# Research on the Numerical Accuracy of Equipotential Ground Model Based on Method of Moment

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*Abstract:* - In numerical calculations of grounding grid based on the method of moment, accurately calculation of the resistance matrix is an important step. The direct numerical integration method, Heppe's method and Kouteynikoff's method have the most broad influence in the previous publications, and the accuracy of these algorithms require further evaluation. A software for simulating grounding systems has been developed based on the method of moments in this paper, it implements the above several important algorithms. Under the assumption of uniform soil and ignoring the conductor impedance, the calculation results of the above methods and CDEGS are compared, and sources of error are analyzed. Furthermore, since the grounding electrode need to be cut into small segments properly when using the method of moment, the effect of the conductor's different divisions for accuracy is analyzed quantitatively, and the rule of the calculation accuracy of grounding resistance with the change of segment numbers is obtained. After comparison of the above methods in the case study, theoretical analysis and program test reveal the theoretical flaw of the Kouteynikoff's method.

*Key-Words:* - Grounding grid, numerical calculation, method of moment, Hepe's method, Kouteynikoff's method, CDEGS, Numerical Accuracy.

## 1 Introduction

With the development of computer technology, numerical simulation is more and more widely applied in the design of the grounding grid. The research on the numeric algorithm of grounding grids began in 1970's [1-7]. Nowadays, a number of novel algorithms are still proposing [8-11]. These methods are classified based on their different principles. Generally, they can be divided into method of moment (MoM), finite element method (FEM) and finite difference time domain method (FDTD). By using the Green's function, MoM can strictly describe the formulation relationship between any two unknowns in the domain space. However, the other two methods pass relationship by means of a series of intermediate variables, which can result the accumulation of dispersion errors, As a result, MoM possesses the highest accuracy in the low-frequency field [12]. In practice, most of the ground related algorithms are based on MoM.

The grounding numerical calculation methods can be classified into methods based on equipotential model and methods based on unequal-potential model. Equipotential model ignores the voltage drop on the grounding conductor, it takes the whole grounding grid as an Equipotential filed. Oppositely, for the unequal-potential model, the potential drop caused by the interior resistance of conductors is taken into account. In the condition of lower soil resistivity or larger conductor impedance, unequal-potential model is more practical. Nevertheless, because the equipotential model is the basis of the unequal-potential model, equipotential model can be converted into unequal-potential model by adding in some circuit analysis. Consequently, the equipotential model still has typical significance. In this paper, the equal-potential model with uniform soil model is investigated.

The remainder of this paper is organized as follows. Section II introduce the basic calculation method by MoM, In Section III, the calculation results of the direct numerical integration method, Heppe's method, Kouteynikoff's method and result from CDEGS are compared in two different grounding grid , Section IV state the mistake of the Kouteynikoff's method, and Section V concludes the findings.

# 2 Introduction of the MoM Algorithms

## 2.1 Process Description of Equal-potential Model Calculation Based on MoM

With the known information of grounding pole shape and soil conditions, the grounding resistance, ground potential rise (GPR), the step and touch voltages and other security parameters can be computed.

The basic idea of using MoM to compute grounding grid is dividing grounding conductor into several small line segments, and then obtain the leakage current of each conductor by solving linear equations, finally the potential at any point can be obtained based on superposition Method [13]. The general procedure is shown in Figure 1.

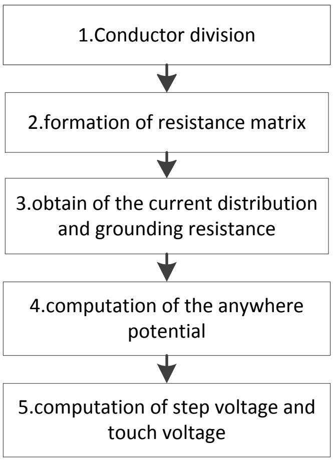


Fig. 1. Calculation Process of Equal-potential Grounding Model

Using Green's function concept, a buried electrode dissipating a current I in the earth will produce at any point P a potential

$$V_p = \iint_s G(P,Q)J(Q)dS \qquad (1)$$

Where, G(P,Q) is the Green's function, which may be viewed as the potential induced at point P by unit current flowing away from the electrode surface at point Q.

