Reexamining a Specific Vendor-buyer System with Rework and an Improving Delivery Plan Using an Alternative Approach

YUAN-SHYI PETER CHIU[a], FENG-TSUNG CHENG[b], KUANG-KU CHEN[c], HUEI-HSIN CHANG[d,*]

[a]Department of Industrial Engineering and Management, [d]Department of Finance,
Chaoyang University of Technology, Taichung 413, TAIWAN
[b]Department of Industrial Engineering and Systems Management,
Feng Chia University, Taichung 407, TAIWAN
[c]Department of Accounting
National Changhua University of Education, Changhua 500, TAIWAN
[*]E-mail: chs@cyut.edu.tw (HUEI-HSIN CHANG) http://www.cyut.edu.tw/

Abstract: - This paper uses an alternative approach to reexamine a prior study on joint determination of manufacturing lot-size and shipment policy in a vendor-buyer system with rework process and an improving delivery plan. The proposed method is a straightforward approach in terms of algebraic derivations. It is different from the conventional method that needs to apply the first-order and second-order differentiations to the system cost function for proof of convexity before derivation of the optimal production-shipment policy. The research result obtained in this paper is confirmed to be identical to what was derived by the use of the conventional method. The proposed algebraic approach may assist practitioners who may not have sufficient knowledge of differential calculus in understanding the solution procedures of such a real world integrated production-shipment problem in supply chain environments.

Key-Words: - Manufacturing lot-size, Multiple deliveries, Algebraic approach, Production-shipment policy, Optimization, Production planning

1 Introduction

In most inventory replenishment systems, ‘when to order?’ and ‘how many to order?’ are the two fundamental questions to be answered most often. The objective of these issues are obvious, that is to minimize total related cost [1-2]. In manufacturing firms when products are made in the plant, the questions become ‘when to start a production run?’ and ‘what will be the production lot-size?’ [3-4].

Chiu et al. [5] studied joint determination of replenishment lot-size and shipment policy for a vendor-buyer integrated system with rework and an improving delivery plan. Their work is as extension of conventional economic production quantity (EPQ) model [6] with additional considerations on product quality assurance and a special cost-saving delivery policy. The classic EPQ model assumes that all items produced are of perfect quality. However, in real world systems due to process deterioration and/or many other factors, generation of defective items is inevitable. Many research articles have been carried out to address the imperfect quality issue in production systems [7-14]. Shih [8] extended two inventory models to the case where the proportion of defective units in the accepted lot is a random variable with known probability distributions. Optimal solutions to the modified system were developed and comparisons with the traditional models were also presented via numerical examples. de Kok [9] considered a lost-sales production/ inventory control model with two adjustable production rates to meet demand. He obtained the practical approximations for optimal switch-over levels to such a model under the service level constraints. Mak [10] developed a mathematical model for an inventory system in which the number of units of acceptable quality in a replenishment lot is uncertain and the demand is partially captive. It was assumed that the fraction of the demand during the stock-out period which can be backordered is a random variable whose probability distribution is known. The optimal replenishment policy is synthesized for such a system. A numerical example was used to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Makis [11] studied the optimal lot sizing and inspection policy for an EMQ model with imperfect inspections. He assumed that the process can be monitored through inspections, and both the lot size and the inspection schedule are subject to control. The in-control periods are assumed to be generally distributed and the inspections are imperfect (that is the true state of the process is not necessarily revealed through an inspection). By using the Lagrange's method and by solving a nonlinear equation, a two-dimensional search procedure was proposed and employed to find the optimal lot sizing and inspection policy. Chiu et al. [14] employed mathematical modeling along with a searching algorithm to solve the manufacturing run time problem with defective rate and random machine breakdown. They derived the optimal run time that minimized the long-run average cost for their proposed model.

