

# Computation of Reactive Power by Power Components in Practical Power Systems

M.SUSITHRA, R. GNANADASS  
Electrical and Electronics Engineering  
Pondicherry University  
Pondicherry Engineering College  
INDIA

E-mail: ersusithra@gmail.com & gnanadass@pec.edu

*Abstract:* - In order to meet the growing load demand, the power utilities are transferred to restructuring process. The reactive power is the supporting power which is necessary for real power transfer. Line charging capacitance, Capacitors, Generators, Synchronous condensers can produce reactive power. So, it is mandatory to know the contribution of reactive power delivered by these regulating devices for the particular load condition. This paper presents the intuition of power flow tracing techniques to determine the amount of reactive power delivered by the regulating devices. The results are compared with using three different reactive power evaluation methods namely Equivalent reactive compensation method, Modified Ybus method, Virtual power flow approach method. The contribution of individual power sources towards the reactive power is demonstrated by using IEEE30 bus system as test system.

*Key-Words:*-Equivalent reactive compensation method, Modified Ybus method, Virtual power flow approach method, Modified power flow tracing method, Upstream algorithm, Downstream algorithm

## 1 Introduction

The modern power industry is changing from one based on vertically integrated monopolies to a new form based on competition and privatization. This results in the unbundling of the vertically integrated functions of generation, transmission, distribution and retail sales. The aspiration is always lowering of the average consumer price and introduction of competition. The transmission and distribution functions remains regulated whereas competition occurs in both the generation and retail electricity services market. Competition is a preferred practice in order to bring fairness and open access to the transmission network. Fairness can only be achieved by adopting a fair and transparent usage allocation methodology acceptable to all parties.

In a competitive electricity market, apart from main transactions of energy and power, system operator(SO) has to arrange or procure certain services required for maintaining the quality and security of supply. These services may be required for maintaining acceptable voltage level and frequency in the system reserve to ensure secure operation of the system, power balancing, handling emergency situations such as black start etc. and are known as Ancillary services. Reactive power service is one of the key ancillary services and its trading is becoming a reality for deregulated electricity markets

[1-7]. with deregulation of electricity sector each electric power service should be economically valued and the fair rules for evaluation and compensation should be established. This has resulted in a need to quantify the value and to compensate the service of reactive power support. Some methods for evaluating the reactive power are given in [8-9].

In view of market operation it becomes more important to know the role of individual generators and loads to the networks and power transfer between individual generators to loads. This is necessary for the competitive market to operate economically, efficiently and for the guarantee of open access to all system users. Several schemes have been developed to solve the allocation problem in the last few years. Methods based on the system Y-bus or Z-bus matrix methods can integrate the network characteristics and circuit theories [10].

Starting from the load flow solution, branch currents are determined as a function of generators' injected currents by using information from the bus impedance matrix. Similarly, contribution to bus voltages is computed as a function of each generator current injection by decomposing the network into different networks [11]. Evaluation of reactive power flow in the lines of the network due to individual sources and its contribution to each load are determined by using virtual flow approach. Counter

flow components are easily determined and loop flows are handled without any difficulty [12&13].

Tracing of electricity gains importance as its solution could enhance the transparency in the operation of the transmission system. Recently a novel electricity tracing method has been proposed in [13- 14] which, assumes that nodal inflows are shared proportionally between the nodal outflows. This allows one to trace the flow of electricity in a meshed network. A similar approach has been proposed in [15] around same time.

Bialek proposed upstream and downstream looking algorithms for tracing reactive power flow [16]. In case of reactive power this approach introduces fictitious nodes on transmission line which act as of these fictitious reactive power source or sink [17]. Due to the addition of this fictitious node the network size increases, thus requiring more computation memory. To overcome this problem a modify methodology for tracing reactive power is proposed in [18-20]. In a restructured power system Reactive power service is one of the ancillary services. This service is provided by a combination of generator capability, reactive compensating devices such as capacitor bank and SVC. The effect of shunt admittance in large scale power system can also be estimated by using this Modified power flow tracing method.

This paper shows the four different methods to solve the reactive power allocation problem. A brief introduction of reactive power valuation by Equivalent Reactive Power Compensation Method is presented in section 2. The effectiveness of Modified Ybus Matrix method is given in section 3. Section 4 gives the method adopted for determining virtual flows. In section 5 Modified reactive power tracing methodology is described. Simulation results and comparison of this method is shown in section 6. Finally a discussion with concluding remarks is given in section 7, while section 8 lists the relevant references.

## 2 Equivalent Reactive Compensation (ERC) Method

For evaluation of source's reactive power to system security and voltage stability and to find which reactive power source is the most important to the system Equivalent Reactive Compensation Method is used. According to the authors [2] of this method the basic form of the idea is as follows: if a Var source changes its output, the network voltage profile and stability levels will change. To maintain a same degree of network security, Var compensations can

be added (at all load buses). The total amount of fictitious compensation added is a direct measure of the value of the missing Var output from the source. The fictitious injected reactive power is termed as Equivalent Reactive Compensation. The procedure of this method used for our system model is given below:

1. Add fictitious synchronous condensers  $Q_{s,i}$  (generators with zero active power output) to each load bus. There are no reactive power limits for the condensers. In further research the fictitious condensers can be placed in a given zone or area of a system, instead of all over the system. Reactive output limits could be added to the fictitious condensers. The Var limits could be set in proportion to the size of bus MVA load.

2. The reactive power output of generators is kept at the base case level, that is almost zero, and Automatic Voltage Regulator is turned off. So generators are represented as PQ buses.

3. For the dynamic reactive power source to be studied, we increase its reactive power output  $Q_{G,i}$  from the  $Q_{Gmin}$  to  $Q_{Gmax}$ . QERCs are calculated in the process as reactive power output of all fictitious condensers:

$$Q_{ERC} = \sum_{i=L} Q_{s,i} \quad (1)$$

4. QERC as a function of  $Q_{G,i}$  the output of the study source can be plotted.

5. QERC curves for each reactive power source can be transformed into the value curves.

$$v(Q_{G,i}) = Q_{ERC,i}(Q_{Gmin,i}) - Q_{ERC,i}(Q_{G,i}) \quad (2)$$

The value curve represents the system-wide reactive power savings that one can achieve if the output of any dynamic Var source is increased. More savings a generator can give to the system implies more efficient it is to support system security [9].

