Improved Gaussian Mixture PHD Smoother for Multi-target Tracking

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Abstract: - The Gaussian mixture probability hypothesis density (GM-PHD) smoother proposed recently can yield better state estimates than the GM-PHD filter. However, there are two major problems with it. First, the smoothed PHD distribution can not provide a more accurate target number estimate due to the target number estimation bias becoming larger by smoothing. Second, the computational complexity of computing the smoothed PHD distribution increases with the cardinality of measurement set, which can be very time-consuming when the clutter rate is high. To solve these problems an improved GM-PHD smoother is proposed that improves the target number estimation performance by using the estimated target number of forward GM-PHD filter and reduces the computational cost of GM-PHD smoother by the rectangular gating method. Simulated results show that the improved GM-PHD smoother is superior to the GM-PHD smoother in both the aspects of target number estimate and computational cost, so this improved GM-PHD smoother will have an applicable potential in related fields.

Key-Words: - Gaussian Mixture, Probability Hypothesis Density, Filtering, Smoothing, Target Tracking, Random Finite Set, Sequential Monte Carlo

1 Introduction

Multi-target tracking (MTT) is an important research issue with wide applications in both civilian and military fields, such as passive radar tracking, terrain vehicle tracking, sonar image tracking, multi-target visual tracking, etc. The purpose of MTT is to jointly estimate the number of targets and their states from a sequence of observation sets in the presence of clutter, data association uncertainty, detection uncertainty and noise. The classical methods to the MTT problem requires data association that operates in conjunction with filtering [1-3], such as nearest neighbor (NN) [4], joint probabilistic data association (JPDA) [5], probabilistic data association (PDA) [6], and multiple hypothesis tracking (MHT) [7], etc. The data association problem in the classical MTT algorithms results in a huge computational load due to its combinatorial nature, so MTT still remains challenges in theory and application.

In recent years, the random finite set (RFS) theory [8] has been used by more and more researchers to tackle with the MTT problem. The probability hypothesis density (PHD) multi-target filter [9] that propagates the posterior intensity function of the RFS of targets in time avoids the combinatorial problem that arises from data association and has attracted considerable interest. The PHD filter is a suboptimal but computationally tractable alternative to the multi-target Bayes filter in RFS framework, it still requires solving multiple integrals that have no closed-form solutions in general. Sequential Monte Carlo implementation of the PHD (SMC-PHD) filter has paved the way for its application to realistic nonlinear non-Gaussian filtering problems [10, 11]. Besides, a closed-form solution to the PHD recursion has also been derived for linear target dynamic and measurement models, called the Gaussian mixture PHD (GM-PHD), which estimates the PHD distribution as a mixture of Gaussian densities [12, 13]. Afterward, many extensions have been developed to solve different MTT problems [14-17].

To improve the capability of PHD-based filters, a forward-backward PHD smoother has been proposed recently. Similar to the PHD filter, there are two major implementation methods of the forward-backward PHD smoother known as the SMC-PHD smoother [18, 19], and the Gaussian mixture PHD (GM-PHD) smoother [20, 21]. The GM-PHD smoother involves a forward multi-target filtering using the standard GM-PHD filter recursion and then a backward smoothing recursion. The backward smoothing recursion is the key step of the forward-backward GM-PHD smoother. In the backward smoothing step, the backward corrector, which involves the innovation from forward filtering using measurements beyond the current time, is used to adjust the filtered PHD distribution at current time step.

The target number of GM-PHD smoother is estimated by rounding the volume of the smoothed intensity function to the nearest integer, and the target state estimates are generated from the smoothed intensity function by extracting the means from the smoothed Gaussian components with weights greater than some threshold [21]. Compared with the GM-PHD filter, GM-PHD smoother provides more accurate target state estimates by the smoothed intensity function. However, there are two issues present in GM-PHD smoother. First, the GM-PHD smoother is, to some extent, of approximation, the target number estimation bias resulted from the forward filtering can not be reduced in the backward smoothing step. Instead, it will become larger with smoothing lag growing. Hence, the target number estimated by summing up the appropriate weights of the Gaussian components in the smoothed PHD is unreliable. Another key issue to the GM-PHD smoother is the computational complexity of computing the smoothed PHD distribution increasing with the cardinality of the measurement set. This can be very time-consuming especially when the clutter rate is high which has become one of the biggest challenges of the GM-PHD smoother.

