Comparison and analysis of GNSS signal tracking performance based on Kalman filter and traditional loop

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Abstract: - As Kalman filter technology has better performance for estimation and prediction of dynamic signal, it is gradually used in GNSS signal tracking. According to the steady-state error, transfer function and equivalent noise bandwidth of Kalman filter and traditional loop in steady status, the tracking performance of these two methods is compared in theory. The theoretical analysis demonstrates that, the dynamic stress error of Kalman filter tracking is less than traditional loop. Kalman filter method can track dynamic signal accurately with small equivalent noise bandwidth. The analysis results are verified by simulation, and the simulation results show that the tracking sensitivity of Kalman filter is similar to that of the traditional loop. The Kalman filter tracking method has higher dynamics performance and better accuracy.

Key-Words: - GNSS, Signal Tracking, Kalman Filter, traditional loop, Phase Locked Loop, steady status.

1 Introduction

Tracking progress of GNSS receiver is essentially a problem of dynamic parameter estimation. Traditional tracking architecture uses two loops, Delay Locked Loop (DLL) and Phase Locked Loop (PLL). PLL is a carrier tracking loop, it adjusts the frequency and phase of local replicated signal to consist with incoming signal. Its basic concept is to model the incoming signal as known dynamics, and determine the optimal filter structure by Wiener filter approach. The details of the traditional PLL method can be found in many references on GNSS signal processing [1].

As Kalman filter (KF) theory can estimate and predict dynamic signal quite well, it is gradually used for GNSS signal tracking process. In addition, the newest GNSS receiver tracking algorithm, vector based tracking algorithm is realized based on KF [2]. Therefore, using KF to track GNSS signal has become a main research direction in navigation field. There are many different KF methods for the GNSS receiver's signal tracking loops, different researchers have different ways to classify these methods, but their fundamentals are the same. If classified by observation, these methods can be divided to two categories [3] : Kalman filter with discriminator output as observation; Extended Kalman filter (EKF) with nonlinear observation of I and Q correlator outputs. Main difference between them is that, the model for using discriminator value as observation is linear, so it uses KF, and that for using baseband I and Q correlation values as observation is nonlinear, so it uses EKF. If classified by filtering state and observation, they can be divided to four categories [4]: Error state EKF for a loop filter with nonlinear observation of I and Q correlator outputs; Error state linear KF for a loop filter with discriminator output as observation; Error state KF for a loop filter with discriminator output as observation; Direct state KF for an entire signal tracking loop with discriminator output as observation residuals. The main difference between the second and third methods is that, the second method uses discriminator output of coherent integration at initial time as observation, and the third method uses the average of discriminator output during the whole coherent integration process. Kalman filter used in this paper is the third method. It uses the average of discriminator output as observation, and signal parameter error as filtering state.

Traditional PLL and Kalman filter both can realize GNSS signal tracking, so the difference and relationship between these two methods gradually becomes an important research point. Certain effect has been obtained by using EKF to realize digital phase locked loop (DPLL) [5]. It has been proved that the traditional PLL structure can be transformed to the similar structure of Kalman filter [6], and

Kalman filter also can be transformed to the similar structure of PLL [7]. The form of steady-state Kalman filter is similar to that of traditional PLL. The equivalent noise bandwidths can be compared for relevant research of these two tracking methods, but it has less research [8]. In order to derive the equivalent loop bandwidth of steady-state Kalman filter, steady-state Kalman gain was written into a form similar to the gain of PLL [9]. Many mathematic approximates were made to illustrate the relationship between them. However, the research results needed to be tested and verified. After that, the equivalent loop bandwidth of EKF tracking method was obtained by experimental performance, and compared with standard PLL [10]. Two approaches were adopted to get it: the first was to find the equivalent PLL that gave the same performance as the steady-state EKF, and the second was to find an equivalent EKF that gave the same performance as the PLL. It was proved that Kalman filter has faster adaption and higher sensitivity than standard PLL.

Taking second order carrier tracking loop for example, the carrier tracking method based on Kalman filter is researched, and the design method of filtering parameters is also given under the stability condition of steady-state Kalman filter. According to the similar architecture of steady-state Kalman filter and traditional second order PLL, the closed loop transfer function of steady-state Kalman filter is derived, and its equivalent noise bandwidth and steady-state error are also calculated. By comparing above parameters with that of traditional PLL, the reason of Kalman filter having better performance than traditional loop is analyzed. Finally, the theoretical analysis results are verified by simulation.

2 Carrier tracking model for Kalman filter

Satellite signal received by satellite navigation receiver antenna can be expressed as follows:

$$r(t) = d(t)\sqrt{2C}\cos(2\pi ft + \theta_0)c(t-\tau) + n(t) \tag{1}$$

where $d(t)$ is data code, C is carrier power, f is carrier frequency including Doppler effects, θ_0 is initial carrier phase, $c(t)$ is ranging code, τ is instantaneous code phase, and $n(t)$ is additive observation noise.