6. Sensitivity coefficients of the value curves for all generators could be established. This method gives information about the most efficient generator to system security and dynamic reactive power supply. But the limitations of this method is not capable to give exact sensitivity of the corresponding generator when subjects to different case studies and does not give any information about contribution of that load and line acquired from generator.

## 3 Modified $Y_{bus}$ Method

In this method, a new modified nodal equation has been developed for identifying reactive power transfer between generators and load. The purpose is to represent each load current as a function of the generator's currents and load voltages. Starting from the concept of circuit theory, it uses the modified

admittance matrix to decompose the load voltage dependent term into components of generator dependent terms. By using these two decompositions of current and voltage terms, the real and reactive power transfer between loads and generators are obtained [10].

The proposed methodology begins with the system node equation. For the convenience of explanation, it is assumed that the power system has a total number of  $n$  buses,  $g$  generators, and  $l$  loads, among which bus number 1 to  $g$  are generation buses and bus number  $g+1$  to  $n$  are load buses. Therefore, the  $Y$  bus of  $n \times n$  dimension can be divided into four sub matrixes as shown in (3).

$$\begin{bmatrix} Y_{1,1} & Y_{1,g} & Y_{1,g+1} & Y_{1,n} \\ & \ddots & & \\ Y_{g,1} & Y_{g,g} & Y_{g,g+1} & Y_{g,n} \\ Y_{g+1,1} & Y_{g+1,g} & Y_{g+1,g+1} & Y_{g+1,n} \\ & \ddots & & \\ Y_{n,1} & Y_{n,g} & Y_{n,g+1} & Y_{n,n} \end{bmatrix} \times \begin{bmatrix} V_1 \\ \vdots \\ V_g \\ V_{g+1} \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_g \\ I_{g+1} \\ \vdots \\ I_n \end{bmatrix} \quad (3)$$

Equation (3) can be briefly represented as the following (4):

$$\begin{bmatrix} YGG & YGL \\ YLG & YLL \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ IL \end{bmatrix}$$

Calculate the equivalent admittance of each load bus with equation (3):

$$Y_{Lj} = \frac{1}{V_{Lj}} \left( \frac{S_{Lj}}{V_{Lj}} \right)^* \quad (4)$$

where,

$S_{Lj}$  is the apparent power of load on bus  $j$ ,

$Y_{Lj}$  is the equivalent admittance of load on bus  $j$ ,

$V_{Lj}$  is the resultant voltage of bus  $j$  of power flow analysis.

We use (4) to calculate the equivalent admittance of every load and then modify the sub matrix  $[YLL]$  in the original  $Y$  bus matrix. The modification is executed by adding the corresponding,  $Y_{Lj}$  to the diagonal elements in the  $[YLL]$  matrix, so the original matrix  $[YLL]$  is replaced by matrix  $[YLL']$ . With the equivalent admittances of loads being represented, the load buses will have no injection current, thus reducing the sub-matrix  $[IL]$  to  $[0]$ .

Hence, (3) is changed as shown:

$$\begin{bmatrix} YGG & YGL \\ YLG & YLL \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ 0 \end{bmatrix} \quad (5)$$

Equation (4) on the lower half part of the matrix is used to arrive at:

$$[YLG][VG] + [YLL'][VL] = 0 \quad (6)$$

And then the relationship functions can be obtained as follows:

$$[YLL'][VL] = -[YLG][VG] \quad (7)$$

$$[VL] = -[YLL']^{-1}[YLG][VG] \quad (8)$$

In (8), it is assumed that

$$[YA] = -[YLL']^{-1}[YLG] \quad (9)$$

and (7) can be rewritten as

$$[VL] = [YA][VG] \quad (10)$$

The voltage of each load bus consisting of the voltages contributed by individual generators is expanded as shown in the following equation:

$$VL_j = \sum_{i=1}^g YA_{j,i} * VG_i \quad (11)$$

and it is assumed that

$$\Delta VL_{i,j} = YA_{j,i} * VG_i \quad (12)$$

Where  $\Delta VL_j$  the voltage contribution is that load acquires from generator. Equation (12) may also be expressed as

$$VL_j = \sum_{i=1}^g \Delta VL_{i,j} \quad (13)$$

With (12), it can be recognized that the voltage contribution of each load bus received from individual generators is  $\Delta VL$ . The reactive power contributions that load acquire from generator  $i$  is as follows:

$$QL_{i,j} = \text{Im} \{ \Delta VL_{i,j} * IL_j^* \} \quad (14)$$

Where,  $IL_j$  is the load current which is to divide the power of the load by known load bus voltage and take the conjugate of the complex number on load bus  $j$ . Reactive Power Contribution that load  $j$  acquires from generator  $i$  can be determined from (14). The calculation results might bring about some differences from those based on other methods. But the contribution of reactive power to the transmission line cannot be calculated [11].

If any static capacitor is added to load bus then the power flows and voltages of this system have been changed. The bus voltage contributions from each generator also changed, reflecting in a change that can be seen as a reduced share on each load bus of the reactive power from existing six generators.

## 4 Virtual Power Flow Approach

This approach presents the concept of virtual flows using the principle of superposition. The concept is

applied to obtain virtual contributions of individual sources to line flows and loads. It is established that the virtual contribution to loads is by each source of the network in some proportion and the actual contribution is the superposition of the all the respective virtual contribution. The procedure of this method to find the contribution of an each generator to the line flow, loads and losses are given below.

1. Perform load flow / state estimation of the network.
2. Read bus voltage phasors, real and reactive power injections at generator buses, loads and network parameters.
3. Convert all the loads to equivalent admittances at the operating point by the relation,

$$y_i^{load} = \frac{(-P_i^{(o)} + jQ_i^{(o)})}{|V_i^{(0)}|^2} \quad i=g+1, g+2 \dots n$$

4. Modify the network Y bus matrix to include loads as admittances.
5. Inject equivalent current from one source at a time to respective bus and obtain corresponding bus voltage profile.

$$I_i^{(o)} = \frac{S_i^{(0)*}}{V_i^{(0)*}} \quad \text{where } S_i^{(0)} = P_i^{(0)} + jQ_i^{(0)}$$

6. Determine all the resulting branch currents for the Voltage profile obtained from this source.
7. The total complex power flow in the line i-j is given by,

$$\begin{aligned} S_{i-j}^{(0)} &= \left\{ (V_i^{(0)} - V_j^{(0)})y_a + V_i^{(0)}y_b \right\}^* V_i^{(0)} \\ &= |V_i^{(0)}|^2 (y_a^* + y_b^*) - V_i^{(0)}V_j^{(0)*} y_a^* \\ &= \Delta S_{i-j}^{(1)} + \Delta S_{i-j}^{(2)}. \end{aligned}$$

Where,  $y_a$  is the series admittance and

$y_b$  is the half line charging susceptance.

