

Equivalence between SLNR and MMSE precoding schemes in the K -user MISO interference channel

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Abstract: - We consider a transmit precoder design for the K -user multiple-input single-output (MISO) interference channel in which each transmitter is equipped with multiple transmit antennas and each receiver employs a single receive antenna. First, we derive a closed-form expression for a minimum mean-square error (MMSE) precoder and an alternative expression for a signal-to-leakage plus noise ratio (SLNR) precoder. Then, the equivalence between the two precoders is proved. Through simulation results, we demonstrate that the sum capacity and bit error rate (BER) performance of the SLNR precoder are the same as the MMSE precoder and that they are better than that of the zero-forcing (ZF) precoder.

Key-Words: - Interference channel, multiple-input single-output, precoding, minimum mean squared error (MMSE), signal-to-leakage-plus noise ratio (SLNR).

1 Introduction

Recently, multiple-input multiple-output (MIMO) technology has received a lot of attention for the next generation communication systems because it can achieve higher system capacity and improved quality of service without the loss of power or bandwidth [1]-[5]. These benefits of MIMO technology can be obtained by using the spatial multiplexing or diversity schemes [6]-[10]. In MIMO systems, multiple antennas are used at both transmitter and receiver.

In multiple-input single-output (MISO) systems, transmitter is equipped with multiple antennas and receiver with single antenna [11]-[13]. One example for the MISO system is the downlink of the mobile communication systems. In that system, transmitter is base station and receiver is user equipment. In general, base station is equipped with multiple transmit antennas. However, user equipment may not be able to be equipped with multiple receive antennas because of size limitation of the user equipment.

Single user MIMO technology is a transmission scheme for point-to-point communication systems where information data of the transmitter is designated to only one receiver. This scheme is very useful to enhance the data rate between the transmitter and the designated receiver. On the other

hand, multiuser MIMO technology is used to improve the sum capacity of the communication network. In the scheme, multiple data streams of the transmitter are simultaneously transmitted to multiple receivers. The precoding matrix for multiuser MIMO is designed to maximize the sum capacity of the network.

In the MIMO broadcasting channels, linear precoding techniques are often considered as practical multiuser MIMO schemes because of their low computational complexity, which involves zero-forcing (ZF) and minimum mean square error (MMSE) techniques [14]. In ZF precoding scheme, multiuser interference can be completely eliminated. However, since this scheme does not consider background noise in designing precoding matrix, it causes an increase of the background noise power and therefore, it is useful especially for high signal-to-noise (SNR) regime. On the contrary, in MMSE precoding scheme, both multiuser interference and background noise are considered and therefore, it has better performance than the ZF scheme.

Among the linear precoding schemes, some schemes use precoders and decoders to maximize the output signal-to-interference plus noise ratio (SINR) [15]. Since it is impossible to find a closed form expression of the optimal solution of these schemes, the solution is usually obtained iteratively in order to reduce the computational complexity.

Recently, to find a closed form solution for the precoder, a signal-to-leakage and noise ratio (SLNR)-based precoding scheme was studied in [16]; in this scheme leakage means interference to all users except a desired user, and it is the measure of how much signal power leaks into the other users. Using the leakage-based criterion, the precoder of a transmitter can be optimally designed without considering the precoders of any other transmitters, and it is easy to find an analytical closed form expression of the optimal solution [16]. Moreover, while the zero-forcing scheme has a dimension limitation on the number of transmit and receive antennas, the SLNR-based scheme does not require any dimension limitation.

For broadcast channels, the equivalence between SLNR-based and MMSE-based precoding schemes with single-antenna receivers was proved in [17]. The generalization to the multiple receiver antennas was given in [19]. For downlink multi-user MIMO systems, the equivalence between SLNR-based and regularized block diagonalization (RBD)-based precoding schemes was shown in [19].

Recently, K-user interference channels have received much attention [20]. There are K pairs of transmitters and receivers and each transmitter sends an independent data stream to the receiver. Therefore, co-channel interference is generated at all receivers that the transmitter is not paired with. To the best of our knowledge, there has been no research on SLNR-based precoding in the K-user interference channel.

