Analysis of global exponential stability of fuzzy BAM neural networks with delays

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Abstract: In this paper fuzzy bi-directional associative memory (BAM) neural networks with constant delays are considered. Some sufficient conditions for existence and global exponential stability of unique equilibrium point are established by using fixed point theorem and differential inequality techniques. The results obtained are easily checked to guarantee existence, uniqueness and global exponential stability of equilibrium point.

Key–Words: Fuzzy BAM neural networks, Equilibrium point, Global exponential stability, Delays

1 Introduction

The bidirectional associative memory neural networks (BAM) models were first introduced by Kosko [1, 2, 3]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers , the X-layer and Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, it performs two-way associative search for stored bipolar vector pairs and generalize the single-layer autoassociative Hebbian correlation to two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks possesses a good applications prospects in the areas of pattern recognition, signal and image process, automatic control. Recently they have been the object of intensive analysis by numerous authors in recent years. In particular, many researchers have studied the dynamics of BAM neural networks with or without delays [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] including stability and periodic solutions. In Refs.[1, 2, 3, 4, 5, 6, 7, 8, 9, 10] the authors discussed the problem of the stability of the BAM networks with or without delays, and obtained sufficient conditions to ensure the stability of equilibrium point. Recently some authors (see,[11, 15, 16]) investigated another dynamical behaviors-periodic oscillatory. some sufficient conditions are obtained to ensure other solution converging the periodic solution. In this paper we would like to integrate fuzzy operations into BAM neural networks and maintain local connectedness among cells. Speaking of fuzzy operations, Yang et al. [25, 26, 27] first combined those operations with cellular neural networks and investigated the stability of fuzzy cellular neural networks(FCNNs). Studies have shown that FCNNs has its potential in image processing and pattern recognition, and some results have been reported on stability and periodicity of FCNNs [25, 26, 27, 28, 29, 30, 31, 32]. Up to now, to the best of our knowledge, dynamical behaviors of fuzzy BAM neural networks are seldom considered. On the other hand, time delays inevitably occurs in electronic neural networks owing to the unavoidable finite switching speed of amplifiers. It is desirable to study the fuzzy BAM neural networks which has a potential significance in the design and applications of stable neural circuits for neural networks with delays.

Motivated by the above discussion, in this paper, we investigate the fuzzy BAM neural networks with constant delays modelled by the following system

$$\left\{\begin{array}{rcl} x_i'(t) & = & -a_i x_i(t) + \bigwedge_{j=1}^{m} \alpha_{ji} f_j(y_j(t-\tau)) \\ & & + \bigvee_{j=1}^{m} \beta_{ji} f_j(y_j(t-\tau)) + \bigwedge_{j=1}^{m} T_{ji} u_j \\ & & + + \bigvee_{j=1}^{m} H_{ji} u_j + I_i \\ \\ y_j'(t) & = & -b_j y_j(t) + \bigwedge_{i=1}^{n} p_{ij} g_i(x_i(t-\sigma)) \\ & & + \bigvee_{i=1}^{n} q_{ij} g_i(x_i(t-\sigma)) + \bigwedge_{i=1}^{n} K_{ij} u_i \\ & & + \bigvee_{i=1}^{n} L_{ij} u_i + J_j \end{array}\right. \tag{1}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, where n and m correspond to the number of neurons in X-layer and Y-layer, respectively. $x_i(t)$ and $y_j(t)$ are the activations of the ith neuron and the jth neurons, respectively, $a_i > 0, b_j > 0$, they denote the rate with which the ith neuron and jth neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $\alpha_{ji}, \beta_{ji}, T_{ji}$ and H_{ji} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in X-layer, respectively; p_{ij}, q_{ij}, K_{ij} and L_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in Y-layer, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation , respectively; u_j and u_i denote external input of the ith neurons in X-layer and external input of the jth neurons in Y-layer, respectively ; I_i and J_j represent bias of the ith neurons in X-layer and bias of the jth neurons in Y-layer, respectively; $\tau > 0$ and $\sigma > 0$ are constants and correspond to the transmission delays, and $f_j(\cdot), g_i(\cdot)$ are signal transmission functions.

The main purpose of this paper is, employing fixed point theorem and differential inequality techniques, to give some sufficient conditions for the the existence, uniqueness and global exponential stability of equilibrium point of system (1). Our results extend and improve the corresponding works in the earlier publications.