$\Delta S_{i-j}^{(1)}, \Delta S_{i-j}^{(2)}$  are called as virtual flows due to source at node1 and node2.

8. The total contributions to given load from all the sources is obtained by the summation of partial contribution by all individual sources and it agrees with load power as in base case. It can be ascertained that the load power.

$$S_i^{(0)} = \sum_{k=1}^g \Delta S_i^{(k)}$$

This method finds the virtual contribution of reactive power to transmissions lines and loads. But the limitation of this method is contribution of reactive power to line losses cannot be calculated. Also this method does not calculate the reactive power generation due to static and dynamic sources.

## 5 Power Flow Tracing Method

The electricity tracing methodology is based on actual flows in the network and proportionality sharing principle. It deals with a general problem of how to distribute flows in a meshed network [12-14]. The proportional sharing principle basically applies Kirchhoff's current law at the node and applies proportionality principle to find the relationship between incoming and outgoing flows. Thus, this method is equally applicable to real and reactive power flows and direct currents. The only assumption that is made in this methodology is that the system is lossless [15, 16]. This is achieved by averaging the sending and receiving end line flows and by - adding half of the line loss to the power injections at each terminal node of the line. Transmission line [π] model is introduced for reactive power tracing method as shown in figure 1.

The main objective of reactive power tracing method is to calculate reactive power loss allocated to each line for particular load by using the following equation

$$QD_{ij,k} = QD_{ij,k} QD_{ij} \tag{15}$$

$$\text{Where, } QD_{ij,k} = \frac{\left( \frac{Q_{ij,k}}{\sin \phi_k} \right)^2}{\sum_{k=1}^l \left( \frac{Q_{ij,k}}{\sin \phi_k} \right)^2}$$

$QD_{ij}$  -total reactive power loss in line i-j.

$QD_{ij,k}$  -reactive power loss distribution factor.

$Q_{ij,k}$  -reactive power flowing in line i-j due to  $k^{\text{th}}$  load.

The procedure to obtain this objective is given below:

1. Obtain the Power Flow solution for given system.
2. Calculate new reactive power in each line due to the reactive power generated by shunt admittance  $Q_{shunt}$  connected to each bus, by assuming that

voltage of shunt admittance is equal to the nearby nodal voltage.

The nodal voltage can be obtained from power flow using the formula:

$$Q_{shunt,i} = V_i^2 B_{sh/2,ij} \quad Q_{ij,New} = Q_{ij} + Q_{shunt,i}$$

$$Q_{shunt,j} = V_j^2 B_{sh/2,ij} \quad Q_{ji,New} = Q_{ji} - Q_{shunt,j}$$

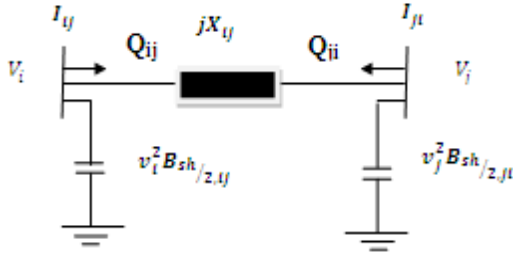


Fig. 1. Transmission line  $\pi$  model and the forward, backward current

3. Form the Lossless Network from a lossy one by Using the following steps:

- a) Calculate the average value of sending and receiving end reactive power of each transmission line.
- b) Add half of the line loss to power injections at each terminal node of the line.

4. Calculate the Upstream Distribution Matrix( $A_u$ ):

These can be calculated using Upstream Looking Algorithm; it states that total flows (inflows and outflows) in bus 'i' i.e.  $P_i$  can be expressed as

$$P_i = \sum_{j \in \alpha_i^{(u)}} c_{ij} |P_{i-j}| + P_{Gi}$$

$$\text{Let } C_{ij} = |P_{j-i}| / P_j$$

$$\text{Therefore, } A_u P = P_{Gi} \quad (16)$$

The Elements of upstream distribution matrix can be calculated by

$$[A_u]_{ij} = \begin{cases} 1 & \text{for } i=j \\ -C_{ji} = -|P_{j-i}| / P_j & \text{for } l \in \alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

5. Find the inverse of upstream distribution matrix

6. The contribution of  $k^{\text{th}}$  generator to  $i^{\text{th}}$  load is found out using

$$P_{Li} = \frac{P_{Li}}{P_i} P_i = \frac{P_{Li}}{P_i} \sum_{k=1}^n [A_u^{-1}]_{ik} P_{GK} \quad \text{for } i=1,2,\dots,n. \quad (18)$$

7.The contribution of  $k^{\text{th}}$  generator to  $i-l$  line is found out using

$$|P_{i-l}| = \frac{|P_{i-l}|}{P_i} P_i = \sum_{k=1}^n D_{i-l,k}^G P_{GK} \quad \text{for all } l \in \alpha_i^{(d)} \quad (19)$$

Where ,

$$D_{i-l,k}^G = |P_{i-l}| [A_u^{-1}]_{jk} / P_i \text{ is generation distribution factor.}$$

8. Calculate the Downstream Distribution Matrix ( $A_d$ ): These can be calculated using Downstream Looking Algorithm, it states that, total flows (inflows and outflows) in bus 'i' i.e.  $P_i$  can be expressed as

$$P_i = \sum_{l \in \alpha_i^{(d)}} |P_{l-i}| + P_{Li} = \sum_{l \in \alpha_i^{(d)}} C_{li} P_l + P_{Li} \quad (20)$$

$$\text{Let } C_{li} = |P_{l-i}| / P_l$$

Therefore

$$P_i - \sum_{l \in \alpha_i^{(d)}} C_{li} P_l = P_{Li} \text{ (or) } A_d P = P_L$$

The Elements of Downstream distribution matrix can be calculated by

$$[A_d]_{il} = \begin{cases} 1 & \text{for } i=l \\ -C_{li} = -|P_{l-i}| / P_l & \text{for } l \in \alpha_i^{(d)} \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

9. Find the inverse of downstream distribution matrix ( $A_d^{-1}$ )

10. Calculate reactive power loss allocated to each line for particular load by using

$$Q_{Dij,k} = Q_{Dij,k} Q_{Dij}$$

The advantage of this method is that the introduction of fictitious node in each transmission line is avoided. Thus the size of the system remains the same. This method also deals with one of the ancillary services that is power loss and proposes a simple method to allocate transmission line losses to individual loads. The flow diagram of power tracing method is shown in figure2.