J(Q) is the current density at a point Q on the electrode surface such that the total current dissipated by this electrode is

$$I = \iint_s J(Q)dS \qquad (2)$$

A complex grounding electrode comprising linear conductors may be subdivided into a number of reasonably small segments, therefore, the following relations hold:

$$I = \sum_{j=1}^{n} I_j \qquad (3)$$

Considering that potentials in all segments are equal under the equal-potential model, supposing the electrode potential is $V_G$, $V_G$ can be written as

$$V_G = \sum_{j=1}^{n} R_{ij} I_j \qquad (4)$$

$R_{ij}$ is the average potential of the segment *j* when the unit current is leaked only on segment *i*. If *i*= *j*, $R_{ij}$ is defined as self resistance, other, if *i*≠*j*, $R_{ij}$ is defined as mutual resistance. Assuming uniform linear leakage current density over the segment $i^{th}$ surface, the formula to compute $R_{ij}$ is:

$$R_{ij} = \frac{\rho}{4\pi l_i l_j} \iint_{l_i l_j} \frac{dl_i dl_j}{r} \qquad (5)$$

Where, $\rho$ is the soil resistivity, $l_i$ and $l_j$ are the length of conductors *i*, *j*, respectively, and $r$ is the distance between infinitesimal $dl_i$ and infinitesimal $dl_j$.

Convert equation (4) to matrix form,

$$\mathbf{RI} = \mathbf{V} \qquad (6)$$

Where, $\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1n} \\ R_{21} & R_{22} & \dots & R_{2n} \\ \dots & \dots & \dots & \dots \\ R_{n1} & R_{n2} & \dots & R_{nn} \end{bmatrix}$ (7)

$\mathbf{I} = \begin{bmatrix} I_1 & I_2 & \dots & I_n \end{bmatrix}^T$

And, $\mathbf{V} = \begin{bmatrix} V_G & V_G & \dots & V_G \end{bmatrix}^T$.

By combining equations (3) and (6) to form simultaneous equations, we can get the current distribution of each segment $I_1$, $I_2$... $I_n$ and ground potential rise $V_G$, thus the grounding resistance can be obtained by

$$R = V_G / I \qquad (8)$$

## 2.2 Three Methods Based on the MoM Algorithm

Gaussian numerical integral is the direct method to implement equation 5 in computer programing. However, this approach is time consuming and difficult to achieve sufficient accuracy, so the following several methods are proposed.

1) CDEGS's Method

CDEGS is a powerful set of engineering software tools released by SES-tech [14], it has been widely used all over the world, and it plays an important impact on the ground numerical calculation domain. SES-tech's scholar Dawalibi and his team proposed their so-called multi step analysis and iterative analysis in 1970's. The result from CDEGS software is used as the basis for model comparison directly.

2) Heppe's Method

Heppe proposed his algorithm in 1979 [2], his core idea is to use a geometric formula instead of the time-consuming numerical calculation.

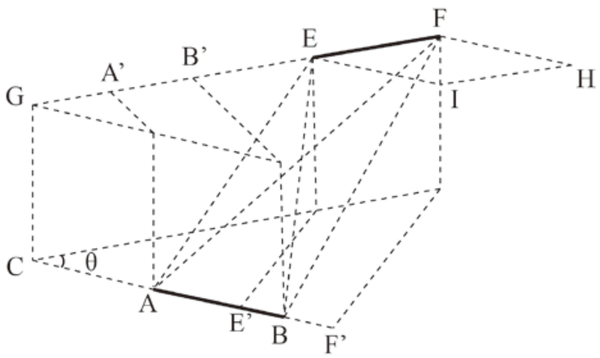


Fig. 2. Geometric Schematic of Heppe's Method

In figure 2, mutual resistance coefficient between conductor AB and conductor EF is

$$M_{12} = \underline{CB}\ln\frac{BF+\underline{B'F}}{BE+\underline{B'E}} - \underline{CA}\ln\frac{AF+\underline{A'F}}{AE+\underline{A'E}} + \underline{GF}\ln\frac{FB+\underline{F'B}}{FA+\underline{F'A}} - \underline{GE}\ln\frac{EB+\underline{E'B}}{EA+\underline{E'A}} - \frac{CG\bullet\Omega}{\sin\theta} \tag{9}$$

