Design of IIR filters with reduced group delay ripple via particle swarm optimization

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Abstract: This paper presents a design method for infinite impulse response (IIR) filters with an approximately linear phase characteristic. The design problem of IIR digital filters is generally expressed as the minimization problem of the complex magnitude error which includes both the magnitude and phase information. However, the group delay response of the filter obtained by solving such design problem may be distant from the desired group delay. In this paper, the filter design problem is formulated as a magnitude error minimization problem having a maximum group delay error constraint and it is optimized using craziness based particle swarm optimization technique. As a result, the proposed method can design the IIR filters that satisfy the prespecified allowable errors of the group delay response. The usefulness of the proposed method is verified through some examples.

Key–Words: IIR filter, Group delay error, Particle swarm optimization, .

1 Introduction

Digital filters are broadly classified into infinite impulse response (IIR) and finite impulse response (FIR). FIR filters find many applications in image processing, waveform transmission, etc. in which phase distortion becomes a problem because the FIR filters with perfect linear phase characteristics can easily be realized and are inherently stable [1, 2]. However, the group delay of the perfect linear phase FIR filters may become unacceptably large when high-order filters or narrow transition bands are required because the resulting delay at the output of the perfect linear phase FIR filters is half of the filter order. On the other hand, it is known that an IIR digital filter requires less computation as compared to FIR filter for the same amplitude response. Hence, the IIR filters are very important for the implementation of a signal processing system with high-speed and with high-precision [3].

The optimal design of the IIR filters in the complex domain with the Chebyshev norm is usually a solution of the following problem:

$$\min_{a,\,b} \max_{\omega \in \Omega} W(\omega)\left|H_d(\omega) - H(\omega)\right|, \tag{1}$$

where $H_d(\omega)$ is the desired frequency response, $H(\omega)$ is the frequency response of actual filter, a and b are the filter coefficients, Ω is the frequency bands of interest (e.g., passband and stopband), and $W(\omega)$ is a wighting function. By solving the above problem, the IIR filters with an approximately linear phase response can be obtained because it is a complex approximation problem which includes the amplitude and the phase informations. Many methods have been developped for the solving this problem [4]-[10]. However, the group delay response of the filter obtained by those methods tends to have relatively large error, especially in the vicinity of the band-edge. This causes distortion of the output signal. Allpass filters are frequently used to suppress the group delay error, but the use of the allpass filter is not necessarily a good policy because the filter coefficients are redundant. Therefore, it is desirable to realize the filter that has an equalized group delay without using the allpass filter. This motivates the investigation on designing the IIR filters with prespecified maximum group delay errors by directly approximating the group delay response.

By the way, it is well known that the error surface of IIR filters is usually nonlinear and multimodal. Therefore, there is a problem that conventional gradient-based design methods may easily get stuck in the local minima of error surface. To overcome this problem, in recent year, many design methods based on modern heuristic optimization algorithms have been researched for IIR filters [11]-[16]. However, most of the work done is concentrated on approximation of magnitude response only of the filters.

In this paper, we propose a new design method using craziness based particle swarm optimization (CRPSO) technique. In the proposed method, the filter design problem is formulated as a magnitude error minimization problem which has a maximum group delay error constraint, and is solved using CRPSO. As a result, the proposed method allows the direct approximation of the group delay response and can design the IIR filters which satisfy a prespecified group delay error. The usefulness of the proposed method is verified through some examples.

2 Proposed method

This section discusses the strategy to design the IIR filters with a prespecified group delay error using craziness based Particle Swarm Optimization(CRPSO).

2.1 Canonical PSO and Craziness based PSO

The PSO algorithm, which was first introduced in 1995 by Kennedy and Eberhart [17], is one of evolutionary computation techniques and can be used to solve real valued and nonlinear continuous optimization problems. Particle Swarm has two primary operators: Velocity update and Position update. During each generation each particle is accelerated toward the particles previous best position and the global best position. The new velocity value is then used to calculate the next position of the particle in the search space.