Unknown parameters in the model are code phase, carrier phase and carrier Doppler frequency. To realize satellite navigation and positioning, it needs to estimate these parameters accurately, and acquire observation information of pseudo-range and its rate. As satellites and vehicle moving with time, these navigation parameters are dynamic. The real-time estimation process of these parameters is referred to as signal tracking. In satellite navigation receiver, the incoming signal after digital sampling is correlated with a local replica to wipe-off the spreading code, and to accumulate signal energy to increase detectability and tracking performance. Carrier tracking is realized by correlating with two quadrature replicas of a locally generated carrier in two branches. After 1ms coherent integral accumulation, the models of I branch and Q branch are expressed as follows.

$$I_k(\Delta) = \frac{N_k \bar{A}_k d_m}{2}\cos(\Delta\theta_k)R(\Delta\tau_k + \Delta) + n_{Ik} \tag{2}$$

$$Q_k(\Delta) = -\frac{N_k \bar{A}_k d_m}{2}\sin(\Delta\theta_k)R(\Delta\tau_k + \Delta) + n_{Qk} \tag{3}$$

where, Δ is code offset of local reference code relative to prompt code (early code $\Delta>0$, and late code $\Delta<0$), N_k is the number of sampling points in coherent integral time, $\bar{A}_k$ is the average carrier phase amplitude in coherent integral time, d_m is navigation bit, $\Delta\theta_k$ is the carrier phase error in coherent integral time, $\Delta\tau_k$ is the code phase error in coherent integral time, n_{Ik} and n_{Qk} are uncorrelated discrete Gaussian white noise sequences.

The phase discriminator output of 1ms coherent integral accumulation is $\Delta\theta_k$, and it is regarded as the average carrier phase error in integral interval. Two-quadrant arctangent phase discriminator is used for second order carrier phase tracking loop, and the output of discriminator is shown as follows:

$$Z(k) = -a\tan\left(\frac{QP(k)}{IP(k)}\right) \approx \Delta\theta(k) + v(k) \tag{4}$$

Where $v(k)$ is random noise of discriminator output.

The output of discriminator $Z(k)$ is the input of Kalman filter.

Basic equations of Kalman filter are as follows:

(a) project the state ahead

$$\hat{X}_{k/k-1} = \Phi_{k,k-1}X_{k-1} \tag{5}$$

(b) update estimate with measurement

$$\hat{\hat{X}}_k = X_{k/k-1} + K_k\left(Z_k - H_k X_{k/k-1}\right) \tag{6}$$

(c) compute the Kalman gain

$$K_k = P_{k/k-1}H_k^T\left(H_k P_{k/k-1}H_k^T + R_k\right)^{-1} \tag{7}$$

(d) project the error covariance ahead

$$P_{k/k-1} = \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + Q_{k-1} \tag{8}$$

(e) update the error covariance

$$P_k = (I - K_k H_k) P_{k/k-1} (I - K_k H_k)^T + K_k R_k K_k^T \tag{9}$$

Kalman filter using in GNSS signal tracking was realized based on these five equations. In this paper, GNSS signal was processed in signal channel. How to set state X, observation Z, observation matrix H, one step transfer matrix Φ will be illustrated in this section. How to set observation noise variance matrix R and driving noise variance matrix Q is the key point of Kalman filter, it will be illustrated in next section.

When second order carrier phase tracking loop of GPS signal is realized by Kalman filter, continuous dynamic carrier is modeled as follows:

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{f} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta f \end{bmatrix} + w(t) \tag{10}$$

Where $\Delta\theta$ is the average carrier phase error in integral interval, Δf is the average Doppler frequency error in integral interval, and $w(t)$ is Gaussian white noise vector with zero mean.

Carrier phase error of discriminator output is used as observation of Kalman filter, the relationship of observation Z and system state variable $\Delta\theta$, Δf is expressed as follows:

$$Z = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta f \end{bmatrix} + v \tag{11}$$

Therefore, observation matrix is $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Continuous state equation is:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{G}(t)w(t) \tag{12}$$

where $\mathbf{F}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $G(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $w(t)$ is driving noise matrix with 2*1 dimension.

The state equation after discretization is:

$$X_k = \Phi X_{k-1} + W_{k-1} \tag{13}$$

Where Φ is one step transfer matrix, and W_{k-1} is driving noise sequence.

System state vector is $X_k = \begin{bmatrix} \Delta\theta_k \\ \Delta f_k \end{bmatrix}$.

One step transfer matrix is:

$$\begin{aligned} \Phi &= I + TF + \frac{T^2}{2!}F^2 + \frac{T^3}{3!}F^3 + ... \\ &\approx I + TF + \frac{T^2}{2!}F^2 \\ &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{14}$$

$Q_k = E\{W_{k-1}W_{k-1}^T\}$ is discrete driving noise variance matrix, it can be calculated according to the continuous driving noise variance matrix q (q is 2*2 matrix).

$$Q_k = \frac{\mathrm{T}}{1!} \cdot M_1 + \frac{\mathrm{T}^2}{2!} \cdot M_2 + \frac{T^3}{3!} \cdot M_3 + ... \tag{15}$$

Only adopting the first and second term of (15), the approximate treatment of discrete driving noise variance matrix is:

$$\begin{aligned} Q_k &\approx \frac{\mathrm{T}}{1!} \cdot M_1 + \frac{\mathrm{T}^2}{2!} \cdot M_2 \\ &= T \cdot \left(GqG^T\right) + \frac{1}{2}T^2 \cdot \left(GqG^T F^T + FGqG^T\right) \end{aligned} \tag{16}$$

Discrete observation of Kalman filter is $Z_k = \Delta\theta_k + V_k$, where V_k is observation noise sequence.

3 Parameter design of Kalman filter tracking

The Kalman filtering model of second order carrier tracking is constant velocity (CV) model with constant coefficient, so the carrier tracking filter is stable Kalman filter (time-invariant or constant coefficient Kalman filter). Comparing with normal Kalman filter, stable Kalman filter should meet following three conditions.

