Existence and Exponential Stability of Anti-periodic Solutions for A Cellular Neural Networks with Impulsive Effects

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Abstract: In this paper, a cellular neural networks with impulsive effects is investigated. By using differential inequality techniques, some very verifiable criteria on the existence and exponential stability of anti-periodic solutions for the model are obtained. Our results are new and complementary to previously known results. An example is included to illustrate the feasibility and effectiveness of our main results.

Key–Words: Cellular neural network, Anti-periodic solution, Exponentially stability, Time-varying delay, Impulse

1 Introduction

Due to the promising potential applications in pattern recognition, associative memory, image processing and reconstruction of moving images, cellular neural networks have been intensively investigated [1-19]. It is well known that high-order neural networks have strong approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order neural networks. In recent years, high-order neural networks have been the object of intensive investigation by numerous authors. Many results on the problem of global stability of equilibrium points and periodic solutions of high-order neural networks have been reported (see [20-28]). In applied sciences, the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations [29-32]. Recently, there are some papers which deal with the problem of existence and stability of anti-periodic solutions (see [33-60]). In addition, we know that many evolutionary processes exhibit impulsive effects which are usually subject to short time perturbations whose durations may be neglected in comparison with durations of the processes [53]. This motivates us to consider the existence and stability of anti-periodic solutions for cellular neural networks with impulses. To the best of our knowledge, very few authors have focused on the problems of anti-periodic solutions for such impulsive cellular neural networks. In this paper, we consider the anti-periodic solution of the following cellular neural networks with delays and impulses

$$\begin{cases} \dot{x}_i(t) = -d_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) \\ \qquad \times f_j(x_j(s))ds + I_i(t), t \neq t_k, \\ x_i(t_k^+) = (1+\delta_{ik})x_i(t_k), k = 1, 2, \ldots, \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, n$, a_{ij}, b_{ij}, c_{ij} are constants, $\tau_{ij}(t)(i, j = 1, 2, \cdots, n$ are nonnegative continuous functions with $0 \leq \tau_{ij}(t) \leq \tau$, for some constant τ, and n-tuple $(x_1(t0, x_2(t), \cdots, x_n(t))^T \in R^n$ denotes the state of the networks at time t. $f(x) = (f_1(x), f_2(x), \ldots, f_n(x))^T : R^n \to R^n$ is a nonlinear vector-valued activation function, $I(t) = (I_1(t), I_2(t), \cdots, I_n(t))^T \in R^n$ is an input vector function. The delay kernel $k_{ij} : R^+ \to R^+$ are real valued nonnegative continuous functions that satisfy the following conditions:

$$(i) \int_0^\infty |k_{ij}(s)|ds \leq k_{ij}^+,$$

where k_{ij}^+ is a positive constant.

The main purpose of this paper is to give the sufficient conditions of existence and exponential stability of anti-periodic solution of system (1). Some new sufficient conditions for the existence, unique and exponential stability of anti-periodic solutions of system (1) are established. Our results not only can be applied directly to many concrete examples of cellular neural networks, but also extend, to a certain extent, the results in some previously known ones. In addition, an example is presented to illustrate the effectiveness of our main results.

For convenience, we introduce some notations as follows.

$$\bar{a}_{ij} = \sup_{t\in R} |a_{ij}(t)|, \bar{b}_{ij} = \sup_{t\in R} |b_{ij}(t)|,$$

$$\bar{c}_{ij} = \sup_{t\in R} |c_{ij}(t)|, \bar{I}_i = \sup_{t\in R} |I_i(t)|,$$

$$d_i^- = \min_{t\in R} |d_i(t)|, \tau = \sup_{t\in R} \max_{1\le i,j\le n} \{\tau_{ij}(t)\}.$$