In order to address the aforementioned problems in GM-PHD smoother, an improved GM-PHD smoother is proposed in this paper. As explained above, we can see that the computational cost of GM-PHD smoother can be reduced by means of reducing the cardinality of measurement set. At each backward smoothing step, based on the forward filtering results and by using the rectangular gating technique, the new measurement set is constructed to compute the smoothed PHD distribution. The rectangular gating may remove all measurements not associated with targets in the measurement set. Thus unnecessary computation can be avoided and the overall processing speed is effectively enhanced. In addition, since the smoothed PHD intensity can not provide a more accurate target number estimate for GM-PHD smoother, thus the target number of the improved GM-PHD smoother estimated by summing up the appropriate weights of the filtered Gaussian components is a better choice. The numerical simulation indicates that the computational cost is reduced and that the target number estimation performance is improved.

The rest of this paper is organized as follows. In Section 2 a brief background of RFS-based MTT is provided. In addition, the forward-backward PHD smoother is reviewed and the GM-PHD smoother is presented. The proposed algorithm is elaborated in Section 3. In Section 4, the simulated results are given and discussed. Finally, some meaningful conclusions are drawn in Section 5.

2 Background

2.1 Multi-target Tracking

In MTT problems, assume that at time step k, there are n_k target states $x_k^1,\cdots,x_k^{n_k}$ in a state space χ and m_k measurements $z_k^1,\cdots,z_k^{m_k}$ received in an observation space Z. Since there is no sequential order on the respective collections of target states and measurements, they can be naturally represented as finite sets, i.e.

$$X_k = \{x_k^1,\cdots,x_k^{n_k}\} \in \chi \tag{1}$$

$$Z_k = \{z_k^1,\cdots,z_k^{m_k}\} \in Z \tag{2}$$

where X_k and Z_k are the target state and measurement sets with n_k targets and m_k measurements, respectively. Some measurements in Z_k may be due to clutter, the number of clutter points is assumed to be Poisson distributed.

Denote the multi-target posterior density function as $p_{k-1|k-1}(X_{k-1}|Z_{1:k-1})$, then, the prediction and update equations for the optimal multi-target Bayes filter are described by

$$p_{k|k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X)p_{k-1|k-1}(X|Z_{1:k-1})\mu_s(dX) \tag{3}$$

$$p_{k|k}(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k)p_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X)p_{k|k-1}(X|Z_{1:k-1})\mu_s(dX)} \tag{4}$$

where the parameter μ_s takes the place of the Lebesgue measure [11], $p_{k|k-1}$ denotes the predicted multi-target density, $f_{k|k-1}$ and g_k are the multi-target transition density function and the multi-target likelihood function, respectively.

2.2 PHD Smoother

PHD smoother involves a forward multi-target filtering recursion using the standard PHD filter and then a backward smoothing recursion [18]. The PHD filter can be derived from the optimal multi-target Bayes filter using finite set statistics. Let $v_{k-1|k-1}$ denote the filtered PHD at time step

$k-1$, the PHD filter involves a prediction step and an update step that propagate the intensity function $v_{k-1|k-1}(x)$ recursively in time, i.e.

$$v_{k|k-1}(x)=\gamma_{k|k-1}(x)+\left\langle p_{S,k|k-1}f_{k|k-1}(x|\cdot),v_{k-1|k-1}(\cdot)\right\rangle \tag{5}$$

$$v_{k|k}(x)=v_{k|k-1}(x)L_k(Z_k;x) \tag{6}$$

where

$$L_k(Z_k;x)=1-p_{D,k}+\sum_{z\in Z_k}\frac{p_{D,k}g_k(z|x)}{\kappa_k(z)+\left\langle p_{D,k}g_k(z|\cdot),v_{k|k-1}(\cdot)\right\rangle} \tag{7}$$

where $f_{k|k-1}$ is the single-target transition density; $\langle\cdot,\cdot\rangle$ denotes inner product, i.e., the operation of multiple integrals; $\gamma_{k|k-1}$ denotes the PHD of spontaneously target birth; $p_{S,k|k-1}$ and $p_{D,k}$ are the survival probability and detection probability, respectively; κ_k denotes the intensity function of the clutter RFS, g_{k+1} is the single-target measurement likelihood.

In the backward smoothing step, the smoothed PHD is propagated backward via the backward smoothing recursion [21], i.e.

$$v_{t|k}(x)=v_{t|t}(x)B_{t|k}(x) \tag{8}$$

where $B_{t|k}(x)$ is the backward corrector, it can be recursively computed as follows

$$B_{t|k}(x)=1-p_{S,t+1|t}+p_{S,t+1|t}\times\left\langle B_{t+1|k}L_{t+1}(Z_{t+1};\cdot),f_{t+1|t}(\cdot|x)\right\rangle \tag{9}$$

Note that the backward corrector starts with $B_{k|k}(x)=1$.