Where, $\theta$ is the angle between AB and EF,

$$\Omega = \arctan\left(\frac{CG}{BF\tan\theta}+\frac{\underline{CB}\bullet\underline{GF}\sin\theta}{CG\bullet BF}\right) - \arctan\left(\frac{CG}{BE\tan\theta}+\frac{\underline{CB}\bullet\underline{GE}\sin\theta}{CG\bullet BE}\right) - \arctan\left(\frac{CG}{AF\tan\theta}+\frac{\underline{CA}\bullet\underline{GF}\sin\theta}{CG\bullet AF}\right) + \arctan\left(\frac{CG}{AE\tan\theta}+\frac{\underline{CA}\bullet\underline{GE}\sin\theta}{CG\bullet AE}\right)$$

.The value of segment with underline can be positive or negative.

The advantage of this method is simple and clear, but it is subjected to severe round off errors which can be catastrophic in numerical process owing to digital calculations. The modified formula proposed by Nagar and Velazquez in 1985 [15] is implemented in this paper's program.

3) Kouteynikoff's Method

Kouteynikoff proposed his method in 1979 [3]. By this method, the grounding electrode is divided into segments, some of which are further sub-divided into micro segments, in such a way that the self resistance of a segment may be calculated more "accurately".

Kouteynikoff's method provides a more precise mathematical expression to determine potentials in the close proximity of an infinitely small micro segment with radius a and length b is:

$$V_p(r,z) = \frac{\rho}{\pi^2 ab}\int_{0^+}^{\infty}\frac{K_0(\lambda r)}{K_1(\lambda a)}\cos z\lambda \sin\frac{b\lambda}{2}\frac{d\lambda}{\lambda^2} \tag{10}$$

where, $r$, $z$ are abscissa and ordinate of the point P, respectively, $K_0$ and $K_1$ are the zero-order and first-order modified second type Bessel function, respectively .

Kouteynikoff's method is implemented completely in this paper's program. It is worth noting that Kouteynikoff proposed a simplified formula to accelerate computation using the point source concept. In this paper, the algorithm is accurately calculated by Eq. (10) with the cost of more computation time. The simplified formula is not employed in this paper's programing.

## 3 Case Study and Comparison

Gaussian numerical integral, Heppe's Method, CDEGS's method and Kouteynikoff's Method are employed in this case study, and the calculation accuracy is compared. Specifically, the shortcoming of Kouteynikoff's Method is investigated on more carefully.

### 3.1 Calculation Based on a Simple Grounding Grid

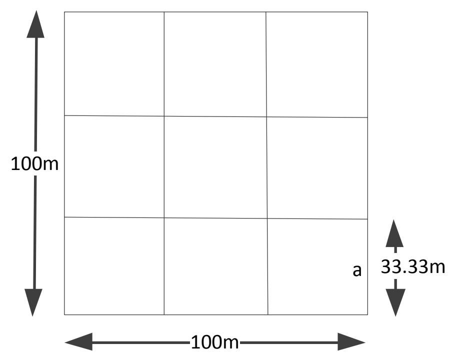


Fig. 3. A simple Grounding Grid

A simple grounding grid is studied first. Figure 3 is a simple square grounding grid, the conductor cross section is circular, and the radius is 0.0067056m, soil resistance is 100Ω·m. For this grounding grid, the basic cutting is performed, that is, cutting is carried only along the conductor intersection. Calculation results based on the above mentioned methods are shown in Table I.

TABLE I
THE BASIC CUTTING CALCULATION RESULTS OF GROUNDING RESISTANCE BASED ON DIFFERENT METHODS

| Computing Method | Grounding Resistance /Ω |
|---|---|
| Gaussian numerical integral（1000point） | 0.572595 |

| | |
|---|---|
| Heppe's Method | 0.557347 |
| Kouteynikoff's Method | 0.556070 |
| CDEGS | 0.546110 |

From Table, it is clearly that the calculation results are different based on different methods. However, the difference is not large, and the largest difference of resistance is not more than 5%.