Throughout this paper, we assume that

(H1) For $i, j = 1, 2, \cdots, n, a_{ij}, b_{ij}, c_{ij}, I_i, f_j : R \to R$, $d_i, \tau_{ij} : R \to [0, +\infty)$ are continuous functions, and there exist a constant $T > 0$ such that

$$\begin{cases} d_i(t+T) = d_i(t), \\ \tau_{ij}(t+T) = \tau_{ij}(t), \\ a_{ij}(t+T)f_j(u) = -a_{ij}(t)f_j(-u), \\ b_{ij}(t+T)f_j(u) = -b_{ij}(t)f_j(-u), \\ c_{ij}(t+T)f_j(u) = -c_{ij}(t)f_j(-u), \\ I_i(t+T) = -I_i(t), \\ c_{ij}(t+T)\displaystyle\int_{-\infty}^{t+T} k_{ij}(t+T-s)f_j(u_j)ds \\ = -c_{ij}(t)\displaystyle\int_{-\infty}^{t} k_{ij}(t-s)f_j(u_j)ds \end{cases}$$

for all $t, u \in R$.

(H2) The sequence of times $\{t_k\}(k \in N$ satisfies $t_k < t_{k+1}$ and $\lim_{k\to+\infty} t_k = +\infty$, and δ_{ik} satisfies $-2 \le \delta_{ik} \le 0$ for $i \in \{1, 2, \ldots, n\}$ and $k \in N$.

(H3) There exists a $q \in N$ such that $\delta_{i(k+q)} = \delta_{ik}, t_{k+q} = t_k + q, k \in N$.

(H4) For each $j \in \{1, 2, \cdots, n\}$, the activation function $f_j : R \to R$ is continuous and there exists a nonnegative constant L_j^f and M_j^f such that

$$f_j(0) = 0, |f_j(u)| \le M_j^f, |f_j(u)-f_j(v)| \le L_j^f|u-v|$$

for all $u, v \in R$.

(H5) There exist constants $\eta > 0, \lambda > 0, i, j = 1, 2, \cdots, n$, such that for all $t > 0$,

$$(\lambda - d_i^-) + \sum_{j=1}^{n} \left((\bar{a}_{ij} + \bar{b}_{ij}k_{ij}^+ + \bar{c}_{ij})L_j^f\right) < -\eta < 0.$$

Let

$$x = (x_1, x_2, \cdots, x_n)^T \in R^n,$$

in which "T" denotes the transposition. We define

$$|x| = (|x_1|, |x_2|, \cdots, |x_n|)^T$$

and

$$||x|| = \max_{1\le i\le n} |x_i|.$$

Obviously, the solution

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$$

of (1) has components $x_i(t)$ piece-wise continuous on $(-\tau, +\infty)$, $x(t)$ is differentiable on the open intervals (t_{k-1}, t_k) and $x(t_k^+)$ exists.

Definition 1 *Let $u(t) : R \to R$ be piece-wise continuous function having countable number of discontinuous $\{t_k\}|_{k=1}^{+\infty}$ of the first kind. It is said to be T-anti-periodic on R if*

$$\begin{cases} u(t+T) = -u(t), \ \ t \ne t_k, \\ u((t_k+T)^+) = -u(t_k^+), \ \ k = 1, 2, \cdots. \end{cases}$$

Definition 2 *Let*

$$x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$$

be an anti-periodic solution of (1) with initial value

$$\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \cdots, \varphi_n^*(t))^T.$$

If there exist constants $\lambda > 0$ and $M > 1$ such that for every solution

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$$

of (1) with an initial value

$$\varphi = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t))^T,$$

$$|x_i(t) - x_i^*(t)| \le M\|\varphi - \varphi^*\|e^{-\lambda t}, \text{for all } t > 0,$$

where $i = 1, 2, \cdots, n$ and

$$\|\varphi - \varphi^*\| = \sup_{-\tau\le s\le 0} \max_{1\le i\le n} |\varphi_i(s) - \varphi_i^*(s)|.$$

Then $x^*(t)$ is said to be globally exponentially stable.

The rest of this paper is organized as follows. In the next section, we give some preliminary results. In Section 3, we derive the existence of T-anti-periodic solution, which is globally exponential stable. In Section 4, we present an example to illustrate the effectiveness of our main results. In Section 5, we a brief conclusion is drawn.

2 Preliminary Results

In this section, we present two important lemmas which are used to prove our main results in Section 3.