Nonconforming products sometimes can be reworked and repaired to reduce total production costs [15-21]. Examples for such situations can be found in plastic injection molding, or in printed circuit board (PCB) assembly, sometimes employs rework as an acceptable process to increase level of quality. Yum and McDowell [15] considered the allocation of inspection effort problem for serial system as a 0-1 mixed integer linear programming (MILP) problem. Their model permitted any combination of scrap, rework, or repair at each station and allowed the problem to be solved using standard MILP software packages. Yu and Bricker [16] presented an informative application of Markov Chain Analysis to a multistage manufacturing problem. They also pointed out an error in the literature which had remained undetected for many years. Teunter and Flapper [18] considered a single-stage single-product production system. Produced units was assumed to be non-defective, reworkable defective, or non-reworkable defective. The system switches between production and rework. After producing a fixed number (N) of units, all reworkable defective units are reworked. They also assumed that the rework time and the rework cost increase linearly with the time that a unit is held in stock. For a given N, they derived an explicit expression for the average profit, using this expression the optimal value for N can be determined numerically. Li [20] presented an overlapping decomposition method to approximate the throughput of production systems with rework loops. He decomposed the system into overlapped serial production lines, with the overlapping machines modified to accommodate the interactions with machines and buffers in other lines. The convergence of the iterative procedure and the uniqueness of the solution were proved analytically. The accuracy of the estimate was demonstrated numerically and illustrated by a case study at an automotive assembly plant. Chiu et al. [21] examined a finite production rate model with scrap, rework and stochastic machine breakdown. Stochastic breakdown rate and random defective rate along with the reworking of nonconforming items were assumed in their study. The objective was to derive the optimal production run time that minimize the long run average production cost.

In vendor-buyer supplier chains environments, multiple or periodic deliveries of finished products are commonly adapted in lieu of continuous issuing policy as assumed by the classic EPQ. Schwarz [22] considered a problem of one-warehouse N-retailer inventory system. The objective was to determine the optimal stocking policy that minimizes system cost. He derived some necessary properties for the optimal policy as well as the optimal solutions. Heuristic solutions were also provided for the general problem and tested against analytical lower bounds. Many studies have since been carried out to address various aspects of supply chains optimization [23-42]. Selected articles are surveyed as follows. Banerjee and Banerjee [24] developed an analytical model for a coordinated, orderless inventory system for the single product, single vendor, multiple purchasers case. Such a system was made practical in electronic data interchange at the time, for the exchange of information between trading partners. On the basis of the potential benefits of this technology, they proposed a common cycle replenishment approach, where the supplier alone makes all replenishment decisions, without ordering on the part of the customers. Their model and concepts were demonstrated by a simple numerical example and concluded that EDI-based inventory control can be attractive from economic, as well as other standpoints. Sarker and Parija [25] considered a manufacturing system which procures raw materials from suppliers and processes them to convert to finished products. They proposed a model that was used to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products, at fixed intervals, to the buyers. Hall [26] examined how attributes of the distribution system affect inventory accounting and EOQ/EPQ decisions. The paper developed a range of "characteristic inventory curves" to represent situations encountered in integrated production/ distribution systems. The paper then showed how system attributes define the inventory curve, and the resulting EOQ/EPQ equation. He concluded: (1) accounting for inventory at both the origin and destination can yield significantly different EOQ/EPQ results, but relatively modest regret; and (2) failure to account for consolidation effects among multiple products sent to a common destination can lead to substantial errors. Sarmah et al. [29] considered coordination between two different business entities is an important way to gain competitive advantage as it lowers supply chain cost, so they reviewed literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. An effort was also made to identify critical issues and scope of future research. Chiu et al. [32] incorporated a multi-delivery policy and quality assurance into an imperfect economic production quantity (EPQ) model with scrap and rework. They assumed the reworking of repairable defective items in each production run and the finished items can only be delivered to customers if the whole lot is quality assured after rework. The expected integrated cost function per unit time was derived. A closed-form optimal batch size solution to the problem was obtained. Hoque [35] considered models of delivering a single product to multiple buyers when the set-up and inventory costs to the vendor are included. His models assume a close relationship between a manufacturer and buyers for a costless way of benefit sharing. Three models were developed, two of which transfer with equal batches and the third with unequal batches of the product. Optimal solution techniques are presented, a sensitivity analysis of the techniques is carried out, and several numerical problems are solved to support the analytical findings. A comparative study of the results shows that the supply by unequal batches performs better. This study also highlights the limitation of methods used in obtaining the least minimal total cost in the single-vendor single-buyer scenario, and the benefit of an integrated inventory is also discussed.