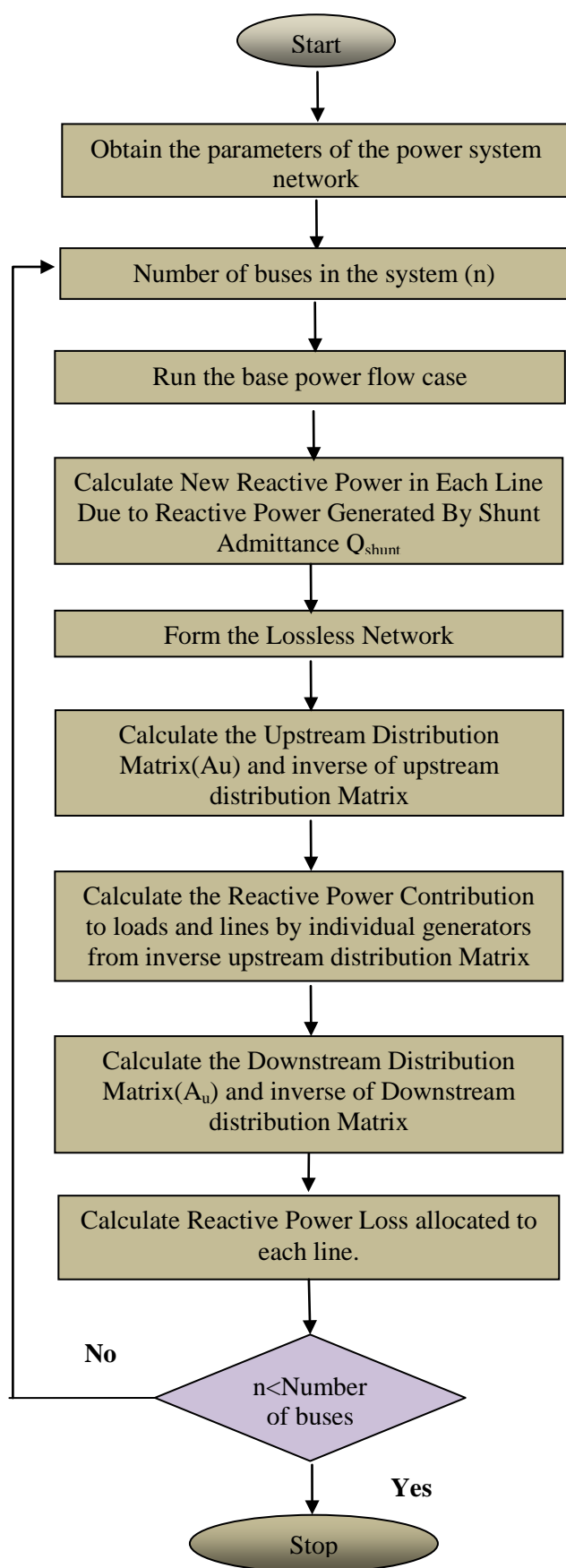


Fig.2. Flow diagram of Power Flow Tracing Method

## 6 Simulation Results and Discussion

The study has been conducted by taking IEEE 30 bus system as test system. The modelling of the power system components (generator, transmission line and loads) of the test system was carried out in the MATLAB environment. Power flows in transmission lines were determined using N-R method. The line flow limits were also checked out during the power flow. In the current market scenario, the real power is traded in the power exchange and the responsibility of system operator is to maintain the reactive power, voltage and security of the system. In this paper, the reactive power contribution of generation sources were determined using ERC, Modified Ybus, Virtual Power Flow Approach and Power flow tracing methods. In this context, the influence of reactive power delivered by the generation sources alone taken for the analysis.

### 6.1 IEEE 30 bus system

The one line diagram of IEEE-30 bus system is shown in figure 1, which consists of six generators and 24 load buses with 41 transmission lines. The following three case studies were carried out to demonstrate contribution of reactive power delivered by the sources using the above four computing methods and total load of 283.4 MW and 126.2 MVAR.

**6.1.1 Base case condition(283.4MW):-**From the power flow result, sensitivity coefficients of ERC method, reactive power generated by each generators were estimated using Modified Ybus, Virtual flow method. The Modified power flow tracing method gives additional information to find reactive power flow in the transmission line and loads at the system base load condition.

**6.1.2. Increased in load condition (120 %):-**When the load is increased to 120% then the contributions of generators to loads are analysed.

**6.1.3. Contingency case (One transmission line contingency):-**In order to analyse the security and stability single line (2-4) outage condition is carried out.

### 6.2.ERC method

The reactive power delivered by the corresponding generators was computed using power flow analysis for the base load condition. Reactive power VAR sources were added to each load bus and their power limits were adjusted from minimum to maximum

limit. The change in reactive power flow of the generators before and after adding the reactive power sources were computed by load flow analysis. Then the graph was plotted between each generators reactive output and limits of the reactive VAR sources. Since it has lot of bumping, the curve was linearised and sensitivity was calculated. To illustrate the vulnerability of the system the contingency analysis (a transmission line outage) and increased load condition were carried out and the results are shown in figure 3.

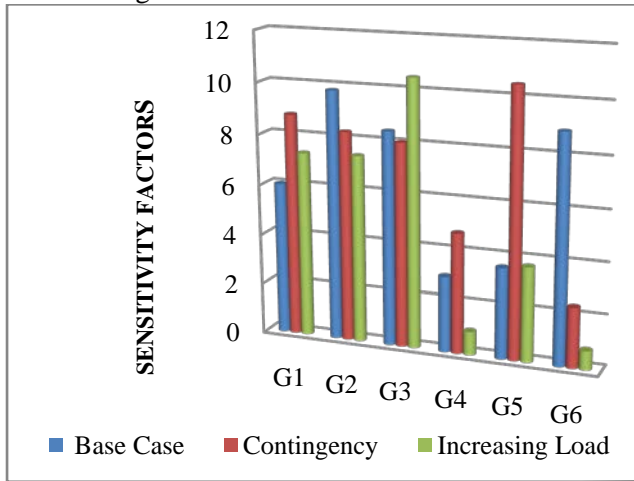


Fig.3. ERC method sensitivity factors of generators

It is inferred that the sensitivity factor of fourth generator (G<sub>4</sub>) is less for the base load condition. It implies that fourth generator may be participating in more reactive power delivery of the system and less sensitive. But in the contingency case and in increased load condition the sixth generator (G<sub>6</sub>) is less sensitive. Hence the limitation of the ERC method is not capable to give exact sensitivity of the corresponding generator when subjects to different case studies.