Let $\boldsymbol{X}_i$ and $\boldsymbol{V}_i$ denote the positions and the corresponding flight speed (velocity) of the particle i in a continuous search space, respectively. Using the personal best position $Pbest_i^k$ of the ith particle at the kth iterations and the best position $Gbest^k$ of the group at the kth iterations, the particle velocity update equations in the simplest form that govern the PSO are given by

$$\begin{aligned}\boldsymbol{V}_i^{k+1} = & w\cdot\boldsymbol{V}_i^k + c_1\cdot rand_1\cdot(\boldsymbol{Pbest}_i^k - \boldsymbol{X}_i^k)\\ & +c_2\cdot rand_2\cdot(\boldsymbol{Gbest}^k - \boldsymbol{X}_i^k)\end{aligned} \quad (2)$$

$$\boldsymbol{X}_i^{k+1} = \boldsymbol{X}_i^k + \boldsymbol{V}_i^{k+1} \quad (3)$$

where w is the inertia weighting function, c_1 and c_2 are the acceleration constant, and $rand_1$ and $rand_2$ are the random parameters taken from interval $[0,1]$. At each iteration, the fitness of each particle is evaluated according to the preselected fitness function.

In [18], the global search ability of above discussed canonical PSO is improved with the help of the following modifications. This modified PSO is termed as craziness based particle swarm optimization(CRPSO). In CRPSO, the velocity can be expressed as follows:

$$\begin{aligned}\boldsymbol{V}_i^{k+1} = & r_2\cdot\mathrm{sign}(r_3)\cdot\boldsymbol{V}_i^k + (1-r_2)\cdot c_1\cdot r_1\cdot(\boldsymbol{Pbest}_i^k - \boldsymbol{X}_i^k)\\ & +(1-r_2)\cdot c_2\cdot(1-r_1)\cdot(\boldsymbol{Gbest}^k - \boldsymbol{X}_i^k)\\ & +P(r_4)\cdot\mathrm{sign}(r_4)\cdot v^{\mathrm{craziness}}\end{aligned} \quad (4)$$

where r_1, r_2, r_3, and r_4 are random parameters uniformly taken from interval $[0,1]$, and $\mathrm{sign}(r_3)$, $P(r_4)$, and $\mathrm{sign}(r_4)$ are a function defined as follows:

$$\mathrm{sign}(r_3) = \begin{cases} -1, & r_3 \le 0.05 \\ 1, & r_3 > 0.05 \end{cases} \quad (5)$$

$$P(r_4) = \begin{cases} 1, & r_4 \le P_{cr} \\ 0, & r_4 > P_{cr} \end{cases} \quad (6)$$

$$\mathrm{sign}(r_4) = \begin{cases} -1, & r_4 \le 0.5 \\ 0, & r_4 > 0.5 \end{cases} \quad (7)$$

Moreover, $v^{\mathrm{craziness}}$ is a random parameter which is uniformly chosen from the interval $[v^{\min}, v^{\max}]$, and P_{cr} is a predefined probability of craziness. In eq.(2), the two random parameters $rand_1$ and $rand_2$ are independent. Therefore, if both are large, both the personal and social experiences are over used and the particle is driven too far away from the local optimum. If both are small then both the social and personal experiences are not used full and convergence speed is extremely slow. On the other hand, in eq.(4), instead of taking independent parameters, one single random parameter r_1 is used so that when r_1 is large, $(1-r_1)$ is small and vice versa. To control the balance between global and local searches, another random parameter r_2 is introduced. A craziness operator $v^{\mathrm{craziness}}$ is introduced to ensure that the particle would have a predefined craziness probability to maintain the diversity of the particles.

A more detailed discussion of CRPSO will appear in [18].