The initial conditions associated with system (1) are of the form

$$x_i(s) = \phi_i(s), \quad s \in (-\sigma, 0], \quad i = 1, 2, \cdots, n$$

$$y_j(s) = \psi_j(s), \quad s \in (-\tau, 0], \quad j = 1, 2, \cdots, m$$

where $\phi_i(\cdot)$ and $\psi_j(\cdot)$ are continuous bounded functions defined on $[-\sigma, 0]$ and $[-\tau, 0]$, respectively.

Throughout this paper, we always make the following assumption.

Assumption A the signal transmission functions $f_j(\cdot), g_i(\cdot)(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ are Lipschtiz continuous on R with Lipschtiz constants μ_j and ν_i, namely, for $x, y \in R$

$$|f_j(x)-f_j(y)| \leq \mu_j|x-y|, \quad |g_i(x)-g_i(y)| \leq \nu_i|x-y|.$$

To be convenience, we introduce some notations. $x = (x_1, x_2, \cdots, x_l)^T \in R^l$ denotes a column vector, in which the symbol $(^T)$ denotes the transpose of vector. For matrix $D = (d_{ij})_{l\times l}$, D^T denotes the transpose of D, and E_l denotes the identity matrix of size l. A matrix or vector $D \geq 0$ means that all entries of D are greater than or equal to zero. $D > 0$ can be defined similarly. For matrices or vectors D and E, $D \geq E$ (resp.$D > E$) means that $D - E \geq 0$ (resp.$D - E > 0$). Let's define that for any $\omega \in R^{n+m}$, $\|\omega\| = \max_{1\leq k\leq n+m} |\omega_k|$.

Definition 1 *Let $Z^* = (x_1^*, \cdots, x_n^*, y_1^*, , \cdots, y_m^*)^T$ be an equilibrium point of system (1.1) with $x^* = (x_1^*, \cdots, x_n^*)^T, y^* = (y_1^*, , \cdots, y_m^*)^T$. If there exist positive constants M, λ such that for any solution $z(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ of system (1) with initial value (ϕ, ψ) and $\phi = (\phi_1(t), \phi_2(t), \cdots, \phi_n(t))^T \in C([-\sigma, 0], R^n), \psi = (\psi_1(t), \psi_2(t), \cdots, \psi_m(t))^T \in C([-\tau, 0], R^m)$,*

$$|x_i(t) - x_i^*| \leq M\|(\phi, \psi) - (x^*, y^*)\|e^{-\lambda t},$$

and

$$|y_j(t) - y_j^*| \leq M\|(\phi, \psi) - (x^*, y^*)\|e^{-\lambda t}$$

where $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$

$$\|(\phi, \psi) - (x^*, y^*)\| = \max\Big\{\max_{1\leq i\leq n} \sup_{-\sigma\leq t\leq 0} |\phi_i(t) - x_i^*|, \max_{1\leq j\leq m} \sup_{-\tau\leq t\leq 0} |\psi_j(t) - y_j^*|\Big\}$$

Then z^* is said to be globally exponentially stable.

Definition 2 *If $f(t) : R \to R$ is a continuous function, then the upper left derivative of $f(t)$ is defined as*

$$D^- f(t) = \lim_{h\to 0^-} \sup \frac{1}{h}(f(t + h) - f(t)).$$

Definition 3 *A real matrix $A = (a_{ij})_{l\times l}$ is said to be an M-matrix if $a_{ij} \leq 0, i, j = 1, 2, \ldots, l, i \neq j$, and all successive principal minors of A are positive.*

Lemma 4 *Let $A = (a_{ij})$ be an $l \times l$ matrix with non-positive off-diagonal elements. Then the following statements are equivalent:*
(i) A is an M-matrix;
(ii) the real parts of all eigenvalues of A are positive;
(iii) there exists a vector $\eta > 0$ such that $A\eta > 0$;
(iv) there exists a vector $\xi > 0$ such that $\xi^T A > 0$;
(v) there exists a positive definite $l \times l$ diagonal matrix D such that $AD + DA^T > 0$.

Lemma 5 *[25] Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^{n} \alpha_{ij}g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij}g_j(y)\right| \leq \sum_{j=1}^{n} |\alpha_{ij}||g_j(x)-g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^{n}\beta_{ij}g_j(x)-\bigvee_{j=1}^{n}\beta_{ij}g_j(y)\right|\leq\sum_{j=1}^{n}|\beta_{ij}||g_j(x)-g_j(y)|$$

Lemma 6 *Let $A\geq 0$ be an $l\times l$ matrix and $\rho(A)<1$, then $(E_l-A)^{-1}\geq 0$, where $\rho(A)$ denotes the spectral radius of A.*

The remainder of this paper is organized as follows. In Section 2 we shall give some sufficient conditions for checking the existence and uniqueness of equilibrium point. In Section 3 we present some sufficient conditions for global exponential stability of the unique equilibrium point of (1). In Section 4 an example will be given to illustrate effectiveness of our results obtained. We will give a general conclusion in Section 5.