2.3 GM-PHD Smoother

The GM-PHD smoother is a closed-form smoothing solution to the forward-backward PHD smoother for the linear Gaussian model. Under linear Gaussian multi-target assumptions, each target follows a linear Gaussian dynamic model and linear Gaussian measurement model, i.e.

$$f_{k|k-1}(x|\zeta)=N(x;F_{k-1}\zeta,Q_{k-1}) \tag{10}$$

$$g_k(z|x)=N(z;H_kx,R_k) \tag{11}$$

where $N(x;m,P)$ denotes a Gaussian density with mean m and covariance P. The parameter F_{k-1} is the state transition matrix, Q_{k-1} is the process noise covariance; H_k is the observation matrix and R_k is the observation noise covariance.

If the PHD at time step $k-1$ is a Gaussian mixture, then the predicted and filtered PHD at time step k are expressed as a Gaussian mixture of the form

$$v_{k|k-1}(x)=\sum_{i=1}^{J_{k|k-1}}w_{k|k-1}^{(i)}N\left(x;m_{k|k-1}^{(i)},P_{k|k-1}^{(i)}\right) \tag{12}$$

$$v_{k|k}(x)=\sum_{i=1}^{J_{k|k}}w_{k|k}^{(i)}N\left(x;m_{k|k}^{(i)},P_{k|k}^{(i)}\right) \tag{13}$$

Suppose that the backward corrector at time step $t+1$ from time step k is given by

$$B_{t+1|k}(x)=\sum_{i=1}^{J_{t+1|k}}\omega_{t+1}^{(i)}N_{C_{t+1}^{(i)},D_{t+1}^{(i)}}\left(\zeta_{t+1}^{(i)};x\right) \tag{14}$$

Then according to Eq. (9), the generic backward corrector recursion at time step t from time step k is given by

$$B_{t|k}(x)=1-p_S+p_S(1-p_D)B'_{t+1|k}(x)+p_Sp_D\sum_{z\in Z_{k+1}}\frac{B''_{t+1|k}(x;z)}{\kappa_{t+1}(z)+p_Dr_{t+1}(z)} \tag{15}$$

where $r_{t+1}(z)$, $B'_{t+1|k}(x)$ and $B''_{t+1|k}(x;z)$ are given, respectively, by

$$r_{t+1}(z)=\left\langle N_{H_{t+1},R_{t+1}}(z;\cdot),v_{t+1|t}(\cdot)\right\rangle=\sum_{i=1}^{J_{t+1|t}}w_{t+1|t}^{(i)}N_{H_{t+1},R_{t+1}+H_{t+1}P_{t+1|t}^{(i)}H_{t+1}^T}(z;m_{t+1|t}^{(i)}) \tag{16}$$

$$B'_{t+1|k}(x)=\sum_{i=1}^{J_{t+1|k}}\omega_{t+1}^{(i)}N_{\dot{C}_{t+1}^{(i)},\dot{D}_{t+1}^{(i)}}\left(\zeta_{t+1}^{(i)};x\right) \tag{17}$$

$$\dot{C}_{t+1}^{(i)}=C_{t+1}^{(i)}F_t,\quad \dot{D}_{t+1}^{(i)}=D_{t+1}^{(i)}+C_{t+1}^{(i)}Q_tC_{t+1}^{(i)T} \tag{18}$$

$$B''_{t+1|k}(x;z)=\sum_{i=1}^{J_{t+1|k}}\omega_{t+1}^{(i)}N_{\ddot{C}_{t+1}^{(i)},\ddot{D}_{t+1}^{(i)}}\left(\begin{bmatrix}\zeta_{t+1}^{(i)}\\ z\end{bmatrix};x\right) \tag{19}$$

$$\ddot{C}_{t+1}^{(i)}=\begin{bmatrix}C_{t+1}^{(i)}\\ H_{t+1}\end{bmatrix}F_t,\quad \ddot{D}_{t+1}^{(i)}=\begin{bmatrix}D_{t+1}^{(i)} & 0\\ 0 & R_{t+1}\end{bmatrix}+\begin{bmatrix}C_{t+1}^{(i)}\\ H_{t+1}\end{bmatrix}Q_t\begin{bmatrix}C_{t+1}^{(i)}\\ H_{t+1}\end{bmatrix}^T \tag{20}$$