**6.3. Modified Ybus method**

From the power flow analysis results the voltages, real power and reactive powers were obtained. The Ybus of n\*n dimensions can be divided into four submatrices [Y<sub>GG</sub>, Y<sub>GL</sub>, Y<sub>LG</sub>, Y<sub>LL</sub>]. Then the equivalent admittance YL<sub>j</sub> of every load was calculated. The submatrix [Y<sub>LL</sub>] is modified by adding YL<sub>j</sub> to the diagonal elements. The voltages of each load buses (VL<sub>j</sub>) were calculated. The reactive power contribution that load acquired from generator is calculated using the equations given in section 3. The total reactive power contribution generated by six generators for three different cases are shown in figure 4.

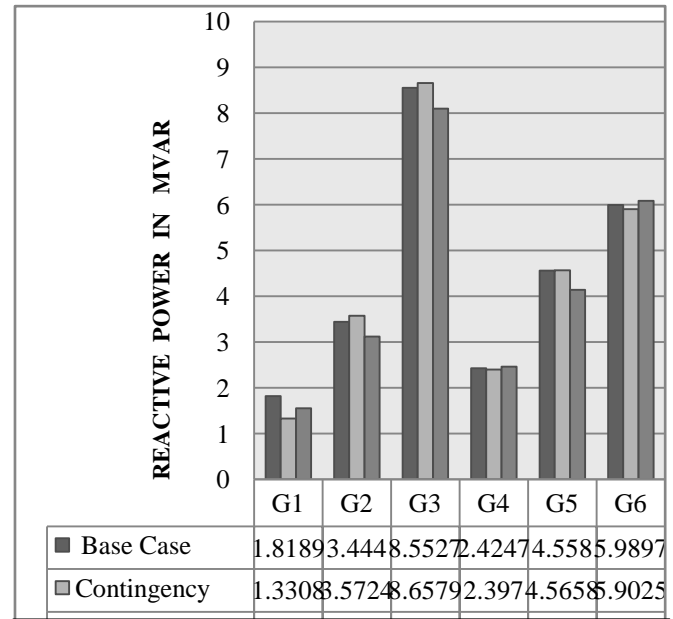


Fig.4. Contribution of reactive power using Modified Y<sub>bus</sub> Method

The above figure shows the result of Modified Y<sub>bus</sub> Method. In this method at different case studies first generator(G<sub>1</sub>) only generates minimum amount of reactive power and third generator(G<sub>3</sub>) contributes maximum amount of reactive power.

This method has the limitation to give the accurate result for larger system. Also, it is not capable to identify counter flow components in a given branch of network produced by some other sources. In order to overcome this above said limitation, virtual power flow approach is used to find out the partial contribution to loads from all the sources.

**6.4. Virtual Power Flow Approach**

The virtual flow tracing approach can identify counter flows and also handles loop flows equally well. The initial data required for evaluation is obtained from power flow analysis. Here all the loads are converted into equivalent complex admittances at the operating point.

The diagonal elements of the admittance bus matrix corresponding to load buses are then modified by including these loads respectively to get the modified bus admittance matrix. The impedance matrix is obtained from the inverse of modified bus admittance matrix. The partial set of bus voltages developed from a particular source, the partial currents in the branches are determined as illustrated in section 4.

Then virtual power flows are obtained for each source in a given line. By knowing the virtual power flows in each branch due to each source, the source contribution to each load can be obtained. It is established that the virtual contribution to loads is by each source of the network in some proportion and the actual contribution is the superposition of the all the respective virtual contribution. The procedure of this method to find the contribution of an each generator to the line load is given in section 4. The results of virtual flow approach for IEEE 30 bus system at different case studies are given in tables 1, 2 and 3.

Table 1. Base case condition

Load	G1	G2	G5	G8	G11	G13
3	0	0	0	0	0	0
4	0.3423	0.315	0.3853	0.0187	0.0765	0.0338
6	0	0	0	0	0	0
7	0.0829	0.2543	0.7467	0.1448	0.1212	0.3548
9	0.2544	0.5667	0.6343	-0.0216	0.1438	-0.0765
10	0.3658	0.8676	1.7982	0.4556	0.3054	2.9908
12	0	0	0	0	0	0
14	0.0622	0.1887	0.4606	0.0926	0.0785	0.3465
15	0.1046	0.2907	0.6842	0.116	0.1165	0.6944
16	0.0835	0.2123	0.5163	0.144	0.0848	0.5757
17	0.2566	0.7077	2.0032	0.7122	0.3184	1.2988
18	0.0347	0.1035	0.2633	0.0288	0.0447	0.2011
19	0.1368	0.3952	1.0573	0.1907	0.1743	0.7575
20	0.0277	0.0828	0.2258	0.0315	0.0374	0.1452
21	0.484	1.4493	4.7997	1.1687	0.7037	1.6697
22	0	0	0	0	0	0
23	0.0709	0.1902	0.5166	0.1068	0.0805	0.4286
24	0.2962	0.8466	2.762	0.6096	0.3964	1.2344
25	0.3526	1.2432	3.2854	0.0885	5.4283	0.1347
26	0.0967	0.291	1.0013	0.1656	0.1416	0.3324
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	0.0336	0.1092	0.3558	0.0409	0.0546	0.1112
30	0.0673	0.223	0.7039	0.0744	0.1117	0.231

Table 2. Contingency Case

Load	G1	G2	G5	G8	G11	G13
4	0.336	0.3224	0.3887	0.0138	0.0798	0.0212
7	0.0781	0.2482	0.7216	0.0753	0.1217	0.3393
9	0.2517	0.5886	0.6389	-0.0413	0.1548	-1.268
10	0.3466	0.8614	1.8105	0.4328	0.3151	2.7239
14	0.0527	0.1813	0.4544	0.0768	0.0806	0.2289
15	0.0927	0.2791	0.6574	0.0796	0.1174	0.5699
16	0.0795	0.2087	0.5069	0.1206	0.0855	0.5383
17	0.2475	0.696	1.9609	0.6133	0.3185	1.2511
18	0.0298	0.0969	0.2447	0.0025	0.0439	0.1608
19	0.1224	0.3739	0.9927	0.0932	0.171	0.6587
20	0.0247	0.0784	0.2116	0.0075	0.0368	0.1268
21	0.4674	1.4277	4.6991	0.993	0.7058	1.6027
23	0.0663	0.1848	0.499	0.0883	0.0805	0.3886
24	0.2913	0.8355	2.7286	0.5955	0.3956	1.2068
25	0.3728	1.2334	3.4371	0.2218	5.299	0.2296
26	0.0911	0.2802	0.9458	0.1433	0.1389	0.3134
29	0.0297	0.101	0.3104	0.0281	0.0522	0.1033
30	0.0574	0.2014	0.5884	0.043	0.1048	0.2105