2 Existence and uniqueness of equilibrium point

In this section, we will derive some sufficient conditions for the existence and uniqueness of equilibrium point for fuzzy BAM neural networks model (1).

Theorem 7 *Suppose that* **Assumption A** *holds and $\rho(D^{-1}EU)<1$, where*

$$D=diag(a_1,\cdots,a_n,b_1,\cdots,b_m),$$

$$U=diag(\mu_1,\cdots,\mu_n,\nu_1,\cdots,\nu_m),$$

$$E=\begin{pmatrix}0_{n\times n} & P^T\\ Q^T & 0_{m\times m}\end{pmatrix},\quad P=(|\alpha_{ji}|+|\beta_{ji}|)_{m\times n},$$

$$Q=(|p_{ij}|+|q_{ij}|)_{n\times m}$$

Then there exists a unique equilibrium point of system (1).

Proof: An equilibrium point $z^*=(x_1^*,\cdots,x_n^*,y_1^*,\cdots,y_m^*)^T\in R^{n+m}$ is a constant vector satisfying system (1), i. e.,

$$\begin{cases} x_i^* = a_i^{-1}\left[\bigwedge_{j=1}^{m}\alpha_{ji}f_j(y_j^*)+\bigvee_{j=1}^{m}\beta_{ji}f_j(y_j^*)\right.\\ \qquad\left.+\bigwedge_{j=1}^{m}T_{ji}u_j+\bigvee_{j=1}^{m}H_{ji}u_j+I_i\right]\\ y_j^* = b_j^{-1}\left[\bigwedge_{i=1}^{n}p_{ij}g_i(x_i^*)+\bigvee_{i=1}^{n}q_{ij}g_i(x_i^*)\right.\\ \qquad\left.+\bigwedge_{i=1}^{n}K_{ij}u_i+\bigvee_{i=1}^{n}L_{ij}u_i+J_j\right]\end{cases}\tag{2}$$

To finish the proof, it suffices to prove that (2) has a unique solution. Consider a mapping $\Phi=(\Phi_i,\Psi_j)^T$: $R^{n+m}\to R^{n+m}$ defined by, for $i=1,2,\cdots,n,j=1,2,\cdots,m.$

$$\begin{aligned}\Phi_i(s_i) &= a_i^{-1}\left[\bigwedge_{j=1}^{m}\alpha_{ji}f_j(v_j)+\bigvee_{j=1}^{m}\beta_{ji}f_j(v_j)\right.\\ &\quad\left.+\bigwedge_{j=1}^{m}T_{ji}u_j+\bigvee_{j=1}^{m}H_{ji}u_j+I_i\right],\end{aligned}\tag{3}$$

$$\begin{aligned}\Psi_j(v_j) &= b_j^{-1}\left[\bigwedge_{i=1}^{n}p_{ij}g_i(s_i)+\bigvee_{j=1}^{m}q_{ij}g_i(s_i)\right.\\ &\quad\left.+\bigwedge_{i=1}^{n}K_{ij}u_i+\bigvee_{i=1}^{n}L_{ij}u_i+J_j\right],\end{aligned}\tag{4}$$

We show that $\Phi:R^{n+m}\to R^{n+m}$ is global contraction mapping on R^{n+m}. In fact, for $s=(s_1,\cdots,s_n,v_1,\cdots,v_m)^T,\overline{s}=(\overline{s}_1,\cdots,\overline{s}_n,\overline{v}_1,\cdots,\overline{v}_m)^T\in R^{n+m}$. By using **Assumption A** and **Lemma 5** , we have

$$\begin{aligned}|\Phi_i(s_i) &- \Phi_i(\overline{s}_i)|\\ &= a_i^{-1}\left[\bigwedge_{j=1}^{m}\alpha_{ji}f_j(v_j)-\bigwedge_{j=1}^{m}\alpha_{ji}f_j(\overline{v}_j)\right]\\ &\quad+a_i^{-1}\left[\bigvee_{j=1}^{m}\beta_{ji}f_j(v_j)-\bigvee_{j=1}^{m}\beta_{ji}f_j(\overline{v}_j)\right]\\ &\leq a_i^{-1}\sum_{j=1}^{m}(|\alpha_{ji}|+|\beta_{ji}|)\mu_j|v_j-\overline{v}_j|,\end{aligned}\tag{5}$$