2.2 Magnitude response approximation with specified group delay error

The frequency response $H(\omega)$ of an IIR digital filter is defined as

$$H(\omega) = \frac{A(\omega)}{B(\omega)} = \frac{\sum_{n=0}^{N} a_n e^{-jn\omega}}{\sum_{m=0}^{M} b_m e^{-jm\omega}}, \quad (8)$$

where N and M are the orders of the numerator and denominator, respectively. a_n and b_m are the filter coefficients, and $b_0 = 1$ in general. The group delay response of the frequency response $H(\omega)$ in eq.(8) is written by

$$\tau(\omega) = \mathrm{Re}\left\{ \frac{\sum_{n=0}^{N} n \cdot a_n e^{-jn\omega}}{\sum_{n=0}^{N} a_n e^{-jn\omega}} + \frac{\sum_{m=0}^{M} m \cdot b_m e^{-jm\omega}}{\sum_{m=0}^{M} b_m e^{-jm\omega}} \right\}. \quad (9)$$

Here, let $H_d(\omega)$ be the desired frequency response which have a desired amplitude response $D(\omega)$ and a desired group delay response $\tau_d(\omega)$, i.e.,

$$H_d(\omega) = D(\omega) e^{-j\tau_d(\omega)\omega}. \quad (10)$$

Then, the minimization problem of the magnitude error for the filters with a maximum group delay error specification is expressed as

$$\begin{aligned} &\min_{a_n,\, b_m} \max_{\omega\in\Omega_a} W_a(\omega)\,|D(\omega) - |H(\omega)|| \\ &\text{subject to } W_g(\omega)\,|\tau_d(\omega) - \tau(\omega)| \le \mu_g, \text{ for } \omega \in \Omega_g \\ &P_{max} < r_m, \end{aligned} \quad (11)$$

where P_{max} is the maximum radius of the obtained filter's pole, Ω_a is the frequency bands of interest of the magnitude response, Ω_g is that of interest of the group delay response. $W_a(\omega)$ and $W_g(\omega)$ are weighting parameters for the magnitude and group delay, respectively. Moreover, μ_g is an allowable error of the group delay response. To solve the minimizaion problem using CRPSO, the fitness function is defined as follows:

$$J(\boldsymbol{X}) = J_a(\boldsymbol{X}) + \phi_g(\boldsymbol{X}) + \phi_p(\boldsymbol{X}) \quad (12)$$

$$J_a(\boldsymbol{X}) = \max_{\omega\in\Omega_g} W_a(\omega)\,|H_d(\omega) - |H(\omega)||, \quad (13)$$

where $\boldsymbol{X} = [a_0, a_1, \cdots, a_N, b_1, b_2, \cdots, b_M]$, $\phi_g(\boldsymbol{X})$ is the penalty function for the maximum group delay error specification, $\phi_p(\boldsymbol{X})$ is the penalty function to gurantee the stability of the obtained filter. In this paper, we use the following penarty functions:

$$\phi_g(\boldsymbol{X}) = \begin{cases} \dfrac{10 \times (E_g - \mu_g)}{\mu_g} & , E_g > \mu_g \\ 0 & , E_g \le \mu_g \end{cases} \quad (14)$$

$$\phi_p(\boldsymbol{X}) = \begin{cases} \dfrac{10 \times (P_{max} - r_m)}{r_m} & , P_{max} \ge r_m \\ 0 & , P_{max} < r_m \end{cases} \quad (15)$$

In eq.(14), E_g is the maximum error to a desired group delay response and is defined as

$$E_g = \max_{\omega\in\Omega_g} W_g(\omega)\,|\tau_d(\omega) - \tau(\omega)| - \mu_g. \quad (16)$$

2.3 Design Procedure

The design procedure of the proposed method is summarized as follows.

Step 0: Generate the initial particle and reset k to 0.

Step 1: If $k = iter\_max$, stop; otherwise, go to step 2.

Step 2: Evaluate the fitness $J(\boldsymbol{X})$ using eq.(12).

Step 3: Search the personal best position $\boldsymbol{p}_i^k$ and group best position $\boldsymbol{p}_g^k$.

Step 4: Update the velocity $\boldsymbol{v}_i^{k+1}$.