(a) State model and observation model are time invariant.

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gw(t) \\ z(t) &= Hx(t) + v(t) \end{aligned} \tag{17}$$

(b) Driving noise and observation noise are wide stable at least.

$$\begin{aligned} \mathrm{cov}\{w(t)w(t+\tau)\} &= q(\tau) = q \cdot \delta(\tau) \\ \mathrm{cov}\{v(t)v(t+\tau)\} &= R(\tau) = R \cdot \delta(\tau) \end{aligned} \tag{18}$$

(c) Observation interval begins at the moment. $t \to -\infty$.

When Kalman filter is stable, its closed loop transfer function and equivalent noise bandwidth can be obtained. According to the driving noise variance matrix and observation noise variance matrix, the state estimation covariance matrix and filtering gain matrix can be calculated.

When Kalman filter arrives to steady status, $P_k = P$, which is:

$$P = \left(I - PH^T R^{-1} H\right)\left(\Phi P \Phi^T + Q\right) \tag{19}$$

The observation of Kalman filter is carrier phase error of discriminator output, so the observation noise variance matrix R is the variance of carrier phase error.

$$R = \sigma_{\delta\varphi}^2 = \frac{1}{2C/N_0 T} \tag{20}$$

When carrier noise ratio of normal signal is 44dB-Hz, and the coherent integral time is 1 ms, the value of R is $R=0.1411^2$

The driving noise variance matrix of Kalman filter mainly depends on the relative dynamic between satellite and receiver and crystal oscillator error [11]. Radial acceleration of GPS satellite relative to earth surface is expressed as follows:

$$a = \frac{dv_d}{d\theta}\cdot\frac{d\theta}{dt} = \frac{vr_e[r_e r_s \sin^2\theta - (r_e^2 + r_s^2)\sin\theta + r_e r_s]}{(r_e^2 + r_s^2 - 2r_e r_s \sin\theta)^{\frac{3}{2}}}\cdot\frac{d\theta}{dt} \quad (21)$$

Where the average radius of earth is $r_e \approx 6368km = 6.368\times10^6\mathrm{m}$, the average radius of satellite orbit is $r_s \approx 26560km = 2.656\times10^7 m$, the angular velocity of satellite is $\frac{d\theta}{dt}=\frac{2\pi}{11\times3600+58\times60+2.05}\approx1.458\times10^{-4} rad/s$, and satellite velocity is $v=\frac{r_s d\theta}{dt}\approx3874m/s$.

The maximum radial acceleration of GPS satellite relative to earth surface is $0.178m/s^2$, and its corresponding Doppler frequency rate is:

$$\Delta f_{doppler} = \frac{a}{c} f_{L1} = \frac{0.178}{3\times10^8}\times1575.42\times10^6 = 0.935Hz/s = 5.875rad/s \quad (22)$$

In GPS receiver, main factors of crystal oscillator error influence on the driving noise of Kalman filter are frequency random walk coefficient h_{-2} and white noise frequency coefficient h_0 [12].

Carrier phase noise variance caused by crystal oscillator error is:

$$q_{\varphi TCXO} = (2\pi f_0)^2\times\frac{h_0}{2} \quad (23)$$

Doppler frequency noise variance caused by crystal oscillator error is:

$$q_{fTCXO} = (2\pi f_0)^2\times 2\pi^2 h_{-2} \quad (24)$$

According to the crystal oscillator type and parameter of GPS receiver, q_φ and q_f can be calculated, and the driving noise variance matrix of continuous Kalman filter is $q=\begin{bmatrix} q_\varphi & 0 \\ 0 & q_f+(\Delta f_{doppler})^2\end{bmatrix}$.

Assume that the solution of steady-state P matrix is $P=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22}\end{bmatrix}$, and take it into equation (19). We can get its final solution by method of undetermined coefficients.

Using matrix P can calculate the gain matrix K of Kalman filter, which is:

$$K = PH^T r^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\frac{1}{r} = \begin{bmatrix}\frac{P_{11}}{r}\\ \frac{P_{21}}{r}\end{bmatrix} \quad (25)$$

In following part, Kalman filter method is used to track static GPS signal with different power, its model is given in the first section, and its observation noise variance matrix R and continuous driving noise variance matrix q can be calculated by (20)-(24). GPS signal is produced by simulator. The simulated signal does not include crystal oscillator error, so the design of q ignores its influence. With fixed values of q and R, K and P of Kalman filter changing with the time are shown in Fig.1 and Fig.2. From these figures we can see that after Kalman filter convergence, the values of K and P don't change with time. When Kalman filter is in steady status, through equation (19) and (25), we can calculate K and P corresponding to different observation noise variance matrix R and continuous driving noise variance matrix q, which are shown in figure 3 and figure 4.