$$\begin{aligned}|\Psi_j(v_j) &- \Psi_j(\overline{v}_j)|\\ &= b_j^{-1}\left[\bigwedge_{i=1}^{n}p_{ij}g_i(s_i)-\bigwedge_{i=1}^{n}p_{ij}g_i(\overline{s}_i)\right]\\ &\quad+b_j^{-1}\left[\bigvee_{i=1}^{n}q_{ij}g_i(s_i)-\bigvee_{i=1}^{n}q_{ij}g_i(\overline{s}_i)\right]\\ &\leq b_j^{-1}\sum_{i=1}^{n}(|p_{ij}|+|q_{ij}|)\nu_i|s_i-\overline{s}_i|.\end{aligned}\tag{6}$$

In view of (5)-(6), it follows that

$$\begin{aligned}|\Phi(s_1,\cdots,s_n,v_1,\cdots,v_m) &- \Phi(\overline{s}_1,\cdots,\overline{s}_n,\overline{v}_1,\cdots,\overline{v}_m)|\\ &\leq F\begin{pmatrix}|s_1-\overline{s}_1|\\ \vdots\\ |s_n-\overline{s}_n|\\ |v_1-\overline{v}_1|\\ \vdots\\ |v_m-\overline{v}_m|\end{pmatrix}\end{aligned}\tag{7}$$

where $F = D^{-1}EU = (w_{ij})_{(n+m)\times(n+m)}$. Let ξ be a positive integer. Then from (7) it follows that

$$|\Phi^{\xi}(s) - \Phi^{\xi}(\overline{s})| \le F^{\xi} \begin{pmatrix} |s_1 - \overline{s}_1| \\ \vdots \\ |s_n - \overline{s}_n| \\ |v_1 - \overline{v}_1| \\ \vdots \\ |v_m - \overline{v}_m| \end{pmatrix} \quad (8)$$

Since $\rho(F) < 1$, we obtain $\lim_{\xi\to+\infty} F^{\xi} = 0$, which implies that there exist a positive integer N and a positive constant $r < 1$ such that

$$\begin{aligned} F^N &= (D^{-1}EU)^N = (h_{ij})_{(n+m)\times(n+m)}, \\ &\sum_{j=1}^{n+m} h_{ij} \le r,\ i = 1, 2, \cdots, n+m. \quad (9) \end{aligned}$$

Noting that (8) and (9), it follows that

$$\begin{aligned} |\Phi^N(s) \quad - \quad \Phi^N(\overline{s})| \le F^N &\begin{pmatrix} |s_1 - \overline{s}_1| \\ \vdots \\ |s_n - \overline{s}_n| \\ |v_1 - \overline{v}_1| \\ \vdots \\ |v_m - \overline{v}_m| \end{pmatrix} \\ \le \quad F^N &\begin{pmatrix} \|s - \overline{s}\| \\ \vdots \\ \|s - \overline{s}\| \\ \|s - \overline{s}\| \\ \vdots \\ \|s - \overline{s}\| \end{pmatrix} \\ = \quad \|s - \overline{s}\| &\begin{pmatrix} \sum_{j=1}^{n+m} h_{1j} \\ \vdots \\ \sum_{j=1}^{n+m} h_{nj} \\ \sum_{j=1}^{n+m} h_{(n+1)j} \\ \vdots \\ \sum_{j=1}^{n+m} h_{(n+m)j} \end{pmatrix} \quad (10) \end{aligned}$$

which implies that $\|\Phi^N(s) - \Phi^N(\overline{s})\| \le r\|s - \overline{s}\|$. Since $r < 1$, it is obvious that the mapping $\Phi^N : R^{n+m} \to R^{n+m}$ is a contraction mapping. By the fixed point theorem of Banach space, Φ possesses a unique fixed point in R^{n+m} which is unique solution of the system (2), namely, there exist a unique equilibrium point of system (1). The proof of Theorem 7 is completed. □

3 Global exponential stability of equilibrium point

In this section, we shall give some sufficient conditions to guarantee global exponential stability of equilibrium point of system (1).