Chiu et al. [5] studies joint determination of manufacturing lot-size and shipment policy in a vendor-buyer system with rework process and an improving delivery plan. They used the differential calculus along with Hessian matrix equations to derive the optimal production batch size and number of deliveries for such a specific vendor-buyer system with rework. This paper employs an algebraic approach [43-46] to reexamine their model. A straightforward algebraic derivation is presented here with the intention of helping practitioners (who may not have sufficient knowledge of differential calculus) on understanding the solution procedures of such a real world integrated production-shipment problem. As future research we would like to use agent based modeling to model the supply chain domain as this techniques provided significant results in other complex domains such as financial markets [47].

2 Problem Description & Modelling

As stated in previous section, this study uses an alternative approach to reexamine model in Chiu et al. [5]. To ease the readability, problem is described below using the exact notation as in [5]. Consider a real life manufacturing system where process may randomly produce a portion x of defective items at a rate d. Under regular operating schedule, the constant production rate P is larger than the sum of demand rate λ and production rate of defective items d, where $(P\text{-}d\text{-}\lambda)>0$. All defective items are considered to be repairable and they are reworked and repaired at a rate P_1 within the same cycle when regular production ends. All end items are delivered to the buyer by a specific cost saving (n+1) shipment plan. Under such a delivery policy, the first installment of finished products is delivered to customer for satisfying demand during uptime t_1 and rework time t_2 (see Fig. 1). Then, after the rework process when the remaining of the production lot is quality assured, fixed quantity n installments of the rest of finished items are delivered to customer at a fixed interval of time during the production downtime t_3.

Figure 1 illustrates vendor's on-hand inventory level of perfect quality items in the proposed n+1 delivery model [5]. Figure 2 depicts the on-hand inventory level of defective items in the proposed n+1 delivery model. Figure 3 depicts buyer's stock level in the proposed model [5].

The cost parameters related to the proposed model include unit production cost C, vendor's unit holding cost h, setup cost K per production run, buyer's unit holding cost h_2, unit rework cost C_R, holding cost h_1 for each reworked item, fixed delivery cost K_1, delivery cost C_T per item shipped. Additional variables are

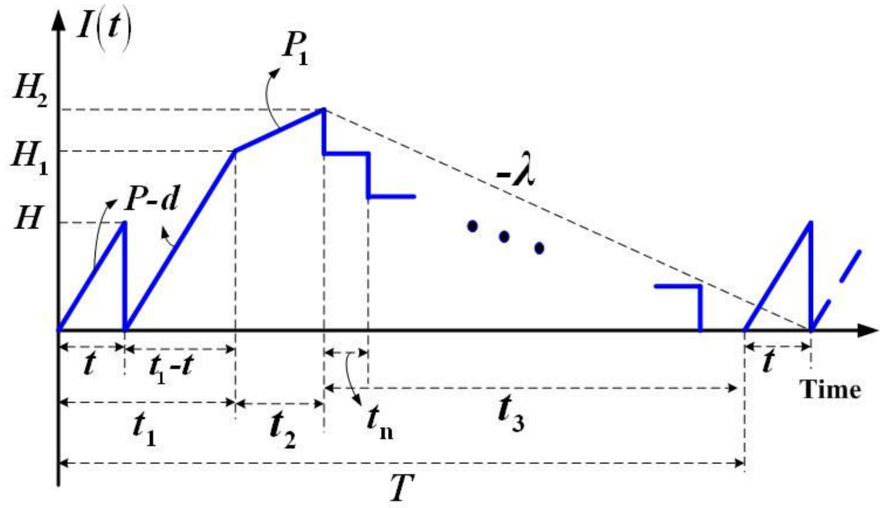

Fig. 1 The vendor's on-hand inventory of perfect quality items for the proposed model with rework and (n+1) delivery policy [5]

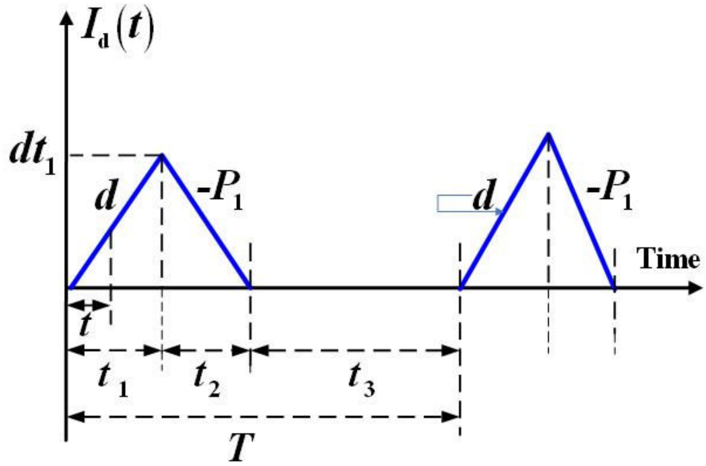

Fig. 2 The on-hand inventory of defective items in the proposed model with rework and (n+1) delivery policy