According to Eqs. (8), (13) and (14), the smoothed PHD at time step $t+1$ from time step k is a Gaussian mixture and is given by

$$v_{t+1|k}(x)=v_{t+1|t+1}(x)B_{t+1|k}(x)=\sum_{i=1}^{J_{t+1|k}}\sum_{j=1}^{J_{t+1|t+1}}\omega_{t+1}^{(i)}w_{t+1|t+1}^{(j)}\mu_{t+1|t+1}^{(i,j)}(\zeta_{t+1}^{(i)})\times N\left(x;\tilde{m}_{t+1|t+1}^{(i,j)}(\zeta_{t+1}^{(i)}),\tilde{P}_{t+1|t+1}^{(i,j)}\right) \tag{21}$$

where

$$\tilde{m}_{t+1|t+1}^{(i,j)}(\zeta_{t+1}^{(i)})=m_{t+1|t+1}^{(j)}+K_{t+1|t+1}^{(i,j)}\left(\zeta_{t+1}^{(i)}-C_{t+1}^{(i)}m_{t+1|t+1}^{(j)}\right) \tag{22}$$

$$\mu_{t+1|t+1}^{(i,j)}(\zeta_{t+1}^{(i)}) = N_{C_{t+1}^{(i)},D_{t+1}^{(i)}+C_{t+1}^{(i)}P_{t+1|t+1}^{(j)}C_{t+1}^{(i)T}}\left(\zeta_{t+1}^{(i)};m_{t+1|t+1}^{(j)}\right) \quad (23)$$

$$\tilde{P}_{t+1|t+1}^{(i,j)} = \left(I - K_{t+1|t+1}^{(i,j)}C_{t+1}^{(i)}\right)P_{t+1|t+1}^{(j)} \quad (24)$$

$$K_{t+1|t+1}^{(i,j)} = P_{t+1|t+1}^{(j)}C_{t+1}^{(i)T} \times \left(C_{t+1}^{(i)}P_{t+1|t+1}^{(j)}C_{t+1}^{(i)T} + D_{t+1}^{(i)}\right)^{-1} \quad (25)$$

Then, the target state estimates of GM-PHD smoother are extracted from the means of the smoothed Gaussian components with weights greater than some threshold, and the estimated target number at time step $t+1$ can be written as

$$\hat{N}_{t+1|k} = \sum_{i=1}^{J_{t+1|k}}\sum_{j=1}^{J_{t+1|t+1}} \omega_{t+1}^{(i)}w_{t+1|t+1}^{(j)}\mu_{t+1|t+1}^{(i,j)}(\zeta_{t+1}^{(i)}) \quad (26)$$

According to Eqs. (8), (13), (14) and (15), assume that the smoothing lag is l time steps, then the GM-PHD smoother requires $J_{t|t}\prod_{i=1}^{l}(1+|Z_{t+i}|)$ Gaussian components to represent $v_{k-l|k}(x)$.

Hence, the computational complexity of computing the smoothed PHD with a lag of l time steps is $O(\prod_{i=1}^{l}(1+|Z_{t+i}|))$, this implies that the computational complexity of GM-PHD smoother can be reduced by means of reducing the cardinality of measurement set.

3 Improved GM-PHD Smoother

In this section, based on the GM-PHD smoother, an improved GM-PHD smoother is proposed to improve the smoothing performance of GM-PHD smoother, in which the rectangular gating method is used to reduce the cardinality of measurement set. We summarize the proposed algorithm as follows.

3.1 Forward Filtering

Initialization step: At time step $k=0$, initialize the algorithm with the weighted sum of J_0 Gaussian components to approximate the initial PHD distribution, i.e.

$$v_{0|0}(x) = \sum_{i=1}^{J_0} w_0^{(i)}N\left(x;m_0^{(i)},P_0^{(i)}\right) \quad (27)$$

Prediction step: Predict existing Gaussian components with the Kalman filter, and we obtain

$$v_{S,k|k-1}(x) = p_S \times \sum_{i=1}^{J_{k-1|k-1}} w_{S,k-1}^{(i)}N\left(x;m_{S,k|k-1}^{(i)},P_{S,k|k-1}^{(i)}\right) \quad (28)$$

Assume that at time step k, the intensity of the birth is

$$\gamma_k(x) = \sum_{i=1}^{J_{\gamma,k}} w_{\gamma,k}^{(i)}N\left(x;m_{\gamma,k}^{(i)},P_{\gamma,k}^{(i)}\right) \quad (29)$$

Then, the predicted intensity $v_{k|k-1}(x)$ at time step k can be written as

$$v_{k|k-1}(x) = v_{S,k|k-1}(x) + \gamma_k(x) \quad (30)$$

Updating step: Update the filtered PHD intensity $v_{k|k}(x)$ via Eq. (6) by the measurement set $Z_k = \{z_k^1, z_k^2, \cdots, z_k^{m_k}\}$ received at time step k.