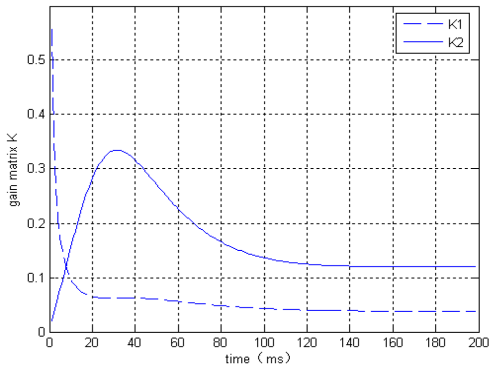

Fig.1 matrix K changing with time

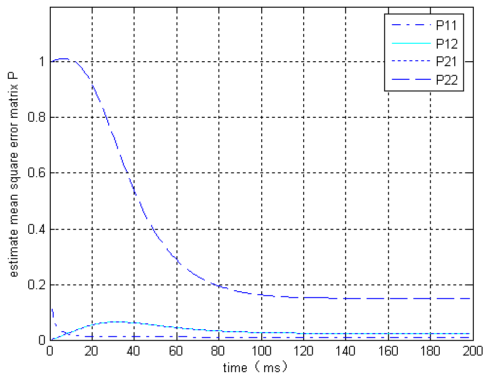

Fig. 2 matrix P changing with time

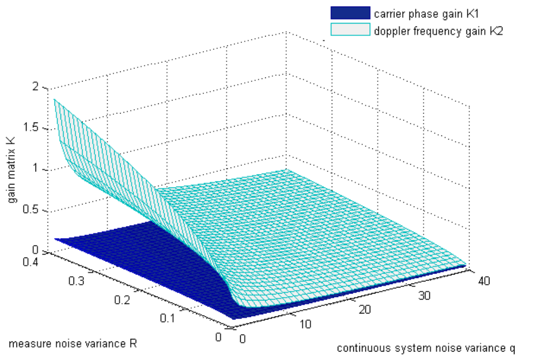

Fig. 3 matrix K corresponding to different q and R

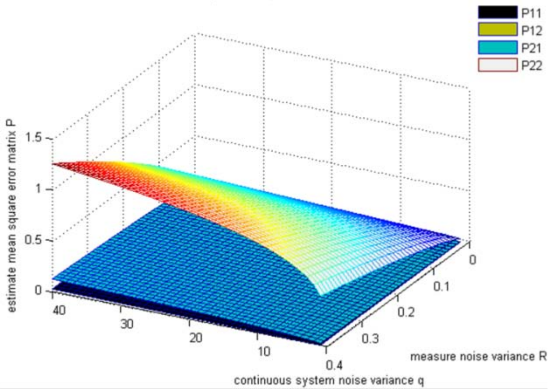

Fig. 4 matrix P corresponding to different q and R

Fig. 3 and Fig. 4 show that under the condition of steady-state Kalman filter, K and P are completely determined by R and q. Once R and q are fixed, Kalman filter will converge to the same fixed value.

4 Theoretical tracking performance comparison and analysis between Kalman filter and traditional PLL

4.1 The steady-state error analysis of discriminator output

According to the analysis of previous section, steady-state estimation error of Kalman filter is shown in equation (19). In order to compare theoretical performance of second order Kalman filter and traditional PLL by locking loss determination, we compare the steady-state discriminator output errors of these two methods.

For Kalman filter tracking method, the error of carrier tracking discriminator output is observation residual (it is also called new message). Under steady-state Kalman filter status, the observation residual is Gaussian noise with zero mean, so the variance of observation residual error is steady-state error of Kalman filter tracking.

$$\sigma_{kalman} = \sqrt{H_k P_{k/k-1} H_k^T + R_k} \tag{26}$$

For traditional PLL tracking loop, the standard deviation of two quadrant arctangent discriminator output phase error caused by thermal noise is:

$$\sigma_{noise} = \frac{360}{2\pi}\sqrt{\frac{1}{2C/N_0 T}} \ (degree) \tag{27}$$

For traditional second order PLL tracking loop, the steady-state error of two quadrant arctangent discriminator output caused by dynamics is:

$$\sigma_{dynamic} = \frac{d^2R/dt^2}{\omega_0^2} = 0.2809\frac{d^2R/dt^2}{B_n^2} \ (degree) \tag{28}$$

Where, for GPS L1 band signal, steady-state error caused by $10m/s^2$ acceleration is:

$$\begin{aligned}\frac{d^2R}{dt^2} &= (10m/s^2) \times (360^\circ / cycle) \times \\ &((1575.42) \times 10^6 cycle/s)/c \\ &= 18905^\circ/s^2\end{aligned} \tag{29}$$

For discriminator output carrier phase errors of Kalman filter and traditional PLL, according to equation (20), we can see that the steady-state errors caused by thermal noise of these two methods are same, but the steady-state errors caused by dynamic stress are different. For different signal power and vehicle dynamics, the steady-state carrier phase errors of Kalman filter and traditional PLL are shown in figure 5.

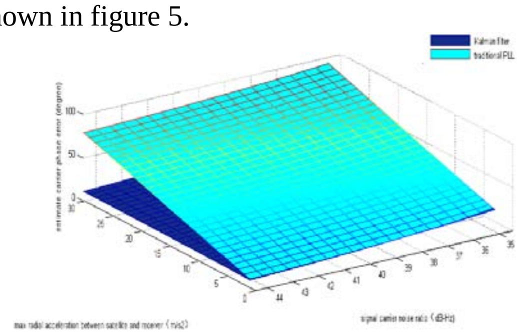

Fig. 5 discriminator output carrier phase errors of Kalman filter and traditional PLL

From Fig. 5 we can see that, when there is no radial dynamics between satellite and receiver, discriminator output errors of Kalman filter and traditional PLL have little difference, and this error is mainly determined by signal power. When there is a high radial dynamics between satellite and receiver, the discriminator output error of Kalman filter is less than that of traditional PLL. If discriminator output error reaches its linear range of 90 degrees, the loop will lose lock. Traditional PLL with equivalent noise bandwidth $B_n = 15Hz$ loses lock when there is $30m/s^2$ acceleration dynamics, but Kalman filter does not lose lock under this dynamic condition. Therefore, Kalman filter has more powerful ability than traditional PLL for tracking dynamic signal.