Table 3. Increasing Load Condition

Load	G1	G2	G5	G8	G11	G13
4	0.3907	0.1084	0.496	0.025	0.098	0.046
7	0.1084	0.1434	0.807	0.146	0.133	0.357
9	0.3419	0.2004	0.846	-0.0214	0.189	-0.832
10	0.4744	0.3931	2.055	0.467	0.356	3.015
14	0.0839	0.0964	0.514	0.092	0.090	0.343
15	0.1388	0.1437	0.767	0.117	0.134	0.694
16	0.1078	0.1057	0.573	0.147	0.096	0.581
17	0.3286	0.3886	2.172	0.723	0.351	1.321
18	0.0464	0.0537	0.291	0.028	0.050	0.200
19	0.1795	0.2099	1.159	0.193	0.195	0.761
20	0.0366	0.0446	0.247	0.031	0.041	0.145
21	0.62	0.8472	5.121	1.190	0.765	1.711
23	0.0917	0.099	0.565	0.109	0.090	0.433
24	0.3783	0.4823	2.955	0.623	0.433	1.262
25	0.4557	0.7907	3.530	0.100	5.484	0.156
26	0.124	0.1701	1.065	0.169	0.154	0.340
29	0.044	0.0638	0.381	0.041	0.059	0.112
30	0.0889	0.1295	0.756	0.075	0.122	0.231



The virtual flow tracing approach can identify counter flows and also handles loop flows equally well. In this method losses are not taken into account. Also this method does not identify the amount of reactive power generated by transmission line and the amount of reactive power generated by static and dynamic reactive power sources. In order to overcome this above mentioned limitations, Modified power flow tracing method is used.

### 6.5. Power Flow Tracing Method

The main objective of modified power flow tracing method is to allocate reactive power loss to each load based upon the proportion of reactive power flow in the transmission line and their power factor. The lossless network was obtained by adding half of the line loss to the power injections at each terminal of the line.

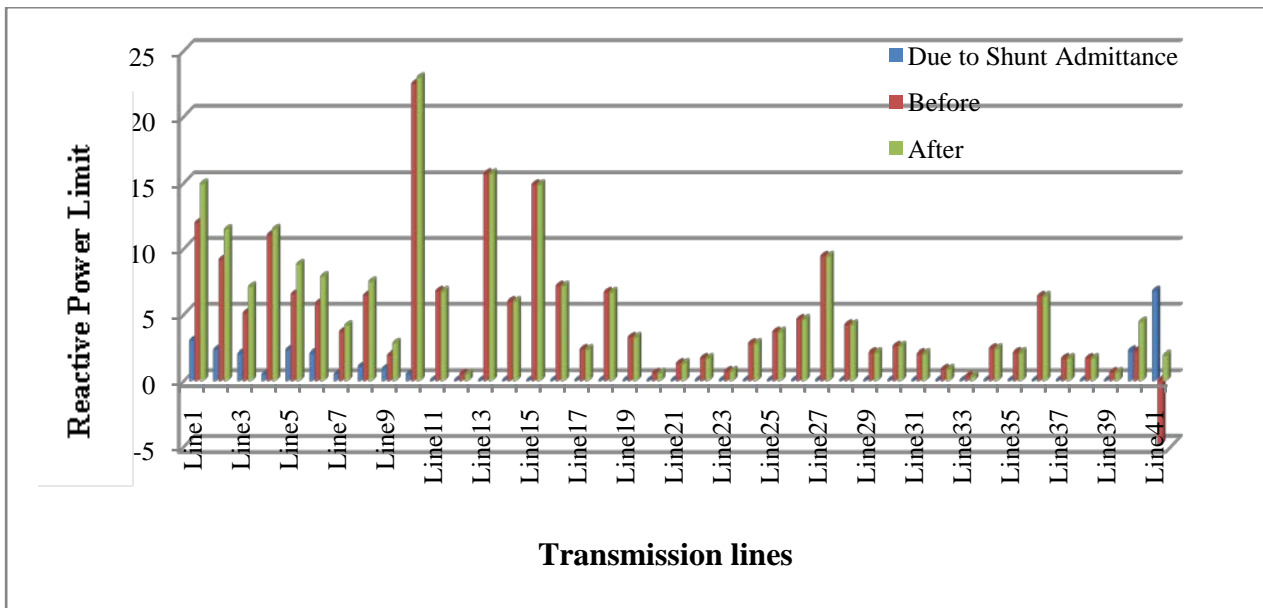


Fig.5. Effect of Shunt Admittance in Each Line

The shunt admittance which present at each end of bus has ability to produce reactive power. It is clearly understood that after considering the effect of shunt admittance, there is change in basic line flow. And the reactive power produced by shunt admittance is considered as fictitious generator. So, it is necessary to include this effect to each line flow. This is clearly shown in figure 5.

Total amount of reactive power delivered to the load from the sources for three case studies by modified power flow tracing method is shown in Table 4. In this method at different case studies the 6<sup>th</sup> generator (G6) only contributes maximum amount of reactive power and generator(G1) gives minimum amount of reactive power.

In each bus there are many types of reactive power producing source are available. So it is necessary to sum up all the reactive power generation at each bus and considered as single reactive power generating source. Table 5 shows the total contribution of reactive power due to each source.