On the basis of the basic cutting, every segment is further subdivided into $m$ micro-segments, then an $24 \times m$-order equations must be solved. With different $m$, a set of grounding resistance can be obtained:

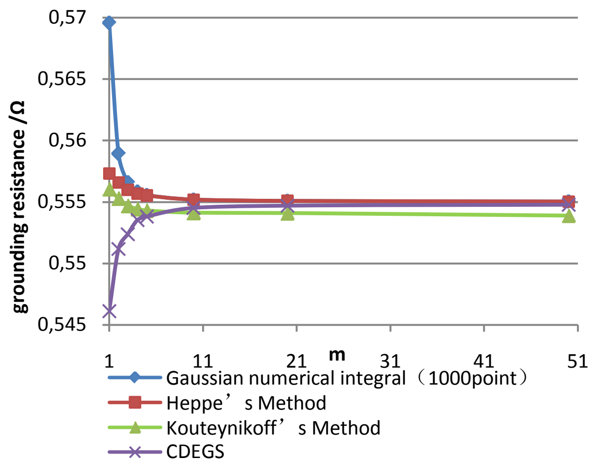


Fig. 4. Grounding Resistance Curve with Different m Based on Four Methods

Figure 3 shows the calculation results of grounding resistance with difference micro-segment numbers. The all four methods tend to be stable along with the increase of micro-segment number of $m$, specifically, Gaussian numerical integral, Heppe's Method and CDEGS's result tended to the same value, while Kouteynikoff's Method tends to another similar value. When $m$=1(no further subdividing is performed), the relative error of 1000 point's Gaussian numerical integral is 3.16%, CDEGS is -1.56%, and Heppe's Method is 0.41%. If the stable value of the above three methods is used as the benchmark value, then the relative error of Kouteynikoff's Method is 0.18% .

## 3.2 Calculation Based on a More Complex Grounding Grid

A more complex grounding grid is chosen in order to fully investigate on the accuracy of the four different methods.

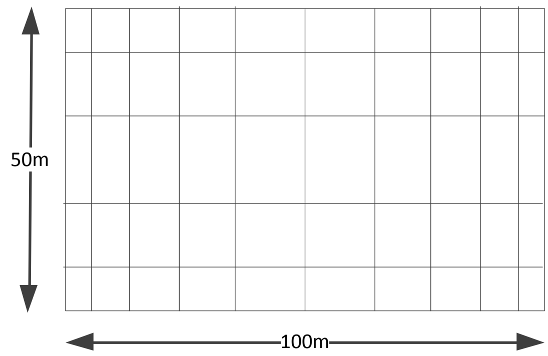


Fig. 5. A Complex Grounding Grid

Figure 5 is an unequally spaced rectangular ground grid with a scale factor of 0.8, the radius is 0.0067056m, and soil resistance is 100Ω·m. With the increase of $m$, calculation results based on the four methods are reported in Figure 6.

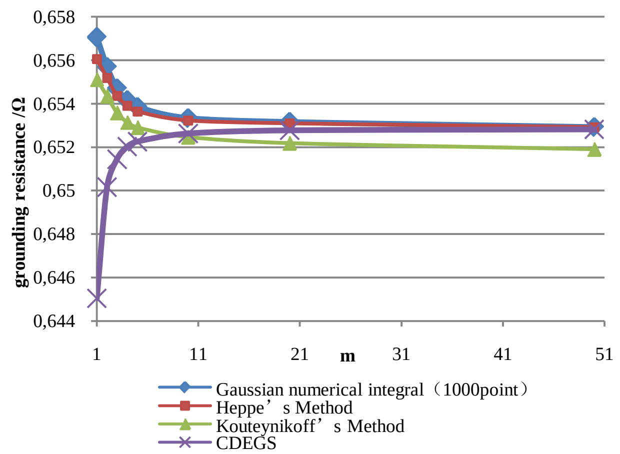


Fig. 6. Grounding Resistance Curve with Increase of $m$ Based on Four Methods

From figure 6, it can be concluded that the tendency of grounding resistance curve do not change in spite of the shape of grounding grid. Compared to the condition of the simple grounding grid, the initial error is lower in the more intensive grid.