Lemma 3 *Let* (H1)–(H4) *hold. Suppose that*

$$x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$$

is a solution of (1) with initial conditions

$$x_i(s) = \varphi_i(s), \quad |\varphi_i(s)| < \gamma, \quad s \in [-\tau, 0], \tag{2}$$

where $i = 1, 2, \cdots, n$. *Then*

$$|x_i(t)| < \gamma, \text{ and } |x_i(t_k^+)| < \gamma, \text{ for all } t \geq 0, \tag{3}$$

where $i = 1, 2, \cdots, n$ *and*

$$\gamma > \frac{\Theta}{d_i^-}, \tag{4}$$

$$\Theta = \sum_{j=1}^n \bar{a}_{ij} M_j^f + \sum_{j=1}^n \bar{b}_{ij} k_{ij}^+ M_j^f + \sum_{j=1}^n \bar{c}_{ij} M_j^f + \bar{I}_i.$$

Proof. For any given initial condition, hypothesis (H4) guarantee the existence and unique of $x(t)$, the solution to (1) in $[-\tau, +\infty)$. By way of contradiction, we assume that (3) does not hold. Notice that $x_i(t_k^+) = (1 + \delta_{ik})x_i(t_k)$ and by the assumption (H2),$-2 \leq \delta_{ik} \leq 0$, then

$$|x_i(t_k^+)| = |(1 + \gamma_{ik})||x_i(t_k)| \leq |x_i(t_k)|.$$

Then if $|x_i(t_k^+)| \geq \gamma$, then $|x_i(t_k)| \geq \gamma$. Thus we may assume that there must exist $i \in \{1, 2, \cdots, n\}$ and $\theta_0 \in (t_k, t_{k+1}]$ such that for all $t \in (-\tau, \theta_0)$,

$$|x_i(\theta_0)| = \gamma, \text{and } |x_j(\theta_0)| < \gamma \tag{5}$$

where $j = 1, 2, \cdots, n$. By directly computing the upper left derivative of $|x_i(t)|$, together with the assumptions (3), (4), (H4) and (5), we deduce that

$$\begin{aligned}
0 &\leq D^+(|x_i(\theta_0)|) \\
&\leq -d_i(\theta_0)x_i(\theta_0) + \Bigg| \sum_{j=1}^n a_{ij}(\theta_0) f_j(x_j(\theta_0)) \\
&\quad + \sum_{j=1}^n b_{ij}(t) f_j(x_j(\theta_0 - \tau_{ij}(\theta_0))) \\
&\quad + \sum_{j=1}^n c_{ij}(\theta_0) \int_{-\infty}^{\theta_0} k_{ij}(\theta_0 - s) f_j(x_j(s)) ds \\
&\quad + I_i(\theta_0) \Bigg| \\
&\leq -d_i^- \gamma + \sum_{j=1}^n |a_{ij}(\theta_0)||f_j(x_j(\theta_0))| \\
&\quad + \sum_{j=1}^n |b_{ij}(t)||f_j(x_j(\theta_0 - \tau_{ij}(\theta_0)))| \\
&\quad + \sum_{j=1}^n |c_{ij}(\theta_0)| \int_{-\infty}^{\theta_0} |k_{ij}(\theta_0 - s)| \\
&\quad \times |f_j(x_j(s))| ds + |I_i(\theta_0)| \\
&\leq -d_i^- \gamma + \sum_{j=1}^n \bar{a}_{ij} M_j^f + \sum_{j=1}^n \bar{b}_{ij} k_{ij}^+ M_j^f \\
&\quad + \sum_{j=1}^n \bar{c}_{ij} M_j^f + \bar{I}_i < 0,
\end{aligned} \tag{6}$$

which is a contradiction and implies that (3) holds. This completes the proof.

Lemma 4 *Suppose that* (H1)–(H5) *hold. Let*

$$x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$$

be the solution of (1) with initial value

$$\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \cdots, \varphi_n^*(t))^T,$$

and

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$$

be the solution of (1) with initial value

$$\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T.$$

Then there exist constants $\lambda > 0$ *and* $M > 1$ *such that for all* $t > 0$,

$$|x_i(t) - x_i^*(t)| \leq M\|\varphi - \varphi^*\| e^{-\lambda t},$$

where $i = 1, 2, \cdots, n$.