Table 4. Contribution of Reactive Power From Modified Power Flow Tracing Method

Generators	Base Case	Contingency Case	Increasing Load
G1	0.371	0	0.569
G2	0.758	0	1.234
G3	9.364	13.349	10.56
G4	8.113	8.358	5.76
G5	3.823	3.528	4.183
G6	16.234	17.545	17.016
G8	3.878	0.907	7.21
G11	4.926	4.436	4.065
G13	7.29	6.734	7.294
G25	3.783	3.744	3.781
G28	4.729	4.86	4.719

Table 5. Contribution of Reactive Power ( in MVAR) due shunt admittance

Bus No.	Base Case			Contingency			Increasing Load Condition		
	Generator	Shunt admittance	Total generation	Generator	Shunt admittance	Total generation	Generator	Shunt admittance	Total generation
1	0.0000	10.0655	10.0655	0.0000	5.6779	5.6779	0.0000	10.0655	10.0655
2	0.0000	17.0125	17.0125	0.0000	18.0125	18.0125	0.0000	18.0125	18.0125
3	5.9723	5.4316	11.4039	11.7664	5.4316	17.1980	7.3513	5.4316	12.7829
4	10.2567	0.0000	10.2567	10.8767	0.0000	10.8767	10.7717	0.0000	10.7717
5	0.0000	6.5988	6.5988	0.0000	6.5988	6.5988	0.0000	6.5988	6.5988
6	17.8266	0.0000	17.8266	19.2123	0.0000	19.2123	18.6470	0.0000	18.6470
8	0.0000	5.1804	5.1804	0.0000	0.8682	0.8682	0.0000	5.1768	5.1768
11	0.0000	5.6618	5.6618	0.0000	5.6069	5.6069	0.0000	5.6575	5.6575
13	0.0000	8.8870	8.8870	0.0000	8.8518	8.8518	0.0000	8.8807	8.8807
25	0.0000	3.8653	3.8653	0.0000	3.8550	3.8550	0.0000	3.8633	3.8633
28	0.0000	5.7247	5.7247	0.0000	5.7071	5.7071	0.0000	5.7164	5.7164

The results shown in table 5 also gives additional information about the reactive power generated by shunt admittance of transmission line.

From the lossless network, it is essential to find upstream and downstream matrix. The upstream matrix is found using the logic that all outflows are taken as zero in the matrix and diagonal element is filled with one and all inflows the corresponding values in matrix is found using the formula which is stated early.

Similarly the elements of downstream matrix is found as same as upstream matrix only difference is all inflow are taken as zero is this case. The downstream-looking algorithm look at the nodal balance of outflows and it gives how the line flows are supplied from individual generators and the amount of power supplied by a particular generator to a particular load and it is given in table6. As from table 6 it can be seen that generator8 (G8) is supplying reactive power only to one load.

The upstream looking algorithm look at the nodal balance of inflows and it is used to determine the line flows supplied by individual load. The procedure of power flow tracing algorithm is given in section 5. Table 7 shows the Reactive power (MVAR) flowing in each line due to generators. From Table7, it can be seen that the generator 8(G8) is not supplying reactive power to any line.

Table 6. Contribution of Each Generator to Each Load

Load Bus No.	GENERATORS					
	Gen 1	Gen 2	Gen 5	Gen 8	Gen 11	Gen 13
2	4.473	8.227	0.000	0.000	0.000	0.000
3	0.909	0.000	0.000	0.000	0.000	0.000
4	0.804	0.323	0.000	0.000	0.000	0.000
5	1.573	2.892	14.535	0.000	0.000	0.000
7	0.863	1.388	5.044	0.000	0.426	0.000
8	3.605	4.366	0.000	3.088	4.838	0.000
10	0.018	0.021	0.000	0.000	1.546	0.000
12	2.440	0.979	0.000	0.000	0.000	2.644
14	0.521	0.209	0.000	0.000	0.000	0.564
15	0.813	0.326	0.000	0.000	0.000	0.881
16	0.586	0.235	0.000	0.000	0.000	0.635
17	0.450	0.213	0.000	0.000	3.508	0.445
18	0.293	0.118	0.000	0.000	0.000	0.317
19	0.242	0.116	0.000	0.000	2.111	0.236
20	0.006	0.007	0.000	0.000	0.541	0.000
21	0.098	0.119	0.000	0.000	8.657	0.000
23	0.521	0.209	0.000	0.000	0.000	0.564
24	0.698	0.313	0.000	0.000	3.619	0.712
26	0.084	0.078	0.000	0.000	0.231	0.031
29	0.024	0.029	0.000	0.000	0.032	0.000
30	0.051	0.061	0.000	0.000	0.068	0.000

Table 7. Contribution of Each Generator to Each Line

From Bus	To bus	Gen 1	Gen 2	Gen 5	Gen 8	Gen 11	Gen 13
1	2	14.172	0.000	0.00	0.00	0.000	0.000
1	3	10.730	0.000	0.00	0.00	0.000	0.000
2	4	2.394	4.403	0.00	0.00	0.000	0.000
3	4	8.577	0.000	0.00	0.00	0.000	0.000
2	5	2.484	4.567	0.00	0.00	0.000	0.000
2	6	2.612	4.803	0.00	0.00	0.000	0.000
4	6	2.026	0.813	0.00	0.00	0.000	0.000
5	7	0.607	1.116	5.61	0.00	0.000	0.000
6	7	0.353	0.428	0.00	0.00	0.474	0.000
6	8	3.528	4.272	0.00	0.00	4.735	0.000
9	6	0.000	0.000	0.00	0.00	6.224	0.000
6	10	0.056	0.067	0.00	0.00	0.075	0.000
11	9	0.000	0.000	0.00	0.00	17.16	0.000
9	10	0.000	0.000	0.00	0.00	4.833	0.000
4	12	7.252	2.910	0.00	0.00	0.000	0.000
13	12	0.000	0.000	0.00	0.00	0.000	7.859
12	14	0.724	0.290	0.00	0.00	0.000	0.784
12	15	2.087	0.837	0.00	0.00	0.000	2.261
12	16	1.026	0.412	0.00	0.00	0.000	1.111
14	15	0.175	0.070	0.00	0.00	0.000	0.190
16	17	0.413	0.166	0.00	0.00	0.000	0.447
15	18	0.530	0.213	0.00	0.00	0.000	0.574
18	19	0.221	0.089	0.00	0.00	0.000	0.240
20	19	0.024	0.029	0.00	0.00	2.144	0.000
10	20	0.031	0.038	0.00	0.00	2.739	0.000
10	17	0.040	0.048	0.00	0.00	3.529	0.000
10	21	0.081	0.098	0.00	0.00	7.156	0.000
10	22	0.036	0.044	0.00	0.00	3.186	0.000
22	21	0.018	0.022	0.00	0.00	1.600	0.000
15	23	0.812	0.326	0.00	0.00	0.000	0.880
22	24	0.017	0.021	0.00	0.00	1.500	0.000
23	24	0.272	0.109	0.00	0.00	0.000	0.295
24	25	0.032	0.014	0.00	0.00	0.166	0.033
25	26	0.085	0.080	0.00	0.00	0.234	0.032
27	25	0.056	0.067	0.00	0.00	0.075	0.000
28	27	0.156	0.190	0.00	0.00	0.210	0.000
27	29	0.042	0.051	0.00	0.00	0.057	0.000
27	30	0.040	0.049	0.00	0.00	0.054	0.000
29	30	0.015	0.019	0.00	0.00	0.021	0.000
28	8	0.118	0.143	0.00	0.00	0.158	0.000
6	28	0.287	0.347	0.00	0.00	0.385	0.000

## 7 Conclusion

The comparison of four different methods of reactive power valuation is reported in this paper. ERC method finds the reactive power source which is the most important to the system based on sensitivity analysis. The modified Ybus Method can identify the source and can calculate the amount of consumed reactive power on each load. Virtual flow approach is used to evaluate real and reactive power flow in the network due to individual sources and its contribution to each load using the principle of superposition.