In this paper, we consider K-user multiple-input single-output (MISO) interference channel. First, we derive a closed-form solution for the MMSE-based precoder design and an alternative expression for the SLNR-based precoder. Then, we derive the equivalence between the MMSE-based and SLNR-based precoding schemes in the K-user MISO interference channel.

The remainder of this paper is organized as follows: Section 2 describes the system model and Section 3 explains the SLNR-based and MMSE-based precoding schemes. The equivalence between the two schemes is proved in Section 4 and simulation results are given in Section 5. Finally, Section 6 concludes this paper.

Notation: Boldface lower- and upper-case letters stand for vectors and matrices, respectively. The superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote transpose, complex conjugate (Hermitian) transpose, complex conjugate, respectively. For any matrix \mathbf{A} , \mathbf{A}^{-1} means inverse matrix and \mathbf{I}_M represents the identity matrix of size $M \times M$. The notations $E[\cdot]$ and \mathbb{C}

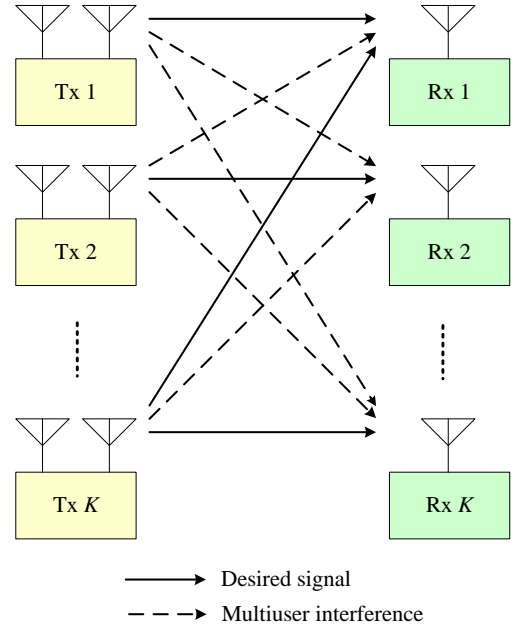


Fig. 1. K-user MISO interference channel.

mean statistical expectation and the complex number field, respectively.

2 System Model

Fig. 1 shows the system model we treat in this paper. We consider the K-user MISO interference channel where K transmitters communicate with K users and each transmitter is equipped with M transmit antennas and each receiver employs a single receive antenna. The transmitted signal from the transmitter k is given by $\mathbf{w}_k s_k$ where $s_k \in \mathbb{C}$ is a data symbol with $E[s_i] = 0$ and an independence property

$E[s_i s_j^*] = \delta(i - j)$ where $\delta(\cdot)$ is the Dirac-delta function, and \mathbf{w}_k is a transmit precoding vector with a power constraint $\mathbf{w}_k^H \mathbf{w}_k \leq P_k$.

The received signal at user k is

$$r_k = \sqrt{\rho_{k,k}} (\mathbf{h}_{k,k}^H \mathbf{w}_k) s_k + \sum_{j=1, j \neq k}^K \sqrt{\rho_{k,j}} (\mathbf{h}_{k,j}^H \mathbf{w}_j) s_j + z_k, \quad (1)$$

where $\rho_{k,j}$ is a path loss from the transmitter j to the user k and $\mathbf{h}_{k,j}$ represents the channel vector of length M from the transmitter j to the user k. The additive noise z_k is an independent complex Gaussian random variable with mean 0 and variance σ_z^2 for all users, i.e., $E[z_i z_j^*] = \sigma_z^2 \delta(i - j)$.

3 SLNR and MMSE Precoding Schemes

In this section, we derive SLNR-based and MMSE-based precoding schemes for the K -user MISO interference channel.