Proof. Let

$$y(t) = \{y_i(t)\} = \{x_i(t) - x_i^*(t)\} = x(t) - x^*(t).$$

Then

$$\begin{aligned}
y_i'(t) &= -d_i(t)[x_i(t) - x_i^*(t)] \\
&\quad + \sum_{j=1}^n a_{ij}(t)[f_j(x_j(t)) - f_j(x_j^*(t))] \\
&\quad + \sum_{j=1}^n b_{ij}(t)[f_j(x_j(t - \tau_{ij}(t))) \\
&\quad - f_j(x_j^*(t - \tau_{ij}(t)))] \\
&\quad + \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t k_{ij}(t - s) \\
&\quad \times [f_j(x_j(s)) - f_j(x_j^*(s))] ds, \qquad (7) \\
y_i(t_k^+) &= (1 + \gamma_{ik}) y_i(t_k), k = 1, 2, \ldots, \qquad (8)
\end{aligned}$$

where $i = 1, 2, \cdots, n$. Next, define a Lyapunov functional as

$$V_i(t) = |y_i(t)|e^{\lambda t}, i = 1, 2, \cdots, n. \tag{9}$$

It follows from (7), (8) and (9) that

$$\begin{aligned}
& D^+(V_i(t)) \\
\leq \; & D^+(|y_i(t)|)e^{\lambda t} + \lambda |y_i(t)|e^{\lambda t} \\
\leq \; & (\lambda - d_i^-)|y_i(t)|e^{\lambda t} \\
& + \left[\sum_{j=1}^{n} |a_{ij}(t)||f_j(x_j(t)) - f_j(x_j^*(t))| \right. \\
& + \sum_{j=1}^{n} |b_{ij}(t)||f_j(x_j(t - \tau_{ij}(t))) \\
& - f_j(x_j^*(t - \tau_{ij}(t)))| \\
& + \sum_{j=1}^{n} |c_{ij}(t)| \int_{-\infty}^{t} |k_{ij}(t-s)| \\
& \left. \times |f_j(x_j(s)) - f_j(x_j^*(s))|ds \right] e^{\lambda t} \\
\leq \; & (\lambda - d_i^-)|y_i(t)|e^{\lambda t} + \sum_{j=1}^{n} \bar{a}_{ij} L_j^f |y_j(t)|e^{\lambda t} \\
& + \sum_{j=1}^{n} \bar{b}_{ij} k_{ij}^+ L_j^f |y_j(t - \tau_{ij}(t))|e^{\lambda t} \\
& + \sum_{j=1}^{n} \bar{c}_{ij} L_j^f |y_j(t)|e^{\lambda t}, t \neq t_k,
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
V_i(t_k^+) &= |y_i(t_k^+)|e^{\lambda t_k} \\
&= |x_i(t_k^+) - x_i^*(t_k^+)|e^{\lambda t_k} \\
&= |1 + \delta_{ik}|y_i(t_k)|e^{\lambda t_k},
\end{aligned} \tag{11}$$

where $i = 1, 2, \cdots, n$. Let $M > 1$ denote an arbitrary real number and set

$$\|\varphi - \varphi^*\| = \sup_{-\tau \leq s \leq 0} \max_{1 \leq j \leq n} |\varphi_j(s) - \varphi_j^*(s)| > 0,$$

where $j = 1, 2, \ldots, n$. Then by (9), we have

$$V_i(t) = |y_i(t)|e^{\lambda t} < M\|\varphi - \varphi^*\|, \text{for all } t \in [-\infty, 0],$$

where $i = 1, 2, \cdots, n$. Thus we can claim that

$$V_i(t) = |y_i(t)|e^{\lambda t} < M\|\varphi - \varphi^*\|, \tag{12}$$

for all $t \in [-\infty, t_1], i = 1, 2, \ldots, n$. Otherwise, there must exist $i \in \{1, 2, \cdots, n\}$ and $\tau_0 \in (-\tau, t_1]$ such that