Theorem 8 *Suppose that* **Assumption A** *holds and* $\rho(D^{-1}EU) < 1$. *Let* $z^* = (x_1^*, \cdots, x_n^*, y_1^*, \cdots, y_m^*)^T$ *be a unique equilibrium point of system (1). Then the unique equilibrium point* z^* *of system (1) is globally exponentially stable.*

Proof: Let $z(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ be an arbitrary solution of system (1) with initial value (ϕ, ψ) and $\phi = (\phi_1(t), \phi_2(t), \cdots, \phi_n(t))^T \in C([-\sigma, 0]; R^n), \psi = (\psi_1(t), \psi_2(t), \cdots, \psi_n(t))^T \in C([-\tau, 0]; R^m)$.Set $\overline{x}_i(t) = x_i(t) - x_i^*, \overline{y}_j(t) = y_j(t) - y_j^*, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$

From (1) and (2), we have

$$\left\{ \begin{aligned} \overline{x}_i'(t) &= -a_i\overline{x}_i(t) + \Big[\textstyle\bigwedge_{j=1}^m \alpha_{ji} f_j(y_j(t-\tau)) \\ &\quad - \textstyle\bigwedge_{j=1}^m \alpha_{ji} f_j(y_j^*)\Big] + \Big[\textstyle\bigvee_{j=1}^m \beta_{ji} \\ &\quad \times f_j(y_j(t-\tau)) - \textstyle\bigvee_{j=1}^m \beta_{ji} f_j(y_j^*)\Big] \\ \overline{y}_j'(t) &= -b_j\overline{y}_j(t) + [\textstyle\bigwedge_{i=1}^n p_{ij} g_i(x_i(t-\sigma)) \\ &\quad - \textstyle\bigwedge_{i=1}^n p_{ij} g_i(x_i^*)] + [\textstyle\bigvee_{i=1}^n q_{ij} \\ &\quad \times g_i(x_i(t-\sigma)) - \textstyle\bigvee_{i=1}^n q_{ij} g_i(x_i^*)] \end{aligned} \right. \quad (11)$$

Using **assumption A, Definition 2** and **Lemma 5**, from (11), we have

$$\left\{ \begin{aligned} & D^-|\overline{x}_i(t)| \\ \le\ & -a_i|\overline{x}_i(t)| + \textstyle\sum_{j=1}^m (|\alpha_{ji}| + |\beta_{ji}|) \\ & \times \mu_j |y_j(t-\tau) - y_j^*| \\ \le\ & -a_i|\overline{x}_i(t)| + \textstyle\sum_{j=1}^m (|\alpha_{ji}| + |\beta_{ji}|)\mu_j \widetilde{y_j(t)} \\ & D^-|\overline{y}_j(t)| \\ \le\ & -b_j|\overline{y}_j(t)| + \textstyle\sum_{i=1}^n (|p_{ij}| + |q_{ij}|) \\ & \times \nu_i |x_i(t-\sigma) - x_i^*| \\ \le\ & -b_j|\overline{y}_j(t)| + \textstyle\sum_{i=1}^n (|p_{ij}| + |q_{ij}|)\nu_i \widetilde{x_i(t)} \end{aligned} \right. \quad (12)$$

where $\widetilde{x_i(t)} = \sup_{t-\sigma\le s\le t}|\overline{x}_i(s)|, \widetilde{y_j(t)} = \sup_{t-\tau\le s\le t}|\overline{y}_j(s)|, i=1,2,\cdots,n; j=1,2,\cdots,m.$

Since $\rho(D^{-1}EU) = \rho(F) < 1$, it follows from **Lemma 4** and **Lemma 6** that $E_{n+m} - D^{-1}EU$ is an M-matrix, therefore there exists a vector $\eta = (\eta_1,\eta_2,\cdots,\eta_n,\zeta_1,\zeta_2,\cdots,\zeta_m)^T > (0,0,\cdots,0,0,0,\cdots,0)^T$ such that

$$(E_{n+m} - D^{-1}EU)\eta > (0,0,\cdots,0,0,0,\cdots,0)^T.$$

Hence, for $i=1,2,\cdots,n;\ \ j=1,2,\cdots,m,$

$$\eta_i - \sum_{j=1}^m a_i^{-1}(|\alpha_{ji}|+|\beta_{ji}|)\mu_j\zeta_j > 0,$$

and

$$\zeta_j - \sum_{i=1}^n b_j^{-1}(|p_{ij}|+|q_{ij}|)\nu_i\eta_i > 0,$$

which implies that

$$\begin{cases} -a_i\eta_i + \sum_{j=1}^m(|\alpha_{ji}|+|\beta_{ji}|)\mu_j\zeta_j < 0, \\ -b_j\zeta_j + \sum_{i=1}^n(|p_{ij}|+|q_{ij}|)\nu_i\eta_i < 0. \end{cases} \tag{13}$$