3.1 SLNR-based Precoding Scheme

In this subsection, we derive an SLNR-based precoding scheme. The SLNR at the user k is given by

$$\begin{aligned} \text{SLNR}_k &= \frac{\rho_{k,k} \mathbf{w}_k^H \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{w}_k}{\sum_{l=1, l \neq k}^K \rho_{l,k} \mathbf{w}_k^H \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H \mathbf{w}_k + \gamma_k \sigma_z^2} \\ &= \frac{\rho_{k,k} \mathbf{w}_k^H \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{Q}_k \mathbf{w}_k}, \end{aligned} \quad (2)$$

where $\mathbf{Q}_k = \sum_{l=1, l \neq k}^K \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H + \gamma_k \sigma_z^2 / P_k \mathbf{I}_M$ and γ_k is a scalar constant and \mathbf{I}_M is an identity matrix of size $M \times M$.

The SLNR precoder at the user k can be obtained from the following optimization problem:

$$\begin{aligned} \mathbf{w}_{k, \text{SLNR}} &= \arg \max_{\mathbf{w}_k} \frac{\rho_{k,k} \mathbf{w}_k^H \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{Q}_k \mathbf{w}_k} \\ &\text{subject to } \mathbf{w}_k^H \mathbf{w}_k \leq P_k. \end{aligned} \quad (3)$$

The closed form solution to (3) can be written as [21]

$$\mathbf{w}_{k, \text{SLNR}} = MV(\rho_{k,k} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H), \quad (4)$$

where $MV(\mathbf{A})$ denotes the eigenvector corresponding to the maximum eigenvalue of a matrix \mathbf{A} . The resultant maximum SLNR value corresponds to the maximum eigenvalue of $\rho_{k,k} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H$.

3.2 MMSE-based Precoding Scheme

In this subsection, we derive an MMSE-based precoding scheme. If we define a receive filter at a user l as g_l , the total mean square error (MSE) of all users is given by

$$J = \sum_{l=1}^K E[|g_l^* r_l - s_l|^2], \quad (5)$$

where $(\cdot)^*$ is a complex conjugate operation.

The optimization problem for designing the transmit precoding vectors, $\mathbf{w}_l, l=1,2,\dots,K$, is formulated to minimize the MSE and it can be written as

$$\min_{\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_K} J \quad \text{subject to } \mathbf{w}_l^H \mathbf{w}_l \leq P_l, l=1,2,\dots,K. \quad (6)$$

Lagrange multiplier can be written as

$$L(\mathbf{w}_1, \dots, \mathbf{w}_K, \lambda_1, \dots, \lambda_K) = J + \sum_{l=1}^K \lambda_l (\mathbf{w}_l^H \mathbf{w}_l - P_l). \quad (7)$$

The derivatives of L with λ_k and \mathbf{w}_k^* are given by

$$\frac{\partial L}{\partial \lambda_k} = \mathbf{w}_k^H \mathbf{w}_k - P_k = 0, \quad (8)$$

$$\frac{\partial L}{\partial \mathbf{w}_k^*} = \sum_{l=1}^K E[(g_l^* r_l - s_l) g_l \sqrt{\rho_{l,k}} \mathbf{h}_{l,k} s_k^*] + \lambda_k \mathbf{w}_k = \mathbf{0}. \quad (9)$$

If we use the independence property of s_l , (9) can be rewritten as

$$\sum_{l=1}^K |g_l|^2 \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H \mathbf{w}_k - g_k \sqrt{\rho_{k,k}} \mathbf{h}_{k,k} + \lambda_k \mathbf{w}_k = \mathbf{0}. \quad (10)$$

From (10), we obtain

$$\sum_{l=1}^K |g_l|^2 \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H \mathbf{w}_k - g_k \sqrt{\rho_{k,k}} \mathbf{h}_{k,k} + \lambda_k \mathbf{w}_k = \mathbf{0}, \quad (11)$$

where $\mathbf{R}_k = \sum_{l=1}^K |g_l|^2 \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H + \lambda_k \mathbf{I}_M$. The

Lagrange multiplier λ_k is selected to satisfy the constraint given in (9).