$$V_i(\sigma_0) = M\|\varphi - \varphi^*\|, V_j(t) < M\|\varphi - \varphi^*\|, \tag{13}$$

for all $t \in [-\tau, \tau_0), j = 1, 2, \ldots, n$. Combining (10), (11) with (12), we obtain

$$\begin{aligned}
0 \; \leq \; & D^+(V_i(\tau_0) - M\|\varphi - \varphi^*\|) \\
= \; & D^+(V_i(\tau_0)) \\
\leq \; & (\lambda - d_i^-)|y_i(\tau_0)|e^{\lambda \tau_0} \\
& + \sum_{j=1}^{n} \bar{a}_{ij} L_j^f |y_j(\tau_0)|e^{\lambda \tau_0} \\
& + \sum_{j=1}^{n} \bar{b}_{ij} k_{ij}^+ L_j^f |y_j(\tau_0 - \tau_{ij}(\tau_0))|e^{\lambda \tau_0} \\
& + \sum_{j=1}^{n} \bar{c}_{ij} L_j^f |y_j(\tau_0)|e^{\lambda \tau_0} \\
= \; & (\lambda - d_i^-)|y_i(\tau_0)|e^{\lambda \tau_0} \\
& + \sum_{j=1}^{n} \bar{a}_{ij} L_j^f |y_j(\tau_0)|e^{\lambda \tau_0} \\
& + \sum_{j=1}^{n} \bar{b}_{ij} k_{ij}^+ L_j^f |y_j(\tau_0 - \tau_{ij}(\tau_0))| \\
& \times e^{\lambda(\tau_0 - \tau_{ij}(\tau_0))} e^{\lambda \tau_{ij}(\tau_0)} \\
& + \sum_{j=1}^{n} \bar{c}_{ij} L_j^f |y_j(\tau_0)|e^{\lambda \tau_0} \\
\leq \; & (\lambda - d_i^-)M\|\varphi - \varphi^*\| \\
& + \sum_{j=1}^{n} \bar{a}_{ij} L_j^f M\|\varphi - \varphi^*\| \\
& + \sum_{j=1}^{n} \bar{b}_{ij} k_{ij}^+ L_j^f M\|\varphi - \varphi^*\| e^{\lambda \tau} \\
& + \sum_{j=1}^{n} \bar{c}_{ij} L_j^f M\|\varphi - \varphi^*\| \\
= \; & \left[(\lambda - d_i^-) + \sum_{j=1}^{n} \Big((\bar{a}_{ij} + \bar{b}_{ij} k_{ij}^+ \right. \\
& \left. + \bar{c}_{ij}) L_j^f\Big)\right] \times M\|\varphi - \varphi^*\|.
\end{aligned} \tag{14}$$

Then

$$(\lambda - d_i^-) + \sum_{j=1}^{n} \Big((\bar{a}_{ij} + \bar{b}_{ij} k_{ij}^+ + \bar{c}_{ij}) L_j^f\Big) > 0,$$

which contradicts (H5), then (12) holds. In view of (12), we know that

$$V_i(t_1) = |y_i(t_1)|e^{\lambda t_1} < M\|\varphi - \varphi^*\|, i = 1, 2, \cdots$$

and

$$V_i(t_1^+) = |1 + \gamma_{i1}||y_i(t_1)|e^{\lambda t_1} \leq |y_i(t_1)|e^{\lambda t_1}.$$

Then

$$V_i(t_1^+) < M\|\varphi - \varphi^*\|. \quad (15)$$

Thus, for $t \in [t_1, t_2]$, we can repeat the above procedure and obtain

$$V_i(t) = |y_i(t)|e^{\lambda t} < M\|\varphi - \varphi^*\|, \text{for all } t \in [t_1, t_2],$$

where $i = 1, 2, \cdots$. Similarly, we have

$$V_i(t) = |y_i(t)|e^{\lambda t} < M\|\varphi - \varphi^*\|, \text{for all } t > 0,$$

where $i = 1, 2, \cdots$. Namely,

$$|x_i(t) - x_i^*(t)| = |y_i(t)| < M\|\varphi - \varphi^*\|, \text{for all } t > 0,$$

where $i = 1, 2, \cdots$. This completes the proof.