Pruning and merging: The Gaussian components are pruned by removing the Gaussian components with negligible weights, and the Gaussian components within a certain distance from each other are also merged into one.

State extraction: Target state estimates of the improved GM-PHD smoother are extracted from the means of filtered Gaussian components with weights greater than 0.5, then the target state estimate set of the forward GM-PHD filter is expressed as $\hat{X}_{k|k} = \{m_{k|k}^1, \cdots, m_{k|k}^{\hat{N}_k}\}$.

Target number estimation: By using the filtered PHD intensity at time step k, the estimated number of targets can be obtained by summing up the appropriate weights, i.e.

$$\hat{N}_{k|k} = round(\sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)}) \quad (31)$$

3.2 Construction of New Measurement Set

Suppose that at time step k, the measurement set received is $Z_k = \{z_k^1, z_k^2, \cdots, z_k^{m_k}\}$, and the true target state set is $X_k = \{x_k^1, x_k^2, \cdots, x_k^{n_k}\}$. Based on the observation measurement model, for each $x_k \in X_k$, since one target can only generate one measurement, the measurement generating from x_k is obtained by

$$z_k = H_k x_k + R_k \quad (32)$$

In practice, the true target state set X_k is unknown, we can use the target state set $\hat{X}_k$ estimated by forward GM-PHD filter to obtain the estimated measurements originating from true targets. Then the measurement estimation set can be expressed as follows

$$\hat{Z}_k = \{H_k m_{k|k}^1 \cdots, H_k m_{k|k}^{\hat{N}_k}\} = \{\hat{z}_k^1 \cdots, \hat{z}_k^{\hat{N}_k}\} \quad (33)$$

where $\hat{z}_k^i = H_k m_{k|k}^i$ is the measurement estimation associated with the ith target.

Given a threshold η, for the ith target, if $\hat{z}_k^i \in \hat{Z}_k$ and $z_k^i \in Z_k$ are its corresponding measurement estimation and true measurement, respectively, then the residual error $\tilde{z}_k^i(j)$ should satisfy the conditions of

$$\tilde{z}_k^i(j) = \left|\hat{z}_k^i(j) - z_k^i(j)\right| \le \eta \tag{34}$$

where $\hat{z}_k^i(j)$ and $z_k^i(j)$ are the jth component of $\hat{z}_k^i$ and z_k^i, respectively.

In order to remove the spurious measurements as much as possible, a threshold η should be empirically set according to the actual situation, with a larger η for a low clutter rate and a smaller η for a high clutter rate.

For clarity, we note the new measurement set as Z_k^{new}. Clutter are spurious measurements that do not carry any useful information. Therefore, the new measurement set Z_k^{new} obtained by rectangular gating method is used to compute the smoothed PHD intensity without affecting the performance of the improved GM-PHD smoother.

3.3 Backward Smoothing

Smoothing step: According to the smoothing lag $l = k - t$ used in smoother, compute the smoothed PHD $v_{t|k}(x)$ with the new measurement sets $Z_{t+i}^{new}, i = 1, \cdots, l$.

State extraction: target states are determined from the means of smoothed Gaussian components with weights greater than a specific threshold, note the smoothed target state estimate set as $\hat{X}_{t|k} = \{m_{t|k}^1, \cdots, m_{t|k}^{\hat{N}_{t|k}}\}$.

Output results: At time step t, the target state estimate set and target number estimate of the improved GM-PHD smoother are $\hat{X}_{t|k} = \{m_{t|k}^1, \cdots, m_{t|k}^{\hat{N}_{t|k}}\}$ and $\hat{N}_{t|k} = \hat{N}_{t|t}$ separately.

In the next section, we analyze the performance of the proposed algorithm compared with the GM-PHD smoother through different metrics using Monte Carlo simulations.

4 Simulation Results

To demonstrate the efficiency of our proposed improved GM-PHD smoother, we consider a two-dimensional scenario with an unknown and time varying number of targets observed in clutter. Each target has survival probability $p_S = 0.99$ and follows a linear Gaussian dynamic model. The target state vector $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^T$ is a vector of planar position and velocity at time k, and the measurement is a noisy version of the position. The sampling period is $\Delta = 1$ s. The simulation environment was as follows: AMD A8-6600K APU with Radeon HD(tm) Graphics 3.9 GHz, 4 GB DDR3 1600 Memory, Windows 7, and MATLAB R2012a.