4.2 Analysis of equivalent noise bandwidth

According to equation (28) we can know that, the steady-state error of traditional PLL caused by dynamic stress is related to equivalent noise bandwidth. Therefore, we will analyze the equivalent noise bandwidth of Kalman filter, and find the reason why Kalman filter has better performance than traditional loop.

According to the analysis of the second section, carrier tracking Kalman filter is stable, so we can get its closed loop transfer function and equivalent noise bandwidth. From continuous Kalman filter equation of carrier tracking loop, state estimation value is:

$$\begin{aligned}\dot{\hat{x}}(t) &= Fx(t) + K(t)[z(t) - Hx(t)] \\ &= [F - K(t)H]\hat{x}(t) + K(t)z(t)\end{aligned} \tag{29}$$

Take Laplace transform on both side of equation (29), we can get:

$$\hat{X}(s) = (sI - F + KH)^{-1}KZ(s) \tag{30}$$

Transfer function of stable Kalman filter is calculated as follows:

$$H(s) = \hat{X}(s)Z^{-1}(s) = (sI - F + KH)^{-1}K \tag{31}$$

Closed loop transfer function of Kalman filter is:

$$\begin{aligned}&\mathbf{H}(s) = (s\mathbf{I} - \mathbf{F} + \mathbf{KH})^{-1}\mathbf{K} \\ &= \begin{bmatrix} H_0(s) \\ H_1(s) \end{bmatrix} = \begin{bmatrix} \dfrac{K_1 s + K_2}{s^2 + K_1 s + K_2} \\ \dfrac{K_2(1-K_1)s + K_1K_2}{s^2 + K_1 s + K_2} \end{bmatrix}\end{aligned} \tag{32}$$

The gain matrix of Kalman filter is $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$

Then, the closed loop transfer function of steady-state Kalman filter is:

$$H_0(s) = \frac{K_1 s + K_2}{s^2 + K_1 s + K_2} \tag{33}$$

The corresponding equivalent noise bandwidth is [13]:

$$B_n = \frac{1}{2|H(0)|^2}\int_{-j\infty}^{j\infty} H(s)H(-s)\frac{ds}{2\pi j} = \frac{1}{4}\left(K_1 + \frac{K_2}{K_1}\right) \tag{34}$$

From equation (34) we can know that the equivalent noise bandwidth of Kalman filter is related to the gain value of Kalman filter. For different signal power and dynamics, we can calculate different Q and R. By equation (19) and (25), we can get the gain matrix K after Kalman filter convergence, and then through equation (34), we can finally get equivalent noise bandwidths of Kalman filter under different signal conditions, which are shown in Fig. 6.

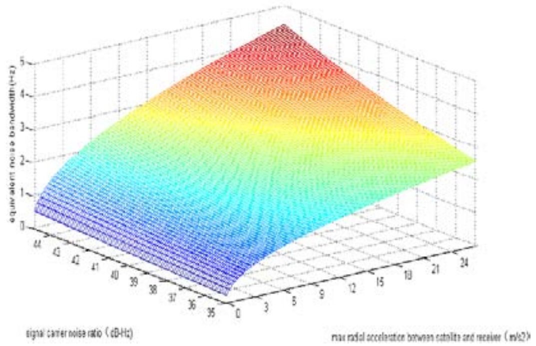

Fig. 6 Equivalent noise bandwidths of Kalman filter under different signal conditions

From Fig. 6 we can see that, when signal power becomes weaker, the equivalent noise bandwidth decreases, and when signal dynamics becomes higher, the bandwidth increases. Therefore, the equivalent noise bandwidth of Kalman filter is determined by signal power and dynamics. Normal GPS signal power is always near 44dB-Hz, and the radial acceleration between receiver and satellite is usually less than for $3m/s^2$ low dynamic vehicle. So the equivalent noise bandwidth is $B_n = 1.2Hz$ for the normal parameter of Kalman filter.

Gain value and equivalent noise bandwidths of steady-state Kalman filter with typical signal power and dynamics are shown in table 1.

Table 1 Bandwidth Bn and gain K with different signal power and dynamics

Signal power (dB-Hz)	Signal dynamics (m/s2)	Phase gain K1	Frequency gain K2	Bandwidth Bn (Hz)
44	0.2	0.017299	0.024087	0.35187
44	1	0.043296	0.153338	0.89214
44	5	0.088755	0.65727	1.8731
44	10	0.120377	1.22994	2.5853
44	30	0.196333	3.428988	4.4121
36	0.2	0.00732	0.003749	0.14824
36	1	0.025075	0.051077	0.51198
36	5	0.056348	0.260985	1.1689
36	10	0.073049	0.441823	1.5287
36	30	0.118199	1.184366	2.5354

It can be seen from Table 1 that, with normal signal power and dynamics, the equivalent noise bandwidth of Kalman filter is less than 5 Hz，and it is much smaller than the bandwidth of traditional PLL. Therefore, Kalman filter tracking has better ability in noise suppression.