Remark 5 *If $x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ is a T-anti-periodic solution of (1), it follows from Lemma 4 and the Definition 2 that $x^*(t)$ is globally exponentially stable.*

3 Main results

In this section,we present our main result that there exists the exponentially stable anti-periodic solution of (1).

Theorem 6 *Assume that* (H1)–(H5) *are satisfied. Then (1) has exactly one T-anti-periodic solution $x^*(t)$. Moreover, this solution is globally exponentially stable.*

Proof. Let $v(t) = (v_1(t), v_2(t), \cdots, v_n(t))^T$ be a solution of (1) with initial conditions

$$v_i(s) = \varphi_i^v(s), |\varphi_i^v(s)| < \gamma, s \in (-\tau, 0], \quad (16)$$

where $i = 1, 2, \cdots, n$. Thus according to Lemma 3, the solution $v(t)$ is bounded and

$$|v_i(t)| < \gamma, \text{for all } t \in R, i = 1, 2, \cdots, n. \quad (17)$$

From (1), we obtain

$$\begin{aligned}
&((-1)^{p+1}v_i(t+(p+1)T))' \\
&= (-1)^{p+1}\Big\{ - d_i(t+(p+1)T) \\
&\times x_i(t+(p+1)T) \\
&+ \sum_{j=1}^{n} a_{ij}(t+(p+1)T) \\
&\times f_j(x_j(t+(p+1)T)) \\
&+ \sum_{j=1}^{n} b_{ij}(t+(p+1)T) \\
&\times f_j(x_j(t+(p+1)T \\
&-\tau_{ij}(t+(p+1)T))) \\
&+ \sum_{j=1}^{n} c_{ij}(t+(p+1)T) \\
&\times \int_{-\infty}^{t+(p+1)T} k_{ij}(t+(p+1)T-s) \\
&\times f_j(x_j(s))ds + I_i(t+(p+1)T)\Big\} \\
&= -d_i(t)(-1)^{p+1}x_i(t+(p+1)T) \\
&+ \sum_{j=1}^{n} a_{ij}(t) f_j((-1)^{p+1}x_j(t+(p+1)T)) \\
&+ \sum_{j=1}^{n} b_{ij}(t) f_j((-1)^{p+1}x_j(t+(p+1)T \\
&-\tau_{ij}(t))) + \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s) \\
&\times f_j(x_j(s))ds + I_i(t), t \neq t_k
\end{aligned} \quad (18)$$

and

$$\begin{aligned}
&(-1)^{p+1}v_i(t_k+(p+1)T)^+) \\
&= (-1)^{p+1}(1+\gamma_{i(k+(p+1)q)} \\
&\times v_i(t_k+(p+1)T)) \\
&= (-1)^{p+1}(1+\gamma_{ik})v_i(t_k+(p+1)T) \\
&= (1+\gamma_{ik})((-1)^{p+1} \\
&\times v_i(t_k+(p+1)T)),
\end{aligned} \quad (19)$$

where $i = 1, 2, \cdots, n, k = 1, 2, \cdots$. Thus $(-1)^{p+1}v(t+(p+1)T)$ are the solutions of (1) on R for any natural number p. Then, from Lemma 4, there exists a constant $M > 1$ such that

$$\begin{aligned}
&|(-1)^{p+1}v_i(t+(p+1)T) \\
&-(-1)^k v_i(t+pT)| \\
\leq\ & Me^{-\lambda(t+pT)} \\
&\times \sup_{-\infty \leq s \leq 0} \max_{1 \leq i \leq n} |v_i(s+T) + v_i(s)| \\
\leq\ & 2e^{-\lambda(t+pT)}M\gamma,
\end{aligned} \quad (20)$$

