r}(t)} \quad (4)$$

So, for steady state, the measured input $r(t)$ and output $y(t)$ are constant, and taking into account the properties of the Hilbert transform [4], the measured deviation is $\Delta\varphi_{yr} = 0, \varphi_{yr} = \varphi_{N0}$. In transients the deviation of the phase delay occurs $\Delta\varphi_{yr} \neq 0$ and defines the structure of the EFCA. The deviation of the phase lag in the stable transients $|\Delta\varphi| < \pi$, there is no need to use the unwrap-function in (4).

Next, we consider the application of the EFCA for excitation control of the synchronous machines, using the vector diagram, Fig. 3:

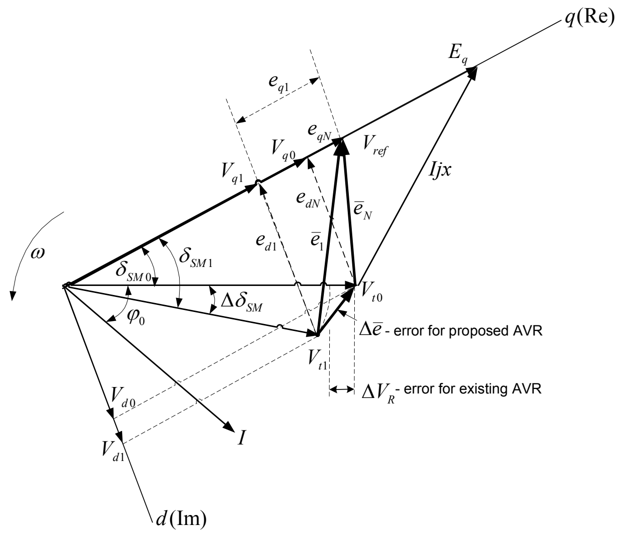


Fig. 3. Vector diagram of the synchronous machine with EFCA.

In the steady state, the setpoint phasor V_{ref} coincides with the synchronous e.m.f. E_q (neglected the delay in the excitation system). The magnitudes of the phasors setpoint and terminal voltage are equal $|V_{ref}| = |V_t|$, but shifted by the rotor angle δ_{SM0}. Thus, it may be determined the NEFCA for steady state of the synchronous machine with terminal voltage $V_{t0} = V_{q0} + jV_{d0}$ as:

$$\begin{aligned}\bar{e}_{N0} &= \overline{V}_{ref} - \overline{V}_{t0} = V_{ref} - (V_{t0}\cos\delta_{SM0} + jV_{t0}\sin\delta_{SM0}) \\ &= (V_{ref} - V_{q0}) - jV_{d0} = e_{qN} + je_{dN}\end{aligned} \tag{5}$$

NEFCA (5) in accordance with the definition 2, characterized the equality of magnitudes the terminal voltage and setpoint of the excitation system and stationary phase state of the inertial element – the rotor of the synchronous machine. At steady state violation due to disturbances in the power system, EFCA $\bar{e}_1 \neq \bar{e}_N$ is:

$$\begin{aligned}\bar{e}_1 &= \overline{V}_{ref} - \overline{V}_{t1} = V_{ref} - (V_{t1}\cos\delta_{SM1} + jV_{t1}\sin\delta_{SM1}) \\ &= (V_{ref} - V_{q1}) - jV_{d1} = e_q + je_d\end{aligned} \tag{6}$$

The deviation of EFCA is:

$$\begin{aligned}\Delta\bar{e} &= \bar{e}_N - \bar{e}_1 = (V_{t1}\cos\delta_{SM1} - V_{t0}\cos\delta_{SM0}) + \\ &+ j(V_{t1}\sin\delta_{SM1} - V_{t0}\sin\delta_{SM0}) = (V_{q1} - V_{q0}) + j(V_{d1} - V_{d0})\end{aligned} \tag{7}$$

EFCA (7) reflects the terminal voltage ΔV_t and rotor angle $\Delta\delta_{SM}$ deviations, and differs from the traditional definition of the error in the AVR:

$$\begin{aligned}\Delta e &= |\overline{V}_{t0} - \overline{V}_{t1}| = \sqrt{V_{ref}^2 + V_{t1}^2 - 2V_{ref}V_{t1}\cos(\Delta\delta_{SM})} = \\ &= \sqrt{(V_{q1} - V_{q0})^2 + (V_{d1} - V_{d0})^2}, \\ \Delta V_t &= V_{ref} - V_{t1} = V_{t0} - V_{t1} = \sqrt{V_{q0}^2 + V_{d0}^2} - \sqrt{V_{q1}^2 + V_{d1}^2}\end{aligned} \tag{8}$$

Next we consider the excitation control with EFCA at terminal voltage phasor and its impact on the small-signal stability for model "synchronous machine -infinite bus" (SMIB).