We can choose a positive constant $\lambda < 1$ such that, for $i=1,2,\cdots,n; j=1,2,\cdots,m$

$$\begin{cases} \lambda\eta_i + \left[-a_i\eta_i + \sum_{j=1}^m(|\alpha_{ji}|+|\beta_{ji}|)\mu_j\zeta_j e^{\lambda\tau}\right] < 0, \\ \lambda\zeta_j + \left[-b_j\zeta_j + \sum_{i=1}^n(|p_{ij}|+|q_{ij}|)\nu_i\eta_i e^{\lambda\sigma}\right] < 0. \end{cases} \tag{14}$$

For all $t \in [-\sigma-\tau, 0]$, we can choose a constant $\gamma > 1$ such that

$$\gamma\eta_i e^{-\lambda t} > 1, \ \ \gamma\zeta_j e^{-\lambda t} > 1. \tag{15}$$

For $\forall\varepsilon > 0$, set

$$\begin{cases} \Delta &= \sum_{i=1}^n \widetilde{x_i(0)} + \sum_{j=1}^m \widetilde{y_j(0)}; \\ V_i(t) &= \gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t}, \\ W_j(t) &= \gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} \end{cases} \tag{16}$$

Caculating the upper left derivative of $V_i(t)$ and $W_j(t)$, respecively, and noting that (14)

$$\begin{aligned} D^- \ \ & V_i(t) \\ = \ \ & -\lambda\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t} \\ > \ \ & \left[-a_i\eta_i + \sum_{j=1}^m(|\alpha_{ji}|+|\beta_{ji}|)\mu_j\zeta_j e^{\lambda\tau}\right] \\ & \times\gamma(\Delta+\varepsilon)e^{-\lambda t} \\ = \ \ & -a_i\gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t} + \sum_{j=1}^m(|\alpha_{ji}|+|\beta_{ji}|) \\ & \times\mu_j\zeta_j\gamma(\Delta+\varepsilon)e^{-\lambda t}e^{\lambda\tau} \\ = \ \ & -a_iV_i(t) + \sum_{j=1}^m(|\alpha_{ji}|+|\beta_{ji}|)\mu_j\overline{W_j}(t) \end{aligned} \tag{17}$$

and

$$\begin{aligned} D^- \ \ & W_j(t) \\ = \ \ & -\lambda\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} \\ > \ \ & \left[-b_j\zeta_j + \sum_{i=1}^n(|p_{ij}|+|q_{ij}|)\nu_i\eta_i e^{\lambda\tau}\right] \\ & \times\gamma(\Delta+\varepsilon)e^{-\lambda t} \\ = \ \ & -b_j\gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} + \sum_{i=1}^n(|p_{ij}|+|q_{ij}|) \\ & \times\nu_i\eta_i\gamma(\Delta+\varepsilon)e^{-\lambda t}e^{\lambda\tau} \\ = \ \ & -b_jW_j(t) + \sum_{i=1}^n(|p_{ij}|+|q_{ij}|)\nu_i\overline{V_i}(t) \end{aligned} \tag{18}$$

where $\overline{W_j}(t) = \sup_{t-\tau\le s\le t}W_j(s), \overline{V_i}(t) = \sup_{t-\tau\le s\le t}V_i(s).i = 1,2,\cdots,n; j = 1,2,\cdots,m.$ from (15) and (16), we have

$$\begin{cases} V_i(t) &= \gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t} > |\overline{x}_i(t)|, t\in[-\sigma,0], \\ W_j(t) &= \gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} > |\overline{y}_j(t)|, t\in[-\tau,0]. \end{cases} \tag{19}$$

On the other hand, we claim that for all $t > 0, i = 1,2,\cdots,n; j=1,2,\cdots,m.$

$$\begin{cases} V_i(t) &= \gamma\eta_i(\Delta+\varepsilon)e^{-\lambda t} > |\overline{x}_i(t)|; \\ W_j(t) &= \gamma\zeta_j(\Delta+\varepsilon)e^{-\lambda t} > |\overline{y}_j(t)|, \end{cases} \tag{20}$$

By contrary, one of the following two cases must occur: (i) there must exist $i\in\{1,2,\cdots,n\}$ and $t_i^* > 0$ such that for $l=1,2,\cdots,n, k=1,2,\cdots,m.$

$$\begin{cases} |\overline{x}_i(t_i^*)| = V_i(t_i^*); |\overline{x}_l(t)| < V_l(t), \forall t\in[-\sigma,t_i^*]; \\ |\overline{y}_k(t)| < W_k(t), \forall t\in[-\tau,t_i^*]. \end{cases} \tag{21}$$