The linear Gaussian dynamic model is used as

$$x_{k+1} = Fx_k + \Gamma q_k \tag{35}$$

where $F = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\Gamma = \begin{bmatrix} \Delta^2/2 & 0 \\ \Delta & 0 \\ 0 & \Delta^2/2 \\ 0 & \Delta \end{bmatrix}$,

$q_k \sim N\left(0, \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.1^2 \end{bmatrix}\right)$.

The linear-Gaussian measurement model is described as

$$z_{k+1} = Hx_{k+1} + r_{k+1} \tag{36}$$

where $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $r_{k+1} \sim N\left(0, \begin{bmatrix} 0.5^2 \\ 0.5^2 \end{bmatrix}\right)$.

New targets can appear spontaneously according to a Poisson point process with intensity function

$$\gamma_k = \sum_{i=1}^{3} w_\gamma N\left(x; m_\gamma^{(i)}, P_\gamma^{(i)}\right) \tag{37}$$

where $P_\gamma^{(1)} = P_\gamma^{(2)} = P_\gamma^{(3)} = \mathrm{diag}([5,1,5,1])$, and $\mathrm{diag}(\cdot)$ denotes the diagonal matrix. $m_\gamma^{(1)} = [0,0.6,0,2.1]^T$, $m_\gamma^{(2)} = [0,0,0,0]^T$, $m_\gamma^{(3)} = [0,2,0,2]^T$, $w_\gamma = 0.2$.

In our simulation, targets can appear or disappear in the scene at any time. Each target is detected with probability $p_D = 0.98$, the maximum number of Gaussian components is $J_{\max} = 100$. The pruning and merging thresholds are $T_p = 10^{-5}$ and $U = 5$, respectively. The clutter is modelled as a Poisson RFS with the uniform density in the surveillance region, and the clutter rate is $r = 5$. The smoothing lag used in this simulation is $l = 2$ time steps. The threshold in the improved GM-PHD smoother is $\eta = 3$.

To evaluate the performance of the proposed improved GM-PHD smoother, an appropriate metric, known as the optimal subpattern assignment (OSPA) distance [22] is employed as follows

$$d_{OSPA}(X,Y) = \left(\frac{1}{n}\left(\min_{\pi\in\prod_k}\sum_{i=1}^{m} d^{(c)}\left(x_i, y_{\pi_i}\right)^p + c^p(n-m)\right)\right)^{\frac{1}{p}} \tag{38}$$

where $X=\{x_1,\cdots,x_m\}$ and $Y=\{y_1,\cdots,y_n\}$ are arbitrary finite subsets, $1\le p<\infty$, $c>0$. In our simulation, the parameters are set to p=2 and c=50m.

In addition, since the target number estimate depends on the weight sum of the Gaussian components, we measure the mean absolute error (MAE) of weight sum on the true number of targets for the improved GM-PHD smoother. The MAE is calculated as follows

$$MAE(S_w, N) = E\{|S_w - N|\} \tag{39}$$

where S_w and N are the weight sum and true target number, respectively.