3 EFCA Excitation Control.

If we use the terminal voltage phasor as the input for excitation control, we can react to changes in both the electromagnetic and electromechanical states of a synchronous machine, as take into account the increments of the terminal voltage and rotor angle in case of violation the steady state. Control of various plants based on the real signals. Therefore, EFCA should be considered as a real function of the complex arguments - the output of the plant. Accordingly, for excitation control, EFCA should be considered as a real function of a complex argument – the terminal voltage phasor. Also, based on the definition 2, it follows that error function between setpoint and the output is always there, even in the steady state, and accordingly the control should be implemented not by the absolute value of EFCA, but to change it. Then, in case the steady state violation of the synchronous machine:

$$\frac{d(\Delta\bar{e})}{dt} = \frac{\partial(\Delta e(\overline{V}_t))}{\partial\overline{V}_t}\frac{d\overline{V}_t}{dt} \tag{9}$$

where $\frac{\partial(\Delta e(\overline{V}_t))}{\partial \overline{V}_t}$ - gradient of the real EFCA at terminal voltage phasors, $\frac{d\overline{V}_t}{dt} = \dot{\overline{V}}_t$ - derivative of the terminal voltage phasors. It is known [5], that the nonconstant real-valued function of a complex variable is nonanalytic and therefore does not differentiate in the accepted sense for the complex variables (Cauchy-Riemann conditions). We prove this statement with respect to the gradient EFCA (9) to introduce the notation for conjugate terminal voltage phasor in the steady state:

$$\overline{V}_{t0} = V_{t0}\cos\delta_{SM0} + jV_{t0}\sin\delta_{SM0} = V_{q0} + jV_{d0}$$
$$\widetilde{V}_{t0} = V_{t0}\cos\delta_{SM0} - jV_{t0}\sin\delta_{SM0} = V_{q0} - jV_{d0} \qquad (10)$$

Then, the following theorem can be proved:

Theorem 1. EFCA is a nonanalytic function for entire domain, where define the complex output of the plant.

Proof: We perform a direct method, verify the feasibility of the Cauchy-Riemann equations [5, 6] for EFCA (6):

$$\frac{\partial e_q(V_q,V_d)}{\partial V_q} = \frac{\partial e_d(V_d,V_q)}{\partial V_d} \rightarrow$$
$$\rightarrow \frac{\partial(V_{ref} - V_t\cos\delta_{SM})}{\partial(V_t\cos\delta_{SM})} = -1 = -1 = \frac{\partial(-V_t\sin\delta_{SM})}{\partial(V_t\sin\delta_{SM})},$$
$$\frac{\partial e_q(V_q,V_d)}{\partial V_d} = -\frac{\partial e_d(V_d,V_q)}{\partial V_q} \rightarrow$$
$$\rightarrow \frac{\partial(V_{ref} - V_t\cos\delta_{SM})}{\partial(V_t\sin\delta_{SM})} = tg\delta_{SM} \neq$$
$$\neq -ctg(\delta_{SM}) = -\frac{\partial(-V_t\sin\delta_{SM})}{\partial(V_t\cos\delta_{SM})} \qquad (11)$$

The second equation of system (11) shows, that for EFCA, Cauchy-Riemann conditions are not met. The gradient (9) has the "anisotropy" at change the direct and quadrature components of the terminal voltage phasor.■

To calculate the gradient and differential EFCA (9) in the whole domain of the terminal voltage phasor, we introduce the definition of a real differentiable [6] EFCA:

Definition 3: EFCA called real differentiable if its components on the direct and quadrature axes defined as differentiable functions of real variables $V_q = \frac{\overline{V}_t + \widetilde{V}_t}{2}$ and $V_d = \frac{\overline{V}_t - \widetilde{V}_t}{2j}$ where $\overline{V}_t = V_q + jV_d = V_t\cos\delta_{SM} + jV_t\sin\delta_{SM} = V_t e^{j(\delta_{SM})}$, $\widetilde{V}_t = V_q - jV_d = V_t\cos\delta_{SM} - jV_t\sin\delta_{SM} = V_t e^{j(-\delta_{SM})}$ and

$$\overline{e}(\overline{V}_t) = e_q(V_q,V_d) + je_d(V_d,V_d) = = e_q\left(\frac{\overline{V}_t + \widetilde{V}_t}{2}\right) + je_d\left(\frac{\overline{V}_t - \widetilde{V}_t}{2j}\right)$$

Thus, EFCA of control system is considered as a map $e: C \mapsto R \times R = R^2$ and can use the properties of the real domain R^2 for the proof [6]:

Theorem 2. Let EFCA $e: C \mapsto R \times R = R^2$ is determined by real variables: $V_q = \frac{\overline{V}_t + \widetilde{V}_t}{2}$ and $V_d = \frac{\overline{V}_t - \widetilde{V}_t}{2j}$ so, that $e = f(V_q,V_d) = f(\overline{V}_t,\widetilde{V}_t)$, where $\overline{V}_t$ and $\widetilde{V}_t$ complex and complex-conjugate, independent terminal voltage phasors. Then:

1) the Wirtinger derivatives [5, 6] of EFCA is:

$$\frac{\partial(\Delta e(\overline{V}_t))}{\partial \overline{V}_t} = \frac{1}{2}\left(\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q} - j\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\right) = = \frac{1}{2}\left[\left(\frac{\partial(\Delta e_q)}{\partial V_q} + \frac{\partial(\Delta e_d)}{\partial V_d}\right) + j\left(\frac{\partial(\Delta e_d)}{\partial V_q} - \frac{\partial(\Delta e_q)}{\partial V_d}\right)\right];$$
$$\frac{\partial(\Delta e(\overline{V}_t))}{\partial \widetilde{V}_t} = \frac{1}{2}\left(\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q} + j\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\right) = = \frac{1}{2}\left[\left(\frac{\partial(\Delta e_q)}{\partial V_q} - \frac{\partial(\Delta e_d)}{\partial V_d}\right) + j\left(\frac{\partial(\Delta e_d)}{\partial V_q} + \frac{\partial(\Delta e_q)}{\partial V_d}\right)\right] \qquad (12)$$

where:

$$V_q = V_t\cos(\Delta\delta_{SM}), V_d = V_t\sin(\Delta\delta_{SM}),$$
$$\Delta e_q = V_{ref} - V_t\cos(\Delta\delta_{SM}), \Delta e_d = -V_t\sin(\Delta\delta_{SM}).$$

2) necessary and sufficient conditions for a stationary point (equilibrium point) $(\overline{V}_{ref}, \overline{V}_{t0}, \overline{e}_{N0})$ of the excitation control system is:

$$\frac{de(\overline{V}_{ref},\overline{V}_{t0},\widetilde{V}_{t0})}{d\overline{V}_t}=0,\frac{de(\overline{V}_{ref},\overline{V}_{t0},\widetilde{V}_{t0})}{d\widetilde{V}_t}=0 \quad (13)$$

Proof: Using the definition of a real differentiable EFCA and the chain rule of differentiation of a composite function, we obtain:

$$\frac{d(\Delta e(\overline{V}_t))}{d\overline{V}_t}=\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q}\frac{\partial V_q}{\partial \overline{V}_t}+\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\frac{\partial V_d}{\partial \overline{V}_t}=$$
$$\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q}\frac{1}{2}+\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\frac{1}{2j}= ,$$
$$=\frac{1}{2}\left(\frac{\partial(\Delta e(\overline{V}_t)}{\partial V_q}-j\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\right)$$

$$\frac{d(\Delta e(\overline{V}_t))}{d\widetilde{V}_t}=\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q}\frac{\partial V_q}{\partial \widetilde{V}_t}+\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\frac{\partial V_d}{\partial \widetilde{V}_t}=$$
$$\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_q}\frac{1}{2}-\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\frac{1}{2j}=$$
$$=\frac{1}{2}\left(\frac{\partial(\Delta e(\overline{V}_t)}{\partial V_q}+j\frac{\partial(\Delta e(\overline{V}_t))}{\partial V_d}\right)$$

Main requirement, necessary for calculation the derivatives of EFCA (12) is the independence the complex and the complex conjugate terminal voltage phasor. ■

For entire complex domain C we define the following equivalence:

$$(\overline{V}_t=V_q+jV_d)\in C\leftrightarrow U=\begin{pmatrix}V_q\\V_d\end{pmatrix}\in R^2 \quad (14)$$

where U - real vector, formed by the direct and quadrature components of the terminal voltage phasor, whose values allow to define his complex conjugate:

$$c=\begin{pmatrix}\overline{V}_t\\\widetilde{V}_t\end{pmatrix}=\begin{pmatrix}V_q+jV_d\\V_q-jV_d\end{pmatrix}\in C^2\cong R^4 \quad (15)$$

Thus, we have three vector spaces, for representation the terminal voltage phasor of the synchronous machine:

A) complex vectors $\overline{V}_t\in C$ (traditional for phasor);

B) components of the real vectors $U\in R^2$;

C) complex and complex conjugate terminal voltage phasor $c\in C^2\cong R^4$.