A dozen of grounding grids with different structures are examined, and the following rules are found:

1) The grounding resistance calculated by all methods tends to be stable along with the increase of the number of micro-segments $m$.

2) Gaussian numerical integral, Heppe's Method and CDEGS's result tended to the same value, while Kouteynikoff's Method tends to another similar value.

3) The calculation results by CDEGS tend to increase along with the increase of $m$, and the calculation results by other methods tend to

decrease along with the increase of *m*.

4) The more intensive grounding grid, the lower the initial error.

From the comparison analysis, it is obvious the Kouteynikoff's method cannot conclude to a converging a same value as the other 3 methods. The principle of the Kouteynikoff's method is further investigated in the following section.

## 4 Investigation on the Kouteynikoff'S Method

The above analysis shows that Kouteynikoff's method different from other methods, it tends to another value along with the increase in the number of micro-segment m. In order to explain this phenomenon, this paper argues that there is an error in Kouteynikoff's method.

Kouteynikoff claimed in his paper that the method can obtain exact values of self-resistance, using a supplementary fragmentation. However, it may be not the case.

To illustrate this question, the segment A in the grounding grid in figure 3 can be used as a typical example. First of all, by dividing the entire ground electrode into n = 2400 (segment A is divided by m = 100), the current density of segment A from CDEGS is obtained and used as a benchmark. Then the current density of n = 24 (m=1) and n=240 (m=10) by Kouteynikoff's method and Heppe's method are calculated and compared. Figure 7 shows the current density of segment with different methods.

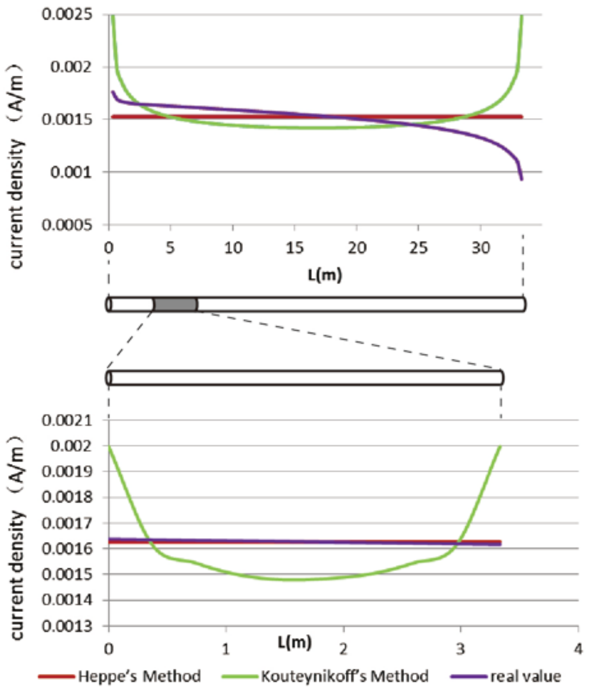


Fig. 7 Different Methods for Handling the Current Density of Segment A

In Figure 7, the upper half of figure 7 show the current density of segment A of $n$ = 24 ($m$=1) , and the lower half of Figure 7 show the current density of a micro segment when $n$ = 240 ($m$=10).

From the density curve comparison in Figure 7, it is clear that Kouteynikoff's method and other methods deal with the current density of segment in different ways, that is to say, different leakage current density assumptions are employed between Kouteynikoff's method and other method. Specifically, Kouteynikoff's method assumes that current flows out of the segment freely without other segment interference, while Heppe's and other's method assumes that current flows out of the segment uniformly along the axis of conductor. In fact, the practical leakage current density of micro-segment is related to the position of the segment. The different assumption of the two types of methods is the source of calculation errors.

When $m$=1, either Kouteynikoff's method or Heppe's method can generate a big gap from the real value. However, with the increase of $m$, for example, when $m$ =10, the assumption of segment's current density from Heppe's method have a good agreement with the real value, while Kouteynikoff's method still has a large gap.