4 Equivalence between SLNR and MMSE Precoding Scheme

In this section, we will show the equivalence between SLNR-based and MMSE-based precoding schemes.

First, we derive an alternative expression on the SLNR-based precoding scheme. Since \mathbf{Q}_k is a

Hermitian positive definite matrix, \mathbf{Q}_k can be rewritten using LU-decomposition by

$$\mathbf{Q}_k = \mathbf{L}_k \mathbf{L}_k^H, \quad (12)$$

where \mathbf{L}_k is a lower triangular matrix and it is invertible.

If we define $\mathbf{u}_k = \mathbf{L}_k^H \mathbf{w}_k$, \mathbf{w}_k can be rewritten as $\mathbf{w}_k = \mathbf{L}_k^{-H} \mathbf{u}_k$. Therefore, the optimization problem in (3) can be rewritten as

$$\begin{aligned} \mathbf{u}_{k,SLNR} = \arg \max_{\mathbf{u}_k} \rho_{k,k} \frac{\mathbf{u}_k^H \mathbf{C}_k \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{u}_k}, \\ \text{subject to } \mathbf{u}_k^H \mathbf{L}_k^{-1} \mathbf{L}_k^{-H} \mathbf{u}_k = P_k, \end{aligned} \quad (13)$$

where $\mathbf{C}_k = \mathbf{L}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{L}_k^{-H}$. The solution of (13) is a scalar multiple of the eigenvector corresponding to the maximum eigenvalue of $\rho_{k,k} \mathbf{C}_k$ [21] and it can be written as

$$\mathbf{u}_{k,SLNR} = \alpha_k \sqrt{\rho_{k,k}} \mathbf{L}_k^{-1} \mathbf{h}_{k,k}, \quad (14)$$

where α_k is a scaling factor to be selected to satisfy the power constraint $\mathbf{w}_{k,SLNR}^H \mathbf{w}_{k,SLNR} = P_k$. Now, $\mathbf{w}_{k,SLNR}$ can be represented by

$$\begin{aligned} \mathbf{w}_{k,SLNR} = \mathbf{L}_k^{-H} \mathbf{u}_{k,SLNR} = \alpha_k \sqrt{\rho_{k,k}} \mathbf{L}_k^{-H} \mathbf{L}_k^{-1} \mathbf{h}_{k,k} \\ = \alpha_k \sqrt{\rho_{k,k}} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}. \end{aligned} \quad (15)$$

By comparing (15) with (11), we can observe the resemblance between the two vectors. Now, let us find the relationship between \mathbf{R}_k^{-1} and \mathbf{Q}_k^{-1} . If we choose $\beta = |g_l|^2$ for all l such that $\lambda_k / \beta = \sigma_z^2 / P_k$, \mathbf{R}_k can be rewritten as

$$\begin{aligned} \mathbf{R}_k &= \beta \left\{ \sum_{l=1}^K \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H + \lambda_k / \beta \mathbf{I}_M \right\} \\ &= \beta \left\{ \sum_{l=1}^K \rho_{l,k} \mathbf{h}_{l,k} \mathbf{h}_{l,k}^H + \sigma_z^2 / P_k \mathbf{I}_M \right\} \\ &= \beta (\mathbf{Q}_k + \rho_{k,k} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H). \end{aligned} \quad (16)$$

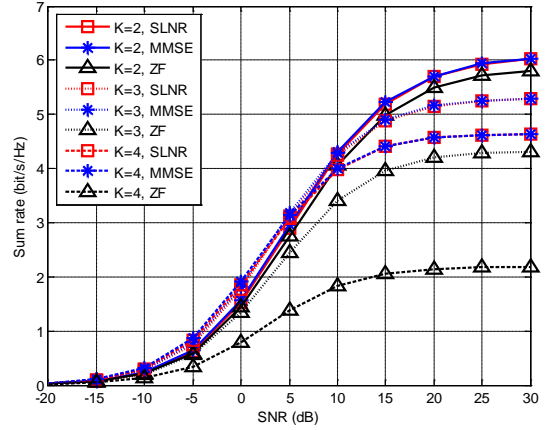


Fig. 2. Sum rate when the number of transmit antenna is $M=4$.