We define a bijection (isomorphism) between sets (14) (15):

$$\begin{pmatrix}\overline{V}_t\\\widetilde{V}_t\end{pmatrix}=\begin{pmatrix}1&j\\1&-j\end{pmatrix}\begin{pmatrix}V_q\\V_d\end{pmatrix}=J\begin{pmatrix}V_q\\V_d\end{pmatrix},J=\begin{pmatrix}1&j\\1&-j\end{pmatrix},c=JU \quad (16)$$

The inverse matrix of the coordinate transform:

$$J^{-1}=\frac{1}{2}\begin{pmatrix}1&1\\j^{-1}&-j^{-1}\end{pmatrix}\to U=J^{-1}c \quad (17)$$

Then can be defined Jacobians:

$$J_c\equiv\frac{\partial}{\partial U}c=\frac{\partial}{\partial U}JU=J,J_U=J_c^{-1}=J^{-1} \quad (18)$$

This allows us to determine the differentials of the terminal voltage phasor as:

$$dc=\frac{\partial c}{\partial U}dU=J_cdU=JdU,$$
$$dU=\frac{\partial U}{\partial c}dc=J_Ydc=J^{-1}dc \quad (19)$$

So as: $\frac{\partial}{\partial c}=\left(\frac{\partial}{\partial \overline{V}_t}\ \frac{\partial}{\partial \widetilde{V}_t}\right)^T,\Delta c=\begin{pmatrix}\Delta\overline{V}_t\\\Delta\widetilde{V}_t\end{pmatrix}$, then for differential of the real EFCA we obtain:

$$\frac{\partial(\Delta\overline{e}(c))}{\partial c}\Delta c=\frac{\partial(\Delta\overline{e})}{\partial\overline{V}_t}\Delta\overline{V}_t+\frac{\partial(\Delta\overline{e})}{\partial\widetilde{V}_t}\Delta\widetilde{V}_t=$$
$$=\frac{\partial(\Delta\overline{e})}{\partial\overline{V}_t}\Delta\overline{V}_t+\overbrace{\frac{\partial(\Delta\overline{e})}{\partial V_t}\Delta V_t}^{\sim}=2\,\mathrm{Re}\left\{\frac{\partial(\Delta\overline{e})}{\partial\widetilde{V}_t}\Delta\overline{V}_t\right\} \quad (20)$$

Then, the differential of the real EFCA is:

$$d(\Delta\overline{e})=2\,\mathrm{Re}\left\{\frac{\partial(\Delta e(\overline{V}_t))}{\partial(\widetilde{V}_t)}\Delta\overline{V}_t\right\}=$$
$$=2\,\mathrm{Re}\left\{\frac{1}{2}\left(\frac{\partial(\Delta e(\overline{V}_t))}{\partial(V_q)}+j\frac{\partial(\Delta e(\overline{V}_t))}{\partial(V_d)}\right)\Delta\overline{V}_t\right\}=$$
$$=\mathrm{Re}\left\{\begin{array}{l}j(ctg(\Delta\delta_{SM})+tg(\Delta\delta_{SM}))\cdot\\\cdot((V_t\cos(\Delta\delta_{SM})-V_{ref})+\\+j(V_t\sin(\Delta\delta_{SM})))\end{array}\right\}= \quad (21)$$
$$=-\frac{2\cos(\Delta\delta_{SM})}{1+\cos(2\Delta\delta_{SM})}V_t.$$

So, the excitation control at EFCA is:

$$u = K\left(V_{ref} - \frac{2\cos(N\Delta\delta_{SM})}{1+\cos(2N\Delta\delta_{SM})}V_t\right) \qquad (22)$$

where K - gain at deviation of the magnitude terminal voltage phasor, and N - gain at deviation of the rotor angle.

We next consider the application of the excitation control (22) for SMIB model (model Heffron - Phillips [1]) with static excitation system (type ST2A standard IEEE Std3.421.5-2005 [2]). Block diagram of the model shown in Fig. 4:

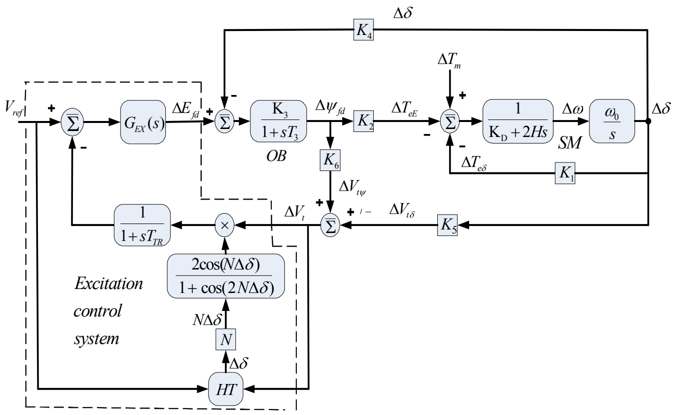


Fig. 4. Block diagram of SMIB-model with EFCA.