Step 5: Calculate the positions $\boldsymbol{X}_i^{k+1}$, and go back to step 1 with $k = k + 1$.

3 Design Examples

In this section, numerical examples are presented to illustrate the effectiveness of the proposed method. In all the following examples, the CRPSO algoritm parameters are set as: particle size = 50, $c_1 = c_2 = 2.0$, $v^{\text{craziness}} = [0.01, 1.0]$, $P_{cr} = 0.3$, $iter\_max = 10000$. The obtained filter coefficients are given in figure 3.

3.1 Example 1

We first consider the filter with following specification.

- $D(\omega) = \begin{cases} 1, & 0 \le |\omega| \le 0.2\pi \\ 0, & 0.4\pi \le |\omega| \le \pi \end{cases}$
- $\tau_d(\omega) = 5, \quad 0 \le |\omega| \le 0.2\pi$
- $W_a(\omega) = \begin{cases} 1, & 0 \le |\omega| \le 0.2\pi \\ 1, & 0.4\pi \le |\omega| \le \pi \end{cases}$
- $W_g(\omega) = \{\ 1, \quad 0 \le |\omega| \le 0.2\pi$
- $N = 4$, $M = 4$
- $r_m = 1.0$

For comparison, the IIR filter with the same design specifications was designed using [6]. Note that the method of [6] cannot specify the maximum group delay allowable error because this method is based on complex Chebyshev approximation.

The main advantage of the proposed method is that it can directly specify the maximum group delay allowable error μ_g. The resulting filters for many different μ_g are summarized in table 1. Moreover, the frequency responses of the obtained filter are depicted in figures 1(a)-(c). In figure 1, the black line, the red line, and the blue line are the frequency response in the case of $\mu_g = 0.75$, $\mu_g = 0.50$, and $\mu_g = 0.25$ of the proposed method, respectively. From figure 1 and table 1, it is seen that the group delay responses of the filter obtained using the proposed method meet the prespecified group delay error constrain, and the magnitude and the group delay errors have the relationship of trade-off. Compared with [6], the proposed filters with $\mu_g = 0.75$ and $\mu_g = 0.50$ are much better both amplitude and group delay responses.

3.2 Example 2

Next, we consider the filter with following specification.

- $D(\omega) = \begin{cases} 1, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.6\pi \le |\omega| \le \pi \end{cases}$
- $\tau_d(\omega) = 9, \quad 0 \le |\omega| \le 0.5\pi$
- $W_a(\omega) = \begin{cases} 1, & 0 \le |\omega| \le 0.5\pi \\ 1, & 0.6\pi \le |\omega| \le \pi \end{cases}$
- $W_g(\omega) = \{ 1, \quad 0 \le |\omega| \le 0.5\pi$
- $N = 12$, $M = 6$
- $r_m = 0.944$

For comparison, the IIR filter with the same design specifications was designed using [8]. Note that the method of [8] cannot specify the maximum group delay allowable error.

Figures 2(a)-(c) show the amplitude and group delay responses of the obtained filters. Table II is the numerical performance comparison between the proposed method and the conventional method[8]. From table II, it is confirmed that the proposed method can realize the filters with much smaller amplitude and group delay ripples than that of the conventional method.

4 Conclusion

In this paper, a new method based on craziness based particle swarm optimization technique was proposed to design stable IIR filters with prespecified group delay errors. In the proposed method, the filter design problem was formulated as a magnitude error minimization problem having a maximum group delay error constraint and it was optimized using craziness based particle swarm optimization technique. The proposed method is possible to directly approximate of the group delay response and can restrict the group delay response within the preselected allowable error. Results showed that the proposed method can design the filters that can not be obtained using the conventional methods based on complex Chebyshev approximation.

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Table 1: Comparison with the conventional method in example 1.