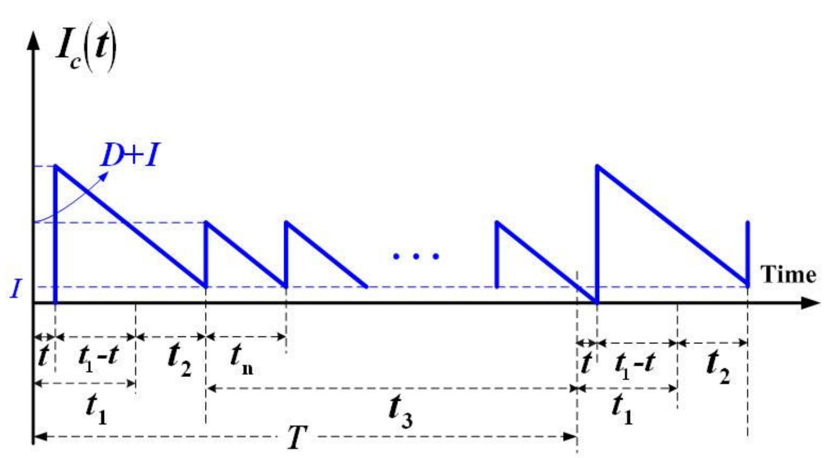

Fig. 3 The stock level at buyer side for the proposed model with rework and (n+1) delivery policy [5]

t = the production time needed for producing enough perfect items for satisfying customer's demand during t_1 and t_2,
t_1 = the production uptime,
t_2 = rework time,
t_3 = production downtime, time to deliver the remaining quality assured finished products,
Q = production lot size per cycle,
t_n = a fixed interval of time between each installment of products delivered during t_3,
T = cycle length,
H = the level of on-hand inventory for satisfying product demand during vendor's uptime t_1 and rework time t_2,
H_1 = maximum level of on-hand inventory in units when regular production ends,
H_2 = the maximum level of on-hand inventory in units when rework process finishes,
n = number of fixed quantity installments of the remaining finished items to be delivered to customer during t_3,
$I(t)$ = the level of on-hand inventory of perfect quality items at time t,
$I_d(t)$ = on-hand inventory of defective items at producer's side at time t,
$I_c(t)$ = on-hand inventory at buyer's side at time t,
$TC(Q,n+1)$ = total production-inventory-delivery costs per cycle for the proposed model,
E[$TCU(Q,n+1)$] = the long-run average costs per unit time for the proposed model.
I = demand during production time t, i.e. $I=\lambda t$.
D = demand during production uptime t_1, i.e. $D=\lambda t_1$.

From Figures 1 and 2, the following basic equations can be obtained.

$$T=\frac{Q}{\lambda} \tag{1}$$

$$t_1=\frac{Q}{P}=\frac{H_1+H_2}{P-d} \tag{2}$$

$$t_2=\frac{xQ}{P_1} \tag{3}$$

$$t_3=nt_n=T-\left(t_1+t_2\right) \tag{4}$$

$$t=\frac{\lambda(t_1+t_2)}{P-d} \tag{5}$$

$$dt_1=xQ \tag{6}$$

$$H=\lambda\left(t_1+t_2\right)=\lambda\left(\frac{Q}{P}+\frac{xQ}{P_1}\right) \tag{7}$$

$$H_1=Q\left(1-x\right)-\lambda\left(t_1+t_2\right) \tag{8}$$

$$H_2=H_1+P_1t_2 \tag{9}$$

Thus, total production-inventory-delivery cost per cycle $TC(Q,n+1)$ of the proposed model consists of the variable manufacturing cost, the setup cost, variable rework cost, the quality assurance costs include variable repairing costs and holding costs for reworked items, (n+1) fixed and variable shipping cost, inventory holding costs for vendor for all end items produced in t_1, t_2, and t_3, and buyer's holding cost.