In this diagram, the notation of the state variables, coefficients and time constants in accordance with [1] and in addition the dotted line marked the excitation control system of synchronous machine which are designated: T_{TR} - summary time constant of the excitation control system, HT - block, that performs a Hilbert transform for the signals setpoint and the terminal voltage and then calculate the deviation of the rotor angle $\Delta\delta$, N - stabilizing coefficient, the purpose of which will be discussed below, $G_{EX}(s) = K_A$ - coefficient of the static excitation system. At constant mechanical torque of a synchronous machine $\Delta T_m = 0$ and the setpoint $\Delta V_{ref} = 0$, model in the state space $\{\Delta\omega \quad \Delta\delta \quad \Delta\psi_{fd} \quad \Delta V_t\}$ is [1]:

$$\begin{bmatrix}\Delta\dot{\omega}\\ \Delta\dot{\delta}\\ \Delta\dot{\psi}_{fd}\\ \Delta\dot{V}_t\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0\\ a_{21} & 0 & 0 & 0\\ 0 & a_{32} & a_{33} & a_{34}\\ 0 & a_{42} & a_{43} & a_{44}\end{bmatrix}\cdot\begin{bmatrix}\Delta\omega\\ \Delta\delta\\ \Delta\psi_{fd}\\ \Delta V_t\end{bmatrix} \qquad (23)$$

The increment of the complete electrical moment ΔT_e of a synchronous machine in case of violation the steady state is determined by three components - synchronising torque $\Delta T_{e\delta}$, proportional to the deviation of the rotor angle, the torque ΔT_{eE}, proportional to the change of the field flux linkage $\Delta\psi_{fd}$ and damper torque ΔT_D, proportional to the speed change $\Delta\omega$ [1]. Damper moment is always positive, and taking into account the complexity of determining the damping coefficient K_D, it neglected (i.e., it determines some margin in the calculation of the stability). Then, the increment of the complete electrical moment:

$$\Delta T_e = \\ = K_1\Delta\delta + K_2\Delta\psi_{fd} + K_D\Delta\omega \rightarrow K_1\Delta\delta + K_2\Delta\psi_{fd} \qquad (24)$$

With (23), deviation of the field flux linkage $\Delta\psi_{fd}$:

$$\Delta\dot{\psi}_{fd} = a_{32}\Delta\delta + a_{33}\Delta\psi_{fd} + a_{34}\Delta V_t \qquad (25)$$

and taking into account the deviation of the terminal voltage [1]:

$$\Delta V_t = K_5\Delta\delta + K_6\Delta\psi_{fd} \qquad (26)$$

and rearranging, we obtain:

$$\Delta\psi_{fd} = \frac{K_3}{1+sT_3}\cdot \\ \cdot\left(-K_4\Delta\delta - \frac{K_A}{1+sT_{TR}}(K_5\Delta\delta + K_6\Delta\psi_{fd})\frac{\cos(N\Delta\delta)}{1+\cos(2N\Delta\delta)}\right) = \\ = \frac{-K_3\left[K_4(1+sT_{TR})(1+\cos(2N\Delta\delta)) + K_5K_A\cos(N\Delta\delta)\right]}{(1+sT_3)(1+sT_{TR})(1+\cos(2N\Delta\delta)) + K_3K_6K_A\cos(N\Delta\delta)}\Delta\delta \qquad (27)$$

where gain 2 with (22) included in K_A. Then the moment ΔT_{eE} is:

$$\Delta T_{eE} = K_2\Delta\psi_{fd} = \Delta T_{eES} + \Delta T_{eED} = \\ = \frac{-K_2K_3\left[K_4(1+j\omega T_{TR})(1+\cos(2N\Delta\delta)) + K_5K_A\cos(N\Delta\delta)\right]}{(1+j\omega T_3)(1+j\omega T_{TR})(1+\cos(2N\Delta\delta)) + K_3K_6K_A\cos(N\Delta\delta)}\Delta\delta = \\ = \left[\frac{-K_2K_3\left[K_4(1+j\omega T_{TR})(1+\cos(2N\Delta\delta) + K_5K_A\cos(N\Delta\delta)\right]}{z}\cdot \\ \cdot\left[(1+\cos(2N\Delta\delta))(1-\omega^2T_3T_{TR}) + K_3K_6K_A\cos(N\Delta\delta)\right]\right]\cdot(\Delta\delta) + \\ + \left[\frac{-K_2K_3\left[K_4(1+j\omega T_{TR})(1+\cos(2N\Delta\delta) + K_5K_A\cos(N\Delta\delta)\right]}{z}\cdot \\ \cdot(T_3+T_{TR})(1+\cos(2N\Delta\delta))(-j\omega\Delta\delta)\right]\cdot(-j\omega\Delta\delta) \qquad (28)$$

where

$$z = [(1+\cos(2N\Delta\delta))(1-\omega^2 T_3 T_{TR}) + K_3 K_6 K_A \cos(N\Delta\delta)]^2 + \\ +[\omega(T_3 + T_{TR})(1+\cos(2N\Delta\delta))]^2$$

In the literature [1] noted, that at the existing parameters of the power systems, coefficients $(K_2, K_3, K_4, K_6) > 0$. Appearance, the negative damping of the synchronous machines is determined by the coefficient K_5, which can be negative, especially for high-speed excitation control. Derivation of the analytical expressions for determining the values K_A and N with equation (28) is rather cumbersome, therefore we considered them to calculate a specific example given in [1].