	Proposed Filter $\mu_g = 0.75$	Proposed Filter $\mu_g = 0.50$	Proposed Filter $\mu_g = 0.25$	Zhang [6]
Maximum passband error in amplitude	1.84×10^{-2}	2.08×10^{-2}	2.43×10^{-2}	2.33×10^{-2}
Maximum stopband error in amplitude	1.84×10^{-2}	2.08×10^{-2}	2.43×10^{-2}	2.34×10^{-2}
Maximum group delay error in passband	0.750	0.5000	0.250	0.970
Maximum pole radius	0.897	0.899	0.894	0.877

Table 2: Performance comparison in example 2.

	Proposed Filter $\mu_g = 0.75$	Proposed Filter $\mu_g = 0.50$	Proposed Filter $\mu_g = 0.25$	Lu [8]
Maximum passband error in amplitude	1.56×10^{-2}	1.76×10^{-2}	2.45×10^{-2}	1.71×10^{-2}
Maximum stopband error in amplitude	1.56×10^{-2}	1.76×10^{-2}	2.45×10^{-2}	1.82×10^{-2}
Maximum group delay error in passband	0.745	0.499	0.250	0.832
Maximum pole radius	0.944	0.943	0.944	0.944

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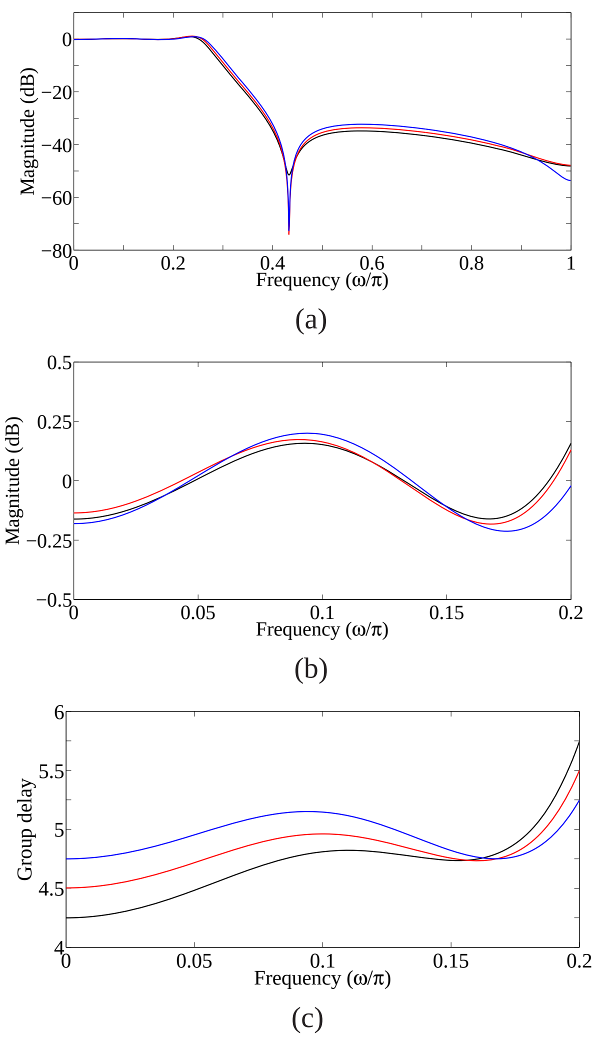

Figure 1: Frequency response of the proposed IIR filters with the order $(N, M) = (4, 4)$. (a) Overall amplitude response (b) Amplitude response in the passband (c) Group delay response in the passband