4.3 Analysis of Doppler frequency estimation accuracy

Doppler frequency estimation error of Kalman filter can be represented by the frequency dimension value of state estimation covariance matrix at steady status. The relationship between frequency estimation error and phase estimation error of traditional PLL is expressed as follows:

$$\sigma_{\varepsilon f}^2 = \frac{\kappa 4\pi^2 B_L^2}{3}\sigma_{\varepsilon\theta}^2 \tag{35}$$

Where the value of coefficient κ approximates to 4 when the equivalent noise bandwidth of PLL is about 10 Hz.

The phase estimation error of traditional PLL is:

$$\sigma_{PLL\theta} = \sqrt{\sigma_{tPLL}^2 + \sigma_v^2 + \theta_A^2} + \frac{\theta_e}{3} \tag{36}$$

Where, it is taken no account to oscillator quiver σ_v caused by shake and oscillator vibration θ_A caused by Allen deviation.

The error caused by thermal noise is:

$$\sigma_{tPLL} = \frac{360}{2\pi}\sqrt{\frac{B_n}{C/N_0}} \tag{37}$$

The error caused by dynamic stress is:

$$\theta_e = \frac{d^2R/dt^2}{\omega_0^2} = 0.2809\frac{d^2R/dt^2}{B_n^2} \tag{38}$$

According to (35)-(38), we can calculate the theoretical Doppler frequency accuracy of traditional PLL. Theoretical Doppler frequency accuracies of Kalman filter and traditional PLL with equivalent noise bandwidth 15 Hz are shown in figure 7.

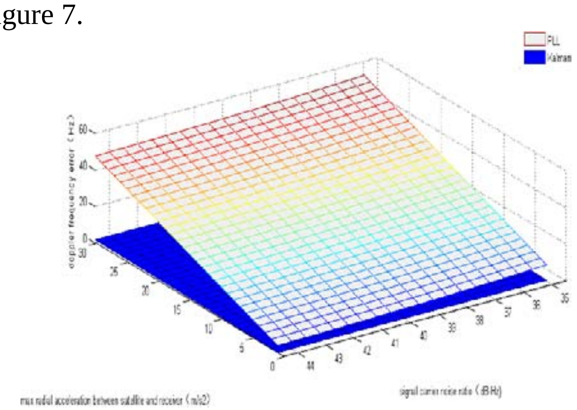

Fig. 7 Doppler frequency accuracies of KF and PLL under different signal conditions

From Fig. 7 we can see that, frequency estimation accuracy of Kalman filter is much higher than that of traditional PLL, especially in high dynamics. The main reason is that Kalman filter has smaller equivalent noise bandwidth than traditional PLL under normal signal conditions.

5 Tracking performance simulation comparison and analysis

In order to validate the theoretical performance analysis results of Kalman filter and traditional PLL tracking methods, in this section, the tracking sensitivity, dynamic adaptability and tracking accuracy of these two methods are tested and analyzed by simulation, and the equivalent noise bandwidth is used as comparing parameter.

(a)Tracking sensitivity analysis

The GPS data is collected from actual environment, and its signal power is reduced 1dB for every 10s after 50s by adding gradually increasing noise into the data. Kalman filter and traditional PLL with different equivalent noise bandwidth are adopted to track the signal, and their output Doppler frequencies are shown in figure 7 and figure 8. When the output Doppler frequency is wrong, the signal power at this time is their tracking sensitivity.

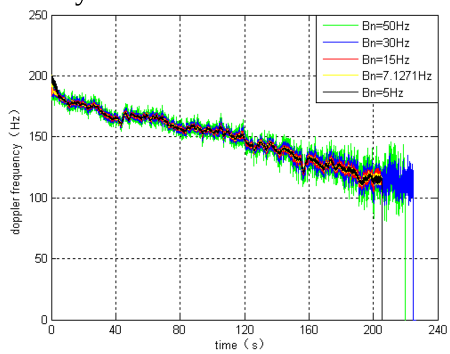

Fig. 8(a) Tracking sensitivity of PLL

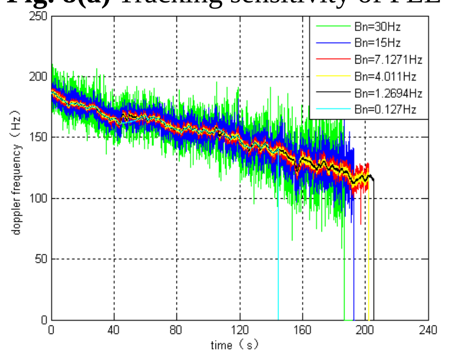

Fig. 8(b) Tracking sensitivity of Kalman filter

Table 2 Tracking sensitivity of Kalman filter and PLL

Bandwidth Bn (Hz)	Tracking sensitivity of Kalman filter(dB-Hz)	Tracking sensitivity of PLL (dB-Hz)
0.127	42.5	Can't work
1.2694	36.5	Can't work
4.011	37	Can't work
7.1271	37	36.5
15	37.5	36.5
30	38.5	35
50		36

It can be seen from figure 8(a), figure 8(b) and table 2 that, Kalman filter with normal parameter ($B_n = 1.27Hz$) and traditional PLL with normal parameter ($B_n = 15Hz$) have approximately same tracking sensitivity $36.5dB\text{-}Hz$. The Kalman filter with normal parameter has the highest tracking sensitivity of all different bandwidths, so choosing $B_n = 1.27Hz$ as normal parameter is reasonable. The situation of PLL is not the same. The PLL with normal parameter $B_n = 15Hz$ has lower tracking sensitivity than that with parameter $B_n = 30Hz$. The main reason of choosing $B_n = 15Hz$ as normal parameter is that the tracking accuracy of $B_n = 30Hz$ is obviously lower than that of $B_n = 15Hz$ (it can be seen from figure 8(a)). In addition, when the equivalent noise bandwidth is less than 4 Hz , PLL cannot work any longer. This is because that the carrier phase error caused by crystal oscillator and radial acceleration between satellite and receiver is too large, so the noise bandwidth of PLL always larger than 5 Hz.