4. Excitation control with EFCA (Example).

Consider SMIB-model, given in [1], with parameters:

$$K_1 = 1.591, K_2 = 1.5, K_3 = 0.333, K_4 = 1.8, K_5 = -0.12, \\ K_6 = 0.3, T_{TR} = 0.02, T_3 = 1.91, H = 3.0, K_D = 0,$$

$$G_{ex}(s) = K_A\left(V_{ref} - \frac{2\cos(N\Delta\delta)}{1+\cos(2N\Delta\delta)}V_t\right) = \\ = 200\left(V_{ref} - \frac{2\cos(N\Delta\delta)}{1+\cos(2N\Delta\delta)}V_t\right)$$

Using these values, and taking into account (28), we calculate the complete synchronization and damper moments, when the rotor angle deviation is $\Delta\delta = 1$ rad. and $T_{TR} = 0$. Further, we define the conditions, under which the synchronization and the damper moments, proportional to change of the field flux linkage $\Delta\psi_{fd}$ of the synchronous machine is positive:

$$\begin{aligned} &\Delta T_{eES} > 0 \rightarrow \\ &-K_2 K_3 [K_4(1+\cos(2N\Delta\delta) + K_5 K_A \cos(N\Delta\delta)] \cdot \\ &\cdot [1+\cos(2N\Delta\delta) + K_3 K_6 K_A \cos(N\Delta\delta)] > 0 \\ &\Delta T_{eED} > 0 \rightarrow \\ &-K_2 K_3 T_3 [K_4(1+\cos(2N\Delta\delta) + K_5 K_A \cos(N\Delta\delta)] \cdot \\ &\cdot [1+\cos(2N\Delta\delta)] < 0 \end{aligned} \quad (29)$$

For model parameters we obtain the next conditions:

$$1.8\cos^2(2N\Delta\delta) - 2.4\cos^2(N\Delta\delta) - \\ -23.82\cos(2N\Delta\delta)\cos(N\Delta\delta) + \\ +3.6\cos(2N\Delta\delta) - 23.82\cos(N\Delta\delta) + 1.8 < 0$$

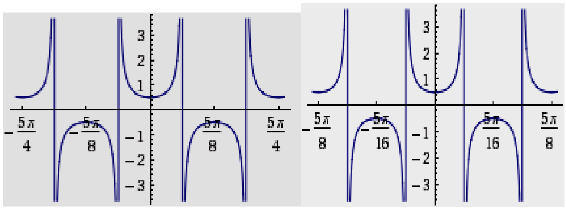
and

$$1.8\cos^2(2N\Delta\delta) - 24\cos(2N\Delta\delta)\cos(N\Delta\delta) + \\ +3.6\cos(2N\Delta\delta) - 24\cos(N\Delta\delta) + 1.8 > 0$$

with domain of admissible values N:

$$0.5(12.56n + 3.14) < N < 2(3.14n + 0.81), n \in Z$$

We define $N = 1.6$. The influence of the gain N on the AVR sensitivity at the rotor angle deviation shown on Fig. 5:



$N = 1.6(\Delta\delta_{max} = 56^\circ)$ $N = 3.2(\Delta\delta_{max} = 28^\circ)$

Fig. 5. Function $\frac{\cos(N\Delta\delta)}{1+\cos(2N\delta)}$ for different N.

Analysis of the above calculation shows, that the excitation control system at EFCA of the terminal voltage phasor has a variable gain of the feedback, that defined the initial value for its steady state (at small rotor angle deviation) $K_A = 200$ and changes as $\frac{\cos(N\Delta\delta)}{1+\cos(2N\Delta\delta)}$ in transients (for large rotor angle deviation). Thus, the natural coordination of tasks by AVR (maintenance of the terminal voltage in accordance with a prescribed setpoint) and PSS (damping of the electromechanical oscillations) is achieved.