In fact, the Kouteynikoff's method incorrectly believes that the grounding resistance of a single conductor only equal to the self-resistance of the conductor in a total grounding grid. However, since the self-resistance is not essential attribute of a conductor segment, it can be changed with the change of leakage current distribution, and the same conductor segment may have different self-resistance just because of its different positions in the system. This mistake makes the error of Kouteynikoff's method do not decrease with the increase of *m*.

## 5 Conclusion

A software for simulating grounding systems has been developed based on the method of moments, It implements several important algorithms such as direct numerical integration method, Heppe's method, and Kouteynikoff's method.

1) Under the assumption of uniform soil and ignoring the conductor impedance, the calculation results of the above method and CDEGS are compared. The grounding resistance calculated by all methods tends to be stable along with the increase of the number of micro-segments *m*, and

Gaussian numerical integral, Heppe's Method and CDEGS's result tended to the same value, while Kouteynikoff's Method tends to another similar value.

2) Through theoretical analysis and program test, the shortness of Kouteynikoff's method is discussed.

*References:*

[1] Dawalibi F, Mukhedkar D. "Optimum Design of Substation Grounding in a two Layer Earth Structure, Parts I, II and III," *IEEE Transactions on Power Apparatus and Systems,* 1975(2).

[2] R. J. Heppe, "Computation of potential at surface above an energized grid or other electrode, allowing for non-uniform current distribution," *IEEE Trans. Power* App. Syst., vol. PAS-98, no. 6, pp. 1978–1989,Nov./Dec. 1979.

[3] Kouteynikoff P. "Numerical Computation of the Grounding Resistance of Substations and Towers," *IEEE Transactions on Power Apparatus and Systems*, 1980(3).

[4] Dawalibi F P. "Electromagnetic fields generated by overhead and buried short conductors part 2 – ground conductor," *IEEE Transaction on Power Delivery,* 1986, 1(4), pp.112-119.

[5] F. P. Dawalibi, J. Ma, and R. D. Southey, "Behavior of grounding systems in multilayer soils: A parametric analysis," *IEEE Trans. Power Del.,* vol. 9, no. 1, pp. 334–342, Jan. 1994.

[6] F. P. Dawalibi and N. Barbeito, "Measurements and computations of the performance of grounding systems buried in multilayer soils," *IEEE Trans. Power Del.,* vol. 6, no. 4, pp. 1483–1490, Oct. 1991.

[7] Nekhoul B, Guerin C, Labie P, et al. "A finite element method for calculating the electromagnetic fields generated by substation grounding systems," *IEEE Trans. on Magnetics*,1995,31(3), pp. 2150-2153.

[8] Li Zhongxin, Yuan Jiansheng, Zhang Liping, "Numerical analysis of substation grounding systems," *Proceedings of the CSEE,*1999,19(5): 75-79(in Chinese).

[9] LU Zhiwei, WEN Xishan, SHI Yan-ling, et al. "Numerical calculation of large substation grounding grids in industry frequency." *Proceedings of the CSEE*,2003,23(12),pp.89-93.

[10] ZhangBo, Cui Xiang, Zhao Zhibin, et al. "Analysis of the complex grounding grids in frequency domain considering mutual inductances," *Proceedings of the CSEE,* 2003, 23(4),pp.77-80(in Chinese).

[11] Gan Yan，Ruan Jiangjun，Chen Yunping．Application of unidimensional finite element method (FEM) coupled with three dimensional FEM in characteristics analysis of grounding mesh property [J]．*Power System Technology*，2004，28(9)：62-66(in Chinese)．

[12] Xinqing S. *A Brief Treatise on Computational Electromagnetics* [M]. Second Edition. China AnHui: Press of University of Science and Technology of China, 2008, pp.160-161.

[13] He Jinliang, Zeng Rong. *Power System Grounding Techniques* [M]. First Edition. Science Press, 2007.

[14] CDEGS Software Package, Safe Engineering Services & technologies ltd., Montreal, Quebec, Canada, 2006.

[15] Nagar R P, Velazquez R. "Review of Analytical Methods For Calculating The Performance Of Large Grounding Electrodes PART 1 and PART 2," *IEEE Transactions on Power Apparatus and Systems,* 1985(11).