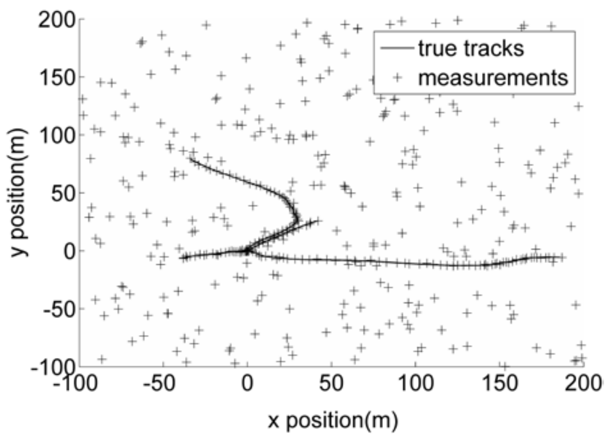

Fig.1. True tracks and measurements

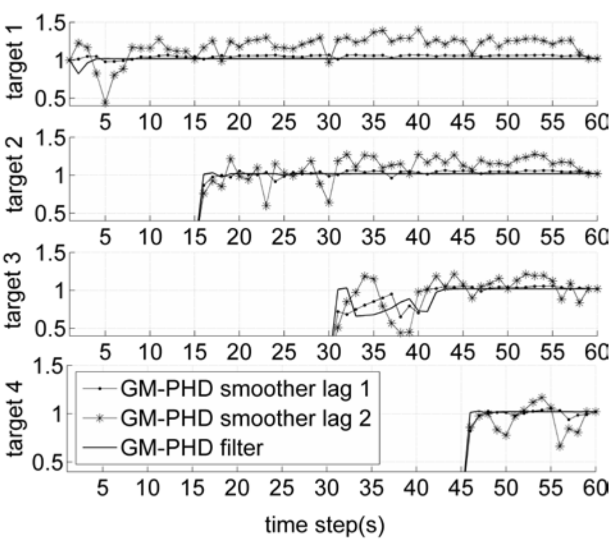

Fig.2. PHD weights of different targets in GM-PHD filter and GM-PHD smoother

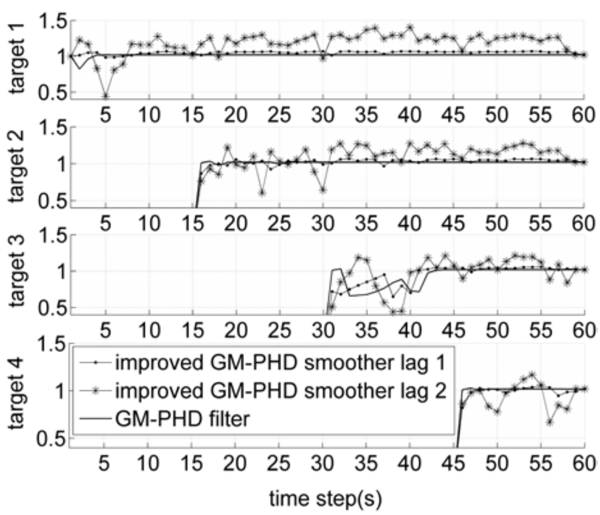

Fig.3. PHD weights of different targets in GM-PHD filter and improved GM-PHD smoother

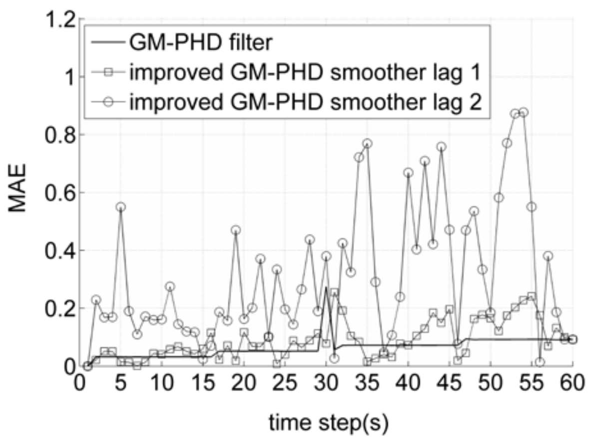

Fig.4. MAE for GM-PHD filter and improved GM-PHD smoother

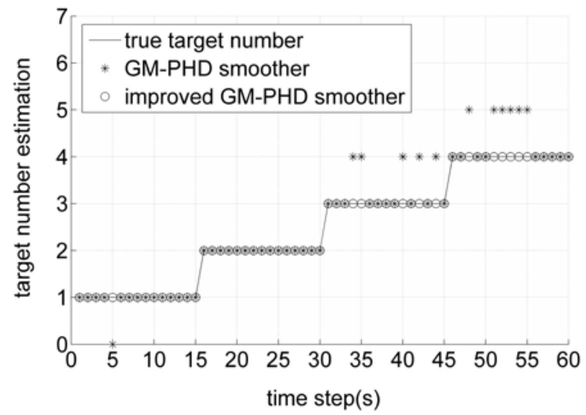

Fig.5. Target number estimations for GM-PHD smoother and improved GM-PHD smoother

Fig.1 shows a simulated scenario with true target tracks and measurements for a duration of 60 time steps in the presence of the clutter, where the solid lines and the plus signs denote the true target tracks and the measurements, respectively.

The smoothed PHD weights of different targets in the GM-PHD smoother and improved GM-PHD smoother are shown in Fig.2 and Fig.3, respectively. As seen from Fig.2 and Fig.3, the cluttered measurements have little effect on the adjustment of the filtered PHD weights. A comparison shows that the smoothed weights become fluctuant and unreliable.