and

$$\begin{aligned}
&|(-1)^{p+1}v_i((t_k+(p+1)T)^+) \\
&-(-1)^p v_i((t_k+pT)^+)| \\
=\ & |x_i((t_k+(p+1)T)^+) \\
&+x_i((t_k+pT)^+)| \\
=\ & |1+\delta_{ik}||x_i(t_k+(p+1)T) \\
&+x_i(t_k+pT)| \\
\leq\ & 2M\gamma e^{-\lambda(pT+t_k)},
\end{aligned} \quad (21)$$

where $k \in N, i = 1, 2, \cdots, n$. Thus, for any natural number q, we have

$$(-1)^{q+1}v_i(t+(q+1)T)$$

$$= v_i(t) + \sum_{k=0}^{q}[(-1)^{k+1}v_i(t+(k+1)T) \\ -(-1)^k v_i(t+kT)], t \neq t_k. \tag{22}$$

Hence

$$|(-1)^{q+1}v_i(t+(q+1)T)| \\ \leq |v_i(t)| + \sum_{k=0}^{q}|(-1)^{k+1}v_i(t+(k+1)T) \\ -(-1)^k v_i(t+kT)|, t \neq t_k, \tag{23}$$

and

$$|(-1)^{q+1}v_i((t_k+(q+1)T)^+)| \\ = |(1+\delta_{ik})(-1)^{q+1}v_i(t_k+(q+1)T)| \\ \leq |(-1)^{q+1}v_i(t_k+(q+1)T)|. \tag{24}$$

where $i = 1, 2, \cdots, n$. It follows from (20)–(24) that $(-1)^{q+1}v_i(t+(q+1)T)$ is a fundamental sequence on any compact set of R. Obviously, $\{(-1)^q v(t + qT)\}$ uniformly converges to a piece-wise continuous function $x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ on any compact set of R.

Now we show that $x^*(t)$ is T-anti-periodic solution of (1). Firstly, $x^*(t)$ is T-anti-periodic, since

$$x^*(t+T) \\ = \lim_{q\to\infty}(-1)^q v(t+T+qT) \\ = -\lim_{(q+1)\to\infty}(-1)^{q+1}v(t+(q+1)T) \\ = -x^*(t), t \neq t_k, \tag{25}$$

and

$$x^*((t+T)^+) \\ = \lim_{q\to\infty}(-1)^q v((t+T+qT)^+) \\ = -\lim_{(q+1)\to\infty}(-1)^{q+1}v((t+(q+1)T)^+) \\ = -x^*(t_k)^+. \tag{26}$$

In the sequel, we prove that $x^*(t)$ is a solution of (1). Noting that the right-hand side of (1) is piece-wise continuous, (18) and (19) imply that $\{((-1)^{q+1}v(t + (q+1)T))'\}$ uniformly converges to a piece-wise continuous function on any compact subset of R. Thus, letting $q \to \infty$ on both sides of (18) and (19), we can easily obtain

$$\begin{cases} \dot{x}_i^*(t) = -d_i(t)x_i^*(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j^*(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j^*(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n} c_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)f_j(x_j^*(s))ds \\ \qquad + I_i(t), t \neq t_k, \\ x_i{}^*(t_k^+) = (1+\delta_{ik})x_i^*(t_k), k = 1, 2, \ldots, \end{cases} \tag{27}$$

where $i = 1, 2, \ldots, n$. Therefore, $x^*(t)$ is a solution of (1). Applying Lemma 4, we can easily check that $x^*(t)$ is globally exponentially stable. The proof of Theorem 6 is completed.

4 An example

In this section, we give an example to illustrate our main results obtained in previous sections. Let $n = 2$, consider the high-order cellular neural networks with delays and impulses

$$\begin{cases} \dot{x}_1(t) = -d_1(t)x_1(t) + \sum_{j=1}^{2} a_{1j}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{2} b_{1j}(t)f_j(x_j(t-\tau_{1j}(t))) \\ \qquad + \sum_{j=1}^{2} c_{1j}(t)\int_{-\infty}^{t} k_{1j}(t-s)f_j(x_j^*(s))ds \\ \qquad + I_1(t), t \neq t_k, \\ \dot{x}_2(t) = -d_2(t)x_2(t) + \sum_{j=1}^{2} a_{2j}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{2} b_{2j}(t)f_j(x_j(t-\tau_{2j}(t))) \\ \qquad + \sum_{j=1}^{2} c_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)f_j(x_j^*(s))ds \\ \qquad + I_2(t), t \neq t_k, \\ x_1(t_k^+) = (1+\delta_{1k})x_i^*(t_k), k = 1, 2, \cdots, \\ x_2(t_k^+) = (1+\delta_{2k})x_i^*(t_k), k = 1, 2, \cdots, \end{cases} \tag{28}$$