(ii) there must exist $j\in\{1,2,\cdots,m\}$ and $t_j^* > 0$ such that for $l=1,2,\cdots,n, k=1,2,\cdots,m.$

$$\begin{cases} |\overline{y}_j(t_i^*)| = W_j(t_j^*)\ ; |\overline{x}_l(t)| < V_l(t), \ \ \forall t\in[-\sigma,t_j^*]; \\ |\overline{y}_k(t)| < W_k(t), \ \ \forall t\in[-\tau,t_j^*]. \end{cases} \tag{22}$$

Suppose case (i) occurs, we obtain

$$\begin{aligned} 0 &\leq D^-(|\overline{x}_i(t_i^*)| - V_i(t_i^*)) \\ &= \lim_{h\to 0^-}\sup\left\{\frac{|\overline{x}_i(t_i^*+h)| - V_i(t_i^*+h)}{h} - \frac{|\overline{x}_i(t_i^*)| - V_i(t_i^*)}{h}\right\} \\ &\leq \lim_{h\to 0^-}\sup\frac{|\overline{x}_i(t_i^*+h)| - |\overline{x}_i(t_i^*)|}{h} - \lim_{h\to 0^-}\inf\frac{V_i(t_i^*+h) - V_i(t_i^*)}{h} \\ &= D^-|\overline{x}_i(t_i^*)| - D_-V_i(t_i^*). \end{aligned} \tag{23}$$

In view of (12), (17) and (21), we have

$$\begin{aligned} D^- & |\overline{x}_i(t_i^*)| \\ \leq & -a_i|\overline{x}_i(t_i^*)| + \sum_{j=1}^{m}(|\alpha_{ji}| + |\beta_{ji}|)\mu_j \widetilde{y_j(t_i^*)} \\ = & -a_iV_i(t_i^*) + \sum_{j=1}^{m}(|\alpha_{ji}| + |\beta_{ji}|)\mu_j \widetilde{y_j(t_i^*)} \\ \leq & -a_iV_i(t_i^*) + \sum_{j=1}^{m}(|\alpha_{ji}| + |\beta_{ji}|)\mu_j \overline{W_j}(t_i^*) \\ < & D_-V_i(t_i^*) \end{aligned} \tag{24}$$

which contradicts (23).

Suppose case (ii) occurs, we obtain

$$\begin{aligned} 0 &\leq D^-(|\overline{y}_j(t_j^*)| - W_j(t_j^*)) \\ &= \lim_{h\to 0^-}\sup\left\{\frac{|\overline{y}_j(t_j^*+h)| - W_j(t_j^*+h)}{h} - \frac{|\overline{y}_j(t_j^*)| - W_j(t_j^*)}{h}\right\} \\ &\leq \lim_{h\to 0^-}\sup\frac{|\overline{y}_j(t_j^*+h)| - |\overline{y}_j(t_j^*)|}{h} - \lim_{h\to 0^-}\inf\frac{W_j(t_j^*+h) - W_j(t_j^*)}{h} \\ &= D^-|\overline{y}_j(t_j^*)| - D_-W_j(t_j^*). \end{aligned} \tag{25}$$

In view of (12), (18) and (22), we have

$$\begin{aligned} D^- & |\overline{y}_j(t_j^*)| \\ \leq & -b_j|\overline{y}_j(t_j^*)| + \sum_{i=1}^{n}(|p_{ij}| + |q_{ij}|)\nu_i \widetilde{x_i(t_j^*)} \\ = & -b_iW_j(t_j^*) + \sum_{i=1}^{n}(|p_{ij}| + |q_{ij}|)\nu_i \widetilde{x_i(t_j^*)} \\ \leq & -b_jW_j(t_j^*) + \sum_{i=1}^{n}(|p_{ij}| + |q_{ij}|)\nu_i \overline{V_i}(t_j^*) \\ < & D_-W_j(t_j^*) \end{aligned} \tag{26}$$

which contradicts (25). Therefore (20) holds.