From the matrix inversion lemma [21], \mathbf{R}_k^{-1} can be rewritten as

$$\begin{aligned} \mathbf{R}_k^{-1} &= \frac{1}{\beta} \left(\mathbf{Q}_k^{-1} - \frac{\mathbf{Q}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1}}{1 + \rho_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}} \right) \\ &= \frac{1}{\beta} \left(\mathbf{Q}_k^{-1} - \frac{\rho_{k,k} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1}}{1 + \rho_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}} \right). \end{aligned} \quad (17)$$

Then, $\mathbf{w}_{k,MMSE}$ can be rewritten as

$$\begin{aligned} \mathbf{w}_{k,MMSE} &= g_k \sqrt{\rho_{k,k}} \frac{1}{\beta} \left(\mathbf{Q}_k^{-1} - \frac{\rho_{k,k} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1}}{1 + \rho_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}} \right) \mathbf{h}_{k,k} \\ &= \frac{1}{g_k^*} \left(\frac{1}{1 + \rho_{k,k} \mathbf{h}_{k,k}^H \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}} \right) \sqrt{\rho_{k,k}} \mathbf{Q}_k^{-1} \mathbf{h}_{k,k}. \end{aligned} \quad (18)$$

By comparing (18) with (15), we can clearly see that the SLNR-based precoder is equivalent to the MMSE precoder except for a scalar multiple.

5 Simulation results

In this section, we compare the average sum rate and the bit error rate (BER) performance among the SLNR-based, MMSE-based, and zero-forcing (ZF)-based precoding schemes. The channel coefficients are generated by an independent and identically distributed (*i.i.d.*) complex Gaussian function with zero-mean and unit-variance. The average sum rate and BER performances are averaged for 1000 independent channel realizations. The number of transmit antennas is $M = 4$ for all transmitters and

the number of receive antennas is $N=1$ for all receivers.

Fig. 2 shows the average sum rate for different numbers of users: $K = 2, 3,$ and 4 . In the figure, red curves, blue curves, and black curves denote SLNR-based, MMSE-based, ZF-based precoding schemes, respectively. From the figure, we can see that the SLNR-based precoder has the same average sum rate as the MMSE-based precoder for all K and it has better performance than the ZF-based precoder.

Fig. 3 shows the uncoded BER performance when quadrature-phase shift keying (QPSK) modulation is used. The figure confirms that the BER performance of the SLNR-based precoder coincides with that of the MMSE-based precoder and it is better than that of the ZF-based precoder.

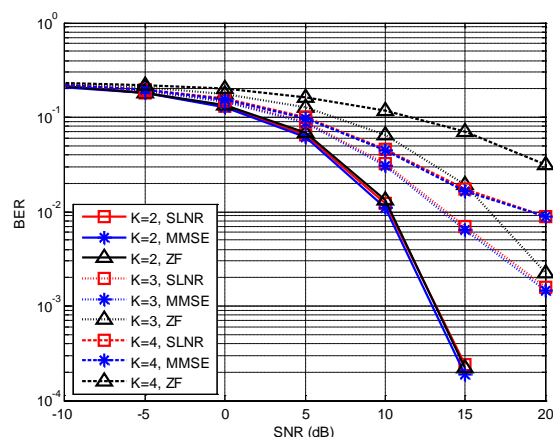


Fig. 3. Average uncoded BER with QPSK modulation when $M=4$.

4 Conclusion

In this paper, we derived the closed-form expression of the MMSE-based precoder and the alternative expression of the SLNR-based precoder in K -user MISO interference channel. We then proved the equivalence between SLNR-based and MMSE-based precoders. Simulation results verified the equivalence and they confirmed that the SLNR-based precoder has a higher sum capacity and lower BER performance than the ZF-based precoder.

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