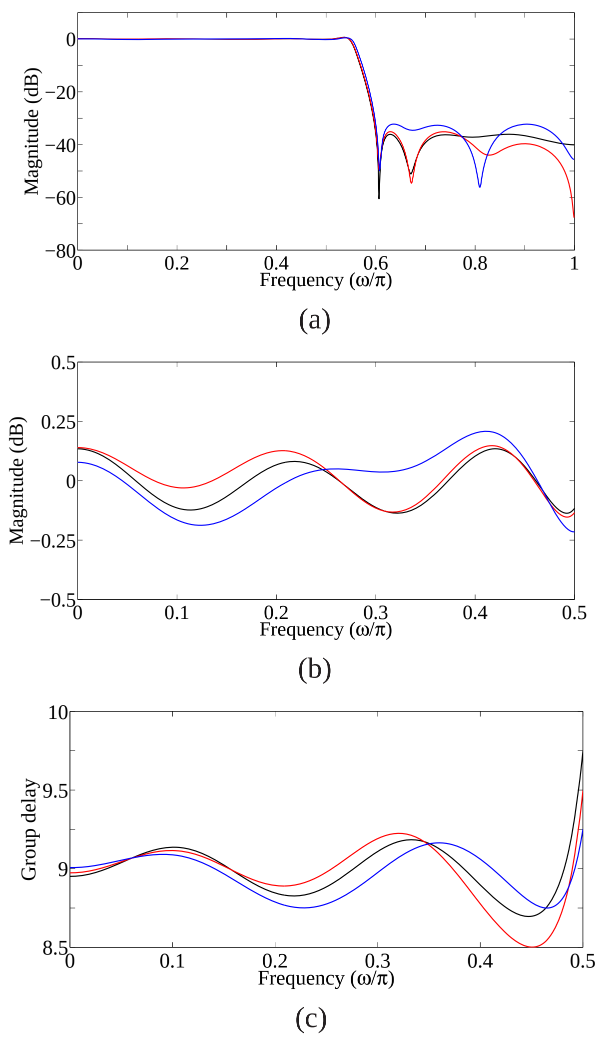

Figure 2: Frequency response of the proposed IIR filters with the order $(N, M) = (6, 12)$ in Example 2. (a) Overall amplitude response (b) Amplitude response in the passband (c) Group delay response in the passband

n	a_n	b_n
0	-0.01573880	1.0
1	0.04318550	-2.57303902
2	0.00803824	2.93936897
3	0.01941263	-1.65907078
4	0.03668810	0.38604360

(a) $\mu_g = 0.75$

n	a_n	b_n
0	-0.02051306	1.0
1	0.04487760	-2.56509613
2	0.00686253	2.93250454
3	0.01850206	-1.66192041
4	0.04244655	0.38813655

(b) $\mu_g = 0.50$

n	a_n	b_n
0	-0.02524043	1.0
1	0.04372168	-2.54312597
2	0.01194024	2.89926703
3	0.01184250	-1.64869803
4	0.05117428	0.38795595

(c) $\mu_g = 0.25$

n	a_n	b_n
0	0.00932661	1.0
1	0.01565213	-0.05543797
2	-0.00265935	1.03796775
3	-0.00905367	-0.29260630
4	0.01331657	0.19736509
5	0.01160070	-0.05185855
6	-0.04527729	-0.00640482
7	-0.01719688	
8	0.20919865	
9	0.51389781	
10	0.60893069	
11	0.40088323	
12	0.14896291	

(d) $\mu_g = 0.75$

n	a_n	b_n
0	0.00858001	1.0
1	0.01617954	-0.06190388
2	-0.00898306	1.00264403
3	-0.01185114	-0.29063712
4	0.01348098	0.17633254
5	0.01461809	-0.03548558
6	-0.04301800	-0.00669035
7	-0.01920925	
8	0.20781092	
9	0.51643860	
10	0.59538867	
11	0.39091191	
12	0.13285242	

(e) $\mu_g = 0.50$

n	a_n	b_n
0	0.00074443	1.0
1	0.02188631	-0.05198591
2	-0.00767932	1.02937640
3	-0.00419166	-0.28896971
4	0.01313683	0.19029972
5	0.00664542	-0.04385430
6	-0.04928880	-0.02364516
7	-0.02248849	
8	0.21978006	
9	0.52030301	
10	0.60605035	
11	0.38486148	
12	0.13773941	

(f) $\mu_g = 0.25$

Figure 3: Filter coefficients: (a)-(c) are that in example 1 and (d)-(f) are that in example 2