(b) Tracking dynamic adaptability analysis

In order to test dynamic adaptability of tracking method, GPS data is produced by simulator. Its signal power is -160dBW, and initial Doppler frequency is 0 Hz, with $2Hz/s^2$ (equivalent to $0.38m/s^3$) jerk all the time. The same test method is used for Kalman filter, but as Kalman filter has much better performance than PLL in tracking dynamic signal, so the GPS data used to test Kalman filter has a large jerk $180Hz/s^2$ (equivalent to $34.28m/s^3$) . The acceleration of dynamic signal gradually increases with time. Kalman filter and PLL with different equivalent noise bandwidths are adopted to track these signals, and the output Doppler frequencies are shown in figure 9 and figure 10. When the output Doppler frequency is wrong, the acceleration at this time is the dynamics limitation of the tracking method.

There is something should be interpreted, for GPS L1 band signal, the relationship between frequency rate and acceleration $\frac{a}{c} = \frac{f'}{f_{L1}}$ is, $1Hz/s$ frequency rate is equivalent to $0.1904m/s^2$ acceleration. All the simulations below use GPS data produced by simulator, and the acceleration is realized by designing frequency rate. Therefore, in the following analysis, frequency rate is used to present acceleration dynamics.

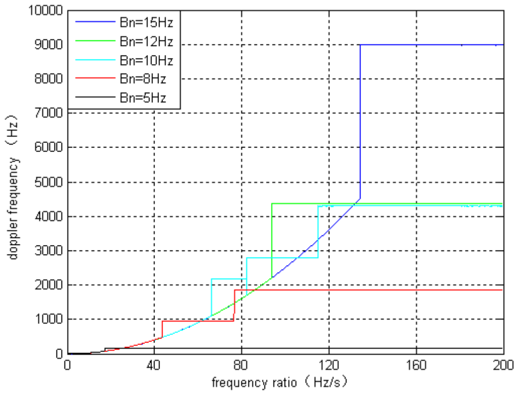

Fig. 9 dynamic adaptability of PLL tracking

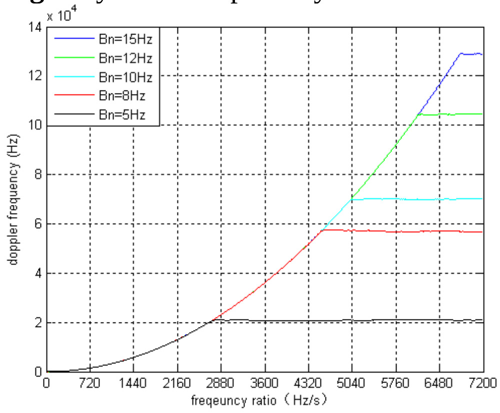

Fig. 10 dynamic adaptability of Kalman filter tracking

Table 3 dynamic adaptability of PLL and Kalman filter tracking

Bandwidth Bn (Hz)	max Doppler frequency rate of PLL tracking (Hz/s)	max Doppler frequency rate of Kalman filter tracking (Hz/s)
5	17.0	2750
8	43.4	4572
10	66.3	4938
12	93.8	6141
15	134.5	6833

From figure9, figure10 and table3 we can see that, Kalman filter has more powerful ability in tracking dynamic signal than traditional PLL. Next, we will analyze the reason why Kalman filter can track higher dynamic signal than traditional PLL, according to the carrier phase errors of these two tracking methods with the same equivalent noise bandwidth.

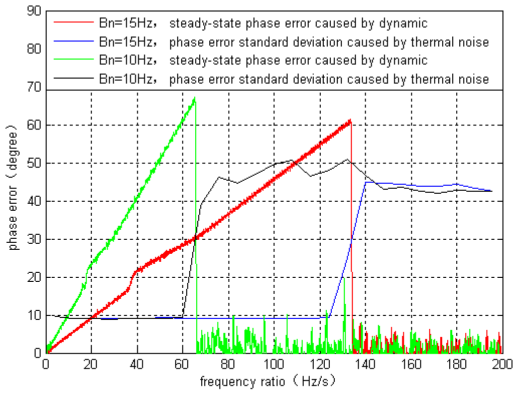

Fig. 11 discriminator output carrier phase error of PLL

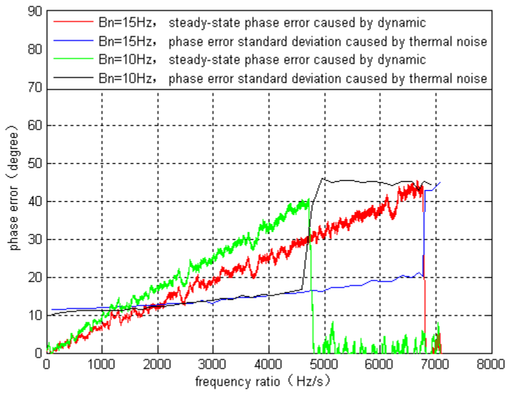

Fig. 12 discriminator output carrier phase error of Kalman filter

It can be seen from figure11 and figure12 that, Kalman filter and PLL have the same principle of losing lock. When the sum of the random error caused by thermal noise and the steady-state error caused by dynamic exceeds 90 degrees, which is the linear range of two-quadrant arctangent discriminator, the loop will lose lock.