Let $\varepsilon \to 0^+$,

$$M = (n+m)\max\left\{\max_{1\leq i\leq n}\{\gamma\eta_i\}, \max_{1\leq j\leq m}\{\gamma\zeta_j\}\right\} + 1,$$

we have from (20) that

$$\begin{cases} |x_i(t) - x_i^*| \leq \gamma\eta_i[\Delta + \varepsilon]e^{-\lambda t} \\ \qquad\qquad \leq M\|(\phi,\psi) - (x^*,y^*)\|e^{-\lambda t} \\ |y_j(t) - y_j^*| \leq \gamma\zeta_j[\Delta + \varepsilon]e^{-\lambda t} \\ \qquad\qquad \leq M\|(\phi,\psi) - (x^*,y^*)\|e^{-\lambda t} \end{cases} \tag{27}$$

for all $t > 0, i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$. The proof of Theorem 8 is completed. □

Corollary 9 *Suppose* **assumption A** *holds, and if there exist some constants* $\eta_i > 0, (i = 1, 2, \cdots, n); \zeta_j > 0, (j = 1, 2, \cdots, m)$ *such that*

$$\begin{cases} -a_i\eta_i + \sum_{j=1}^{m}(|\alpha_{ji}| + |\beta_{ji}|)\mu_j\zeta_j < 0, \\ -b_j\zeta_j + \sum_{i=1}^{n}(|p_{ij}| + |q_{ij}|)\nu_i\eta_i < 0. \end{cases} \tag{28}$$

Then system (1) has a unique equilibrium point z^* *which is globally exponentially stable.*

Corollary 10 *Let* **Assumption A** *hold, and suppose that* $E_{n+m} - D^{-1}EU$ *is an M-matrix. Then system (1) has a unique equilibrium point* z^* *which is globally exponentially stable.*

4 An illustrative example

In this section, we give an example to illustrate effectiveness of our results.

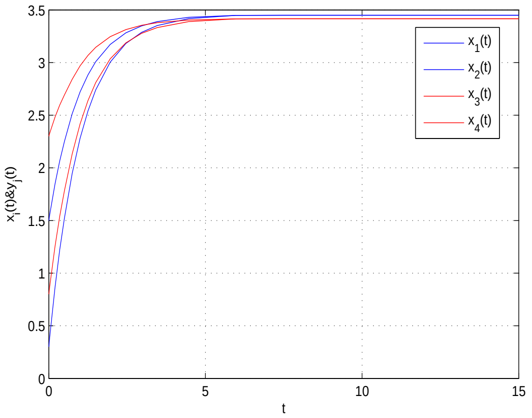
Example 4.1 Considering the following fuzzy BAM neural networks with constant delays.

$$\begin{cases} x_i'(t) = -a_ix_i(t) + \bigwedge_{j=1}^{2}\alpha_{ji}f_j(y_j(t-\tau)) \\ \qquad + \bigvee_{j=1}^{2}\beta_{ji}f_j(y_j(t-\tau)) \\ \qquad + \bigwedge_{j=1}^{2}T_{ji}u_j + \bigvee_{j=1}^{2}H_{ji}u_j + I_i \\ y_j'(t) = -b_jy_j(t) + \bigwedge_{i=1}^{2}p_{ij}g_i(x_i(t-\sigma)) \\ \qquad + \bigvee_{i=1}^{2}q_{ij}g_i(x_i(t-\sigma)) \\ \qquad + \bigwedge_{i=1}^{2}K_{ij}u_i + \bigvee_{i=1}^{2}L_{ij}u_i + J_j \end{cases} \tag{29}$$

where $i,j = 1,2.f_i(x) = g_i(x) = \frac{1}{2}(|x+1| - |x-1|), a_i = b_j = 1, \alpha_{11} = \alpha_{12} = 1/3, \alpha_{21} = \alpha_{22} = 1/4, \beta_{11} = \beta_{12} = 1/5, \beta_{21} = \beta_{22} = 1/6, p_{11} = p_{12} = 1/4, p_{21} = p_{22} = 1/3, q_{11} = q_{12} = 1/6, q_{21} = q_{22} = 1/5, T_{ji} = H_{ji} = K_{ij} = L_{ij} = 1, u_i = u_j = 1, (i,j = 1,2), I_i = J_j = 1, (i,j = 1,2), \sigma = \tau = 2.$

So, by easy computation, we can see that $\rho(D^{-1}EU) = 0.95 < 1$. Therefore, from Theorem 7, system (29) has an unique equilibrium point which is globally exponentially stable (see Fig.1.).

Fig.1. The Dynamics of system (29) with the initial values $x_1(0)=0.3, x_2(0)=1.5, y_1(0)=0.8, y_2(0)=2.3$



5 Conclusion

In this paper, fuzzy BAM neural networks with constant delays has been studied. Some sufficient conditions for existence, uniqueness and global exponential stability of equilibrium point have been obtained. The criteria of stability is simple and independent of time delay. It is only associated with the templates of system (1). Moreover an example is given to illustrate the effectiveness of our results obtained.

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