The above model was simulated in MATLAB [7]. Simulated three-phase synchronous hydrogenerator with rated parameters: power 200 MVA, terminal voltage 13.8 kV, speed 112.5 min^{-1}, connected to the grid with power 10 000 MVA and voltage 230 kV through step-up transformer and long transmission. For grid, ratio of the inductive and active impedance is X/R = 10. AVR with structure (22) was used to control the thyristor excitation system of the synchronous hydrogenerator. We investigated the small-signal and transient stabilities for two different perturbations of the initial steady state:
1) from the control input (setpoint of the terminal voltage) at 5^{th} s of the simulation - the step signal 0.2 p. u. duration 100 ms, which allows to estimate the small-signal stability of the model, 2) from the grid - simulated at 10^{th} s of the simulation

- a three-phase short-circuit duration 300 ms, which allows to estimate the transient stability of the model. The simulated model shown in Fig. 6:

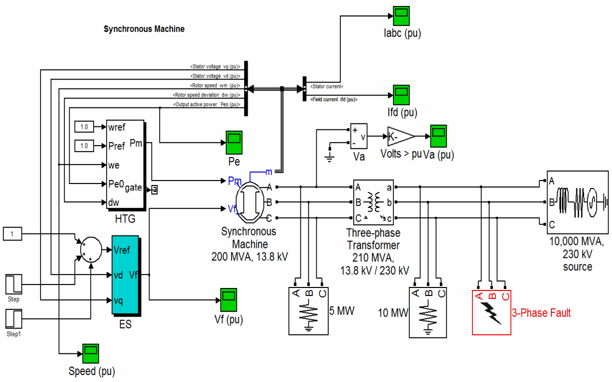


Fig. 6. Block diagram of the simulated SMIB-model in MATLAB.

The electromagnetic state of the synchronous hydrogenerator simulated a system of the differential and algebraic equations in the Park coordinate system [1], taking into account the dynamics of the stator windings, rotor and damper circuits, and ties the voltage V_i, current i_i, flux linkage ψ_i, active resistance R_i and inductance L_i of i[th] loop, neglecting leakage inductance's.

$$STATOR:$$
$$V_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_R \psi_q,$$
$$V_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_R \psi_d,$$
$$ROTOR:$$
$$V_{fd} = R_{fd} i_{fd} + \frac{d\psi_{fd}}{dt},$$
$$DAMPER:$$
$$V_{kd} = R_{kd} i_{kd} + \frac{d\psi_{kd}}{dt},$$
$$V_{kq1} = R_{kq1} i_{kq1} + \frac{d\psi_{kq1}}{dt},$$
$$V_{kq2} = R_{kq2} i_{kq2} + \frac{d\psi_{kq2}}{dt},$$
$$FLUX \quad LINKAGES:$$
$$\psi_d = L_d i_d + L_{md}(i_{fd} + i_{kd}), \psi_q = L_q i_q + L_{mq} i_{kq},$$
$$\psi_{fd} = L_{fd} i_{fd} + L_{md}(i_d + i_{kd}),$$
$$\psi_{kd} = L_{kd} i_{kd} + L_{md}(i_d + i_{fd}),$$
$$\psi_{kq1} = L_{kq1} i_{kq1} + L_{mq} i_q, \quad (30)$$
$$\psi_{kq2} = L_{kq2} i_{kq2} + L_{mq} i_q$$

where the notation of the lower indices: d, q - parameters of the direct d and the quadrature q axes in Park coordinate system, s, f, k - stator, rotor and damper circuits parameters, m - flux linkages parameters. Electromechanical state of the synchronous hydrogenerator was simulated equations (23) - (28). The excitation system of type ST2A simulated in accordance with the standards IEEE Std3.421.5-2005 [2]. Its main elements are the AVR and exciter with the transfer function:

$$\frac{V_{fd}}{u} = \frac{1}{T_e s + K_e} \quad (31)$$

where V_{fd} - excitation (field) voltage, u - output of the AVR, T_e, K_e - time constant and gain of the exciter, s - complex variable. For measurement a rotor angle deviation used block SIMULINK «delta fi», simulated equation (4), Fig. 7:

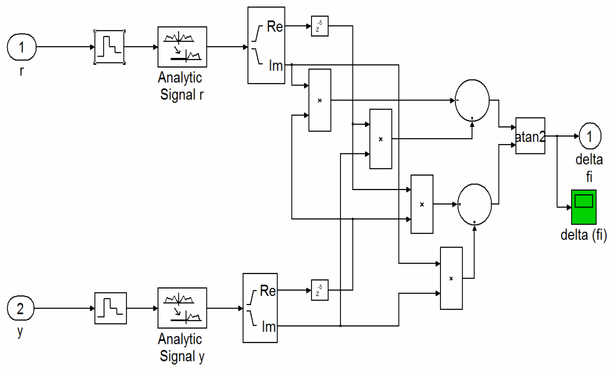


Fig. 7. Block SIMULINK for equation (4).

For correct calculate the deviation of the phase delay $\Delta\varphi$ ($\Delta\delta_{SM}$) introduced the lag $Z/2$ for the real components of analytical signals input and output, where Z - order of the FIR-filter for Hilbert transform. Integration the differential equations of the model is performed by numerical integration method with variable step *ode23tb* with a given relative error 10^{-3}. Simulation results are shown in Appendix 1.