where

$$f_j(u) = \frac{1}{2}(|u-1| - |u-1|)(i = 1, 2),$$

$$k_{ij}(s) = 1, \tau_{ij}(t) = 1, k_{ij}^+ = 1, i, j = 1, 2$$

and

$$\begin{bmatrix} d_1(t) & d_2(t) \\ I_1(t) & I_2(t) \end{bmatrix}$$

$$= \begin{bmatrix} 3+|\cos t| & 3.2+|\sin t| \\ 2\sin t & 3\sin t \end{bmatrix},$$

$$\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{10}|\sin t| & \frac{1}{10}|\cos t| \\ \frac{1}{8}|\cos t| & \frac{1}{6}|\sin t| \end{bmatrix},$$

$$\begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{5}|\sin t| & \frac{1}{6}|\cos t| \\ \frac{1}{4}|\cos t| & \frac{1}{3}|\sin t| \end{bmatrix},$$

$$\begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{2}|\sin t| & \frac{1}{4}|\sin t| \\ \frac{1}{2}|\cos t| & \frac{1}{4}|\cos t| \end{bmatrix}.$$

Then $L_j^f = M_j^f = 1, d_1^- = 2, d_2^- = 2.2$ and

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{1}{10} \\ \frac{1}{8} & \frac{1}{6} \end{bmatrix},$$
$$\begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} \\ \bar{b}_{21} & \bar{b}_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix},$$
$$\begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} \\ \bar{c}_{21} & \bar{c}_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}.$$

Let $\eta = 0.1$ and $\lambda = 0.5$. Then

$$\begin{aligned} &(\lambda - d_1^-) + \sum_{j=1}^{2} \Big((\bar{a}_{1j} + \bar{b}_{1j} k_{1j}^+ \\ &+ \bar{c}_{1j}) L_j^f \Big) \\ &= (0.5-2) + \Big(\frac{1}{10} + \frac{1}{10} + \frac{1}{5} \\ &+ \frac{1}{6} + \frac{1}{2} + \frac{1}{4} \Big) \\ &= -0.1833 < -0.1 < 0, \\ &(\lambda - d_2^-) + \sum_{j=1}^{2} \Big((\bar{a}_{2j} + \bar{b}_{2j} k_{2j}^+ \\ &+ \bar{c}_{2j}) L_j^f \Big) \\ &= (0.5-2.2) + \Big(\frac{1}{8} + \frac{1}{6} + \frac{1}{4} \\ &+ \frac{1}{3} + \frac{1}{2} + \frac{1}{4} \Big) \\ &= -0.075 < -0.1 < 0, \end{aligned}$$

which implies that system (28) satisfies all the conditions in Theorem 6. Thus we can conclude that system (28) has exactly one π-anti-periodic solution. Moreover, this solution is globally exponentially stable.

5 Conclusions

In this paper, we investigate a class of cellular neural networks with impulsive effects. With the aid of differential inequality techniques, a series of very verifiable criteria on the existence and exponential stability of anti-periodic solutions for the cellular neural networks are established. Our results are new and complementary to previously known results. Finally, an example is given to illustrate the feasibility and effectiveness of our main results.

Acknowledgements: The research was supported by This work is supported by National Natural Science Foundation of China(No.11261010, No.11101126), Soft Science and Technology Program of Guizhou Province(No.2011LKC2030), Natural Science and Technology Foundation of Guizhou Province(J[2012]2100), Governor Foundation of Guizhou Province([2012]53), Natural Science and Technology Foundation of Guizhou Province(2014) and Natural Science Innovation Team Project of Guizhou Province ([2013]14).

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