According to equation (26), we can know that the phase error caused by thermal noise of PLL has little relationship with loop parameters. It is mainly determined by signal power and loop update time. The phase error of Kalman filter caused by thermal noise has the same principle with that of PLL. However, Kalman filter in the simulation uses fixed matrix Q and matrix R, and it means the model of Kalman filter is fixed. Therefore, with acceleration gradually increasing, the difference between Kalman filter model and actual signal is becoming larger, so the phase error caused by thermal noise is also becoming larger. When the acceleration dynamics of this signal is small, the phase error of Kalman filter caused by thermal noise is similar to that of traditional PLL.

Steady-state error caused by acceleration dynamics of PLL is expressed as equation (27). With acceleration increasing, discriminator output steady-state error is also increased. The PLL with bandwidth $B_n = 15Hz$ loses lock when frequency rate reaches $134.5Hz/s$, and the PLL with bandwidth $B_n = 10Hz$ loses lock when frequency rate reaches $66.3Hz/s$. The Kalman filter with bandwidth $B_n = 15Hz$ loses lock when frequency rate reaches $6833Hz/s$, and the Kalman filter with bandwidth $B_n = 10Hz$ loses lock when frequency rate reaches $4938Hz/s$. Therefore, with the same equivalent noise bandwidth, Kalman filter has better performance in tracking dynamic signal than traditional PLL.

However, from figure 8 it can be seen that the frequency accuracy of Kalman filter with large bandwidth is low. Based on the analysis of section 3.2, we know that the Kalman filter with normal parameter has small equivalent noise bandwidth (which is 1.2Hz) under normal GPS signal conditions. The bandwidth is much smaller than that of the traditional PLL with normal parameter (which is 15Hz). Therefore, we compare the discriminator output steady-state errors of Kalman filter and PLL with normal parameter in figure 13.

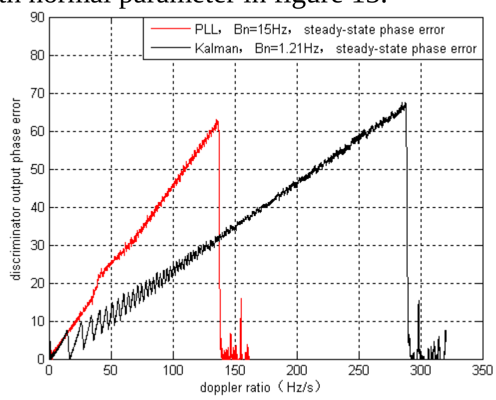

Fig. 13 Discriminator output steady-state errors of PLL and Kalman filter with normal parameter

From figure13 we can see that, when frequency rate reaches $290Hz/s$, Kalman filter with bandwidth $B_n = 1.2Hz$ loses lock. Although the equivalent noise bandwidth of Kalman filter with normal parameter is only 1.2 Hz, it still has more powerful ability in tracking dynamic signal than PLL with bandwidth 15 Hz. This simulation result corresponds to the analysis results of discriminator output steady-state error in section 3.1. The steady-state error of Kalman filter caused by acceleration is less than that of traditional PLL. This is the main reason why Kalman filter can track higher dynamic signal than traditional PLL.

(c) Tracking accuracy analysis

Kalman filter with normal parameter ($B_n = 1.2Hz$) and traditional PLL with $B_n = 15Hz$ are adopted to track static signal whose power is -160dBW. The output Doppler frequencies of these two methods are shown in figure14.

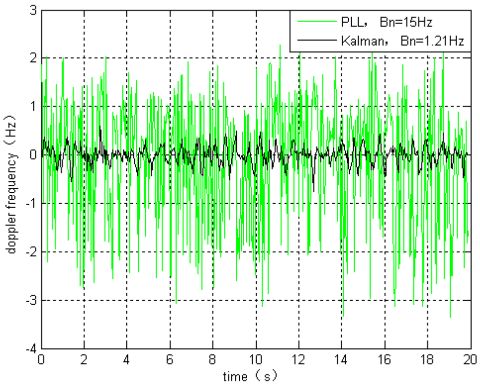

Fig. 14 Doppler frequency accuracies of PLL and Kalman filter with normal parameter

Doppler frequency error's standard deviation of Kalman filter ($B_n = 1.2Hz$) is 0.192 Hz, and that of traditional PLL ($B_n = 15Hz$) is 1.33 Hz. So the Doppler frequency accuracy of Kalman filter is much higher than that of PLL. The main reason is that the less the equivalent noise bandwidth of tracking method is, the higher the tracking accuracy is.

6 Conclusions

In order to compare the performance of Kalman filter and traditional PLL tracking GPS signals, this paper analyses the steady-state error, closed loop transfer function and equivalent noise bandwidth of these two methods in theory. The analysis results show that the dynamic stress error of Kalman filter is less than traditional PLL. Kalman filter can track dynamic signal accurately with lower equivalent noise bandwidth. The theoretical analysis results are also verified by simulation. It can be seen from the simulation results that Kalman filter has the same tracking sensitivity with traditional PLL, and has better dynamic adaptability and tracking accuracy than PLL. Kalman filter can estimate and predict dynamic signal very well, so it can take small equivalent noise bandwidth to track normal GPS signal, and has higher tracking accuracy.

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