Analysis of the transients shows, that proposed excitation control system ensures the accuracy of the terminal voltage at different perturbations, small-signal and transient stabilities, and robustness at change the model parameters.

5. Conclusion.

In paper presented the new type of the feedback control system, that realized by deviations of magnitude and phase delay of the plant output. In fact, the proposed controller (22) is proportional (P-controller), gain of it is depended by deviation the phase delay (deviation of rotor angle) in transients. That provides it high adaptive and robust properties. Excitation control system by terminal voltage phasor of the synchronous machine provides control of it electromagnetic and electromechanical states, and allows us to natural way coordinate the AVR/PSS tasks. The next step should be to determine the method of calculating the required values of the gains *K* and *N*.

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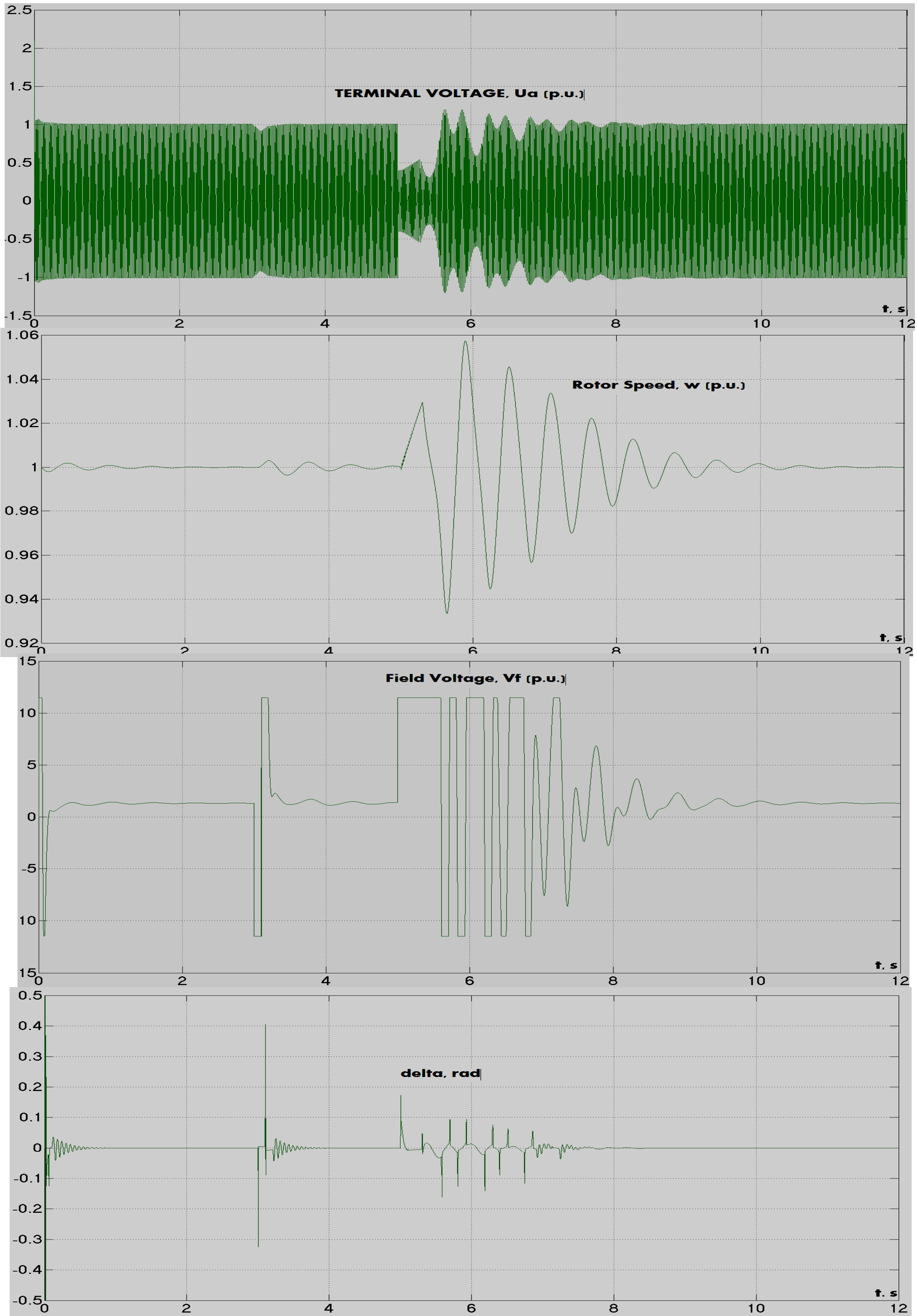
Appendix 1.


TERMINAL VOLTAGE, Ua (p.u.)
Rotor Speed, w (p.u.)
Field Voltage, Vf (p.u.)
delta, rad
t, s