Fig.4 illustrates the MAE for GM-PHD filter and improved GM-PHD smoother. It can be seen that the MAE becomes larger with the smoothing lag increasing, thus the target number estimated by summing up the appropriate weights of the smoothed Gaussian components is unreliable. Fig.5 shows the target number estimates for different algorithms. As seen, the target number estimate of the forward GM-PHD filter used as that of the improved GM-PHD smoother is a more reasonable solution.

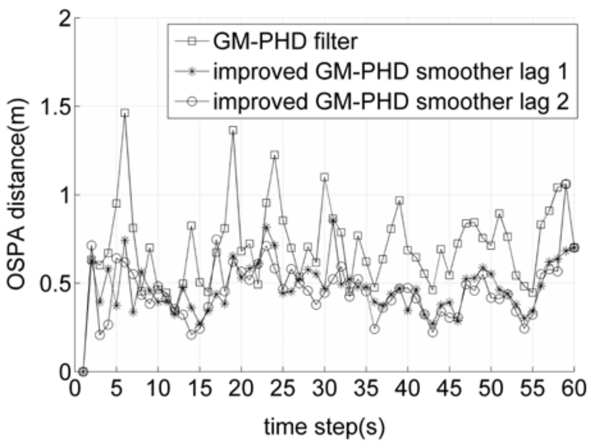

Fig.6. OSPA distances for improved GM-PHD smoother

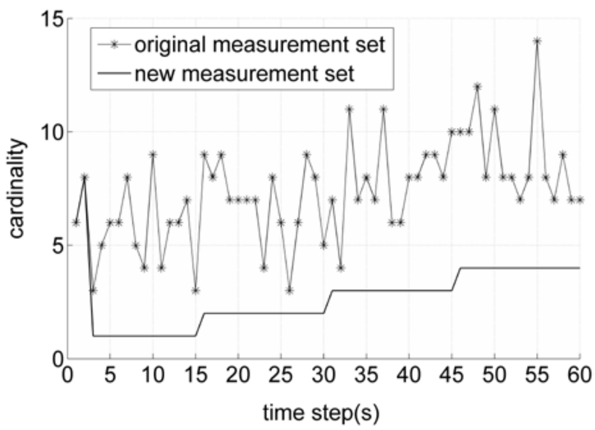

Fig.7. Cardinality comparison to original measurement set and new measurement set

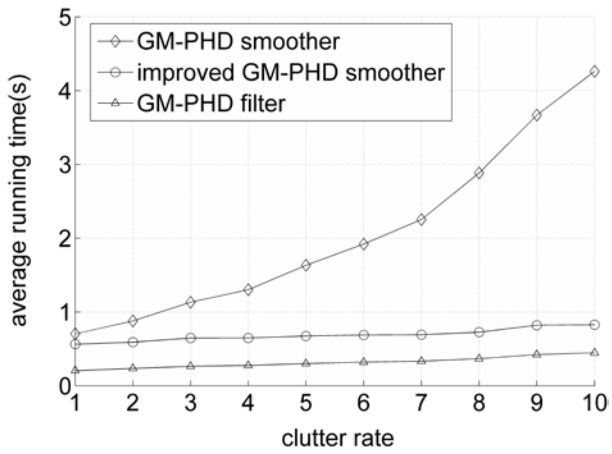

Fig.8. Average running time for GM-PHD smoother and improved GM-PHD smoother

Fig.6 shows the OSPA distances for the improved GM-PHD smoother with lags of 1 and 2 time steps. We can see that the performance of the proposed algorithm is improved over the GM-PHD filter. It can also be observed that the higher the lag we use for smoothing the better the state estimations we get, but improved performance is achieved at the cost of additional computational load.

The cardinality of measurement set is reduced significantly, as shown in Fig.7. Thus the computational cost of the proposed improved GM-PHD smoother can be reduced more significantly. The average running time of one MC trial using clutter rates from 1 to 10 is presented in Fig.8. As seen, the computing loads of two algorithms are growing with the increase of the clutter rate. It is obvious that the proposed improved GM-PHD smoother can achieve a much faster computing speed comparing with the GM-PHD smoother.

5 Conclusion

Based on the analysis of the problems in the GM-PHD smoother, an improved GM-PHD smoother is proposed in this paper, which reduces the computational cost of the GM-PHD smoother by the rectangular gating technique and estimates the target number by summing up the appropriate weights of the filtered Gaussian components rather than those of the smoothed Gaussian components. Simulation results show that the improved GM-PHD smoother can achieve a much faster processing speed and provide a more accurate target number estimation as compared with the GM-PHD smoother. All these indicate that this proposed smoother has a good application prospect.

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