The power flow tracing method trace the flow of electricity in meshed electrical network. This proposed method estimates the contribution of reactive power from individual generators or loads to individual line flows. This method is demonstrated to determine power flows in the network. it can be used to assess the contribution of individual sources of reactive power demands and therefore be used as a tool for reactive power pricing.

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## APPENDIX A

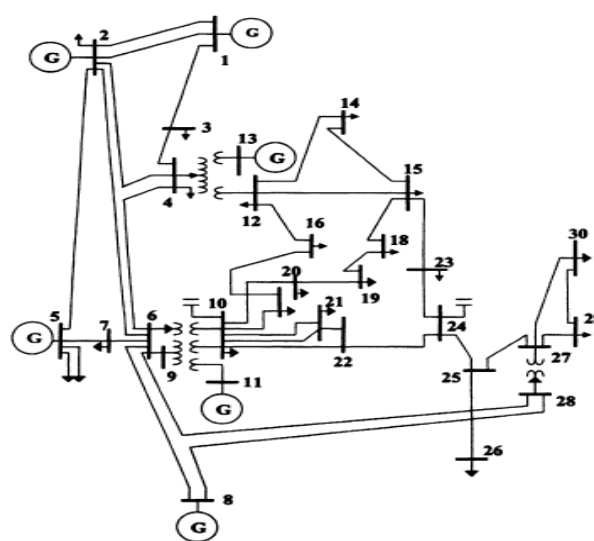


Fig. A.1 One Line diagram of IEEE30 bus system

Table A.1. Transmission Line Data

Line No.	From Bus	To Bus	Series Impedance (p.u)		Half Line Charging susceptance (p.u.)	Tap Setting	MVA Rating
			R	X			
1	1	2	0.0192	0.05750	0.02640	-	130
2	1	3	0.0452	0.18520	0.02040	-	130
3	2	4	0.0570	0.17370	0.01840	-	65
4	3	4	0.0132	0.03790	0.00420	-	130
5	2	5	0.0472	0.19830	0.02090	-	130
6	2	6	0.0581	0.17630	0.01870	-	65
7	4	6	0.0119	0.04140	0.00450	-	90
8	5	7	0.0460	0.11600	0.01020	-	70
9	6	7	0.0267	0.08200	0.00850	-	130
10	6	8	0.0120	0.04200	0.00450	-	32
11	6	9	0.0000	0.20800	0.00000	1.0155	65
12	6	10	0.0000	0.55600	0.00000	0.9629	32
13	9	11	0.0000	0.20800	0.00000	-	65
14	9	10	0.0000	0.11000	0.00000	-	65
15	4	12	0.0000	0.25600	0.00000	1.0129	65
16	12	13	0.0000	0.14000	0.00000	-	65
17	12	14	0.1231	0.25590	0.00000	-	32
18	12	15	0.0662	0.13040	0.00000	-	32
19	12	16	0.0945	0.19870	0.00000	-	32
20	14	15	0.2210	0.19970	0.00000	-	16
21	16	17	0.0824	0.19320	0.00000	-	16
22	15	18	0.1070	0.21850	0.00000	-	16
23	18	19	0.0639	0.12920	0.00000	-	16
24	19	20	0.0340	0.06800	0.00000	-	32
25	10	20	0.0936	0.20900	0.00000	-	32
26	10	17	0.0324	0.08450	0.00000	-	32
27	10	21	0.0348	0.07490	0.00000	-	32
28	10	22	0.0727	0.14990	0.00000	-	32
29	21	22	0.0116	0.02360	0.00000	-	32
30	15	23	0.1000	0.20200	0.00000	-	16
31	22	24	0.1150	0.17900	0.00000	-	16
32	23	24	0.1320	0.27000	0.00000	-	16
33	24	25	0.1885	0.32920	0.00000	-	16
34	25	26	0.2544	0.38000	0.00000	-	16
35	25	27	0.1093	0.20870	0.00000	-	16
36	28	27	0.0000	0.36900	0.00000	0.9581	65
37	27	29	0.2198	0.41530	0.00000	-	16
38	27	30	0.3202	0.60270	0.00000	-	16
39	29	30	0.2399	0.45330	0.00000	-	16
40	8	28	0.0636	0.20000	0.02140	-	32
41	6	28	0.0169	0.05990	0.00650	-	32

Table A.2. Load Bus Data

Bus No.	Load		Bus No.	Load	
	P (MW)	Q (MVAR)		P (MW)	Q (MVAR)
1	0.00	0.00	16	3.50	1.80
2	21.7	12.7	17	9.00	5.80
3	2.40	1.20	18	3.20	0.90
4	7.60	1.60	19	9.50	3.40
5	94.2	19.0	20	2.20	0.70
6	0.00	0.00	21	17.5	11.2
7	22.8	10.9	22	0.00	0.00
8	30.0	30.0	23	3.20	1.60
9	0.00	0.00	24	8.70	6.70
10	5.80	2.00	25	0.00	0.00
11	0.00	0.00	26	3.50	2.30
12	11.2	7.50	27	0.00	0.00
13	0.00	0.00	28	0.00	0.00
14	6.20	1.60	29	2.40	0.90
15	8.20	2.50	30	10.6	1.90

Table A.3. Generator Bus Data

Gen No.	$P_i^{\min}$ (MW)	$P_i^{\max}$ (MW)	$Q_i^{\min}$ (MVar)	$Q_i^{\max}$ (MVar)
1	50	200	-	-
2	20	80	-20	100
3	15	50	-15	80
4	10	35	-15	60
5	10	30	-10	50
6	12	40	-15	60