$$\begin{aligned} TC\left(Q,n+1\right)&=CQ+K+C_R\left[xQ\right]+h_1\cdot\frac{dt_1}{2}\cdot\left(t_2\right)\\ &+\left(n+1\right)K_1+C_TQ\\ &+h\left[\begin{array}{l}\frac{H}{2}\left(t\right)+\frac{H_1}{2}\left(t_1-t\right)+\frac{dt_1}{2}\left(t_1\right)\\ +\frac{H_1+H_2}{2}\left(t_2\right)+\left(\frac{n-1}{2n}\right)H_2t_3\end{array}\right]\\ &+h_2\left[\frac{H(t_1+t_2)}{2}+n\left(\frac{D+2I}{2}\right)t_n\right]\end{aligned} \tag{10}$$

Taking into the randomness of defective rate x, one can use the expected values of x in cost analysis and obtains E[$TCU(Q,n+1)$] as follows (the same as Eq. (7) in [5]).

$$\begin{aligned} E\left[TCU\left(Q,n+1\right)\right]&=\lambda\left\{C+\frac{\left[(n+1)K_1+K\right]}{Q}+C_RE\left(x\right)+C_T\right\}\\ &+\frac{hQ}{2}\left\{\frac{2\lambda^3E_0}{P^3}+\frac{4\lambda^3E_1}{P^2P_1}+\frac{2\lambda^3E_2}{PP_1^2}-\frac{\lambda E_3}{P_1}+1-\frac{\lambda}{P}\right\}\\ &+\frac{(h_2-h)Q}{2n}\left[1-\frac{2\lambda}{P}-\frac{2\lambda E\left(x\right)}{P_1}+\frac{2\lambda^2E\left(x\right)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2}{P_1^2}E_3\right]\\ &+h_2Q\left[\frac{\lambda^2}{P^2}E_0-\frac{\lambda^3}{P^3}E_0-\frac{2\lambda^3}{P^2P_1}\cdot E_1+\frac{\lambda^3}{PP_1^2}\cdot E_2+\frac{\lambda^2}{PP_1}E_1\right]\\ &+\frac{(h_2-h)Q}{2}\left[\frac{\lambda^2}{P^2}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P_1^2}\cdot E_3\right]+\frac{h_1Q\lambda}{2P_1}E_3\end{aligned} \tag{11}$$

where

$$\begin{aligned} &E_0=E\left(\frac{1}{1-x}\right);\ E_1=E\left(\frac{x}{1-x}\right);\\ &E_2=E\left(\frac{x^2}{1-x}\right);E_3=\left[E\left(x\right)\right]^2\end{aligned} \tag{12}$$

3 The Algebraic Approach

In this section, an algebraic approach is presented to derive the optimal replenishment lot size and the optimal number of deliveries. It is noted that no differential calculus is involved in the proposed solution process. It is also noted that Eq. (11) has two decision variables Q and n, and they are in terms of coefficients associated with nQ^{-1}, Q^{-1}, Q, and Qn^{-1}.

First let a_1, a_2, a_3, a_4 and a_5 denote the following:

$$a_1 = \lambda\left[C + C_R E(x) + C_T\right] \tag{13}$$

$$a_2 = \lambda K_1 \tag{14}$$

$$a_3 = \lambda(K_1 + K) \tag{15}$$

$$\begin{aligned} a_4 = {} & \frac{h}{2}\left[\frac{2\lambda^3 E_0}{P^3} + \frac{4\lambda^3 E_1}{P^2 P_1} + \frac{2\lambda^3 E_2}{P P_1^2} - \frac{\lambda E_3}{P_1} + 1 - \frac{\lambda}{P}\right] \\ & + h_2\left[\frac{\lambda^2}{P^2}E_0 - \frac{\lambda^3}{P^3}E_0 - \frac{2\lambda^3}{P^2 P_1}E_1 + \frac{\lambda^3}{P P_1^2}E_2 + \frac{\lambda^2}{P P_1}E_1\right] \\ & + \frac{(h_2 - h)}{2}\left[\frac{\lambda^2}{P^2} + \frac{2\lambda^2 E(x)}{P P_1} + \frac{\lambda^2}{P_1^2}E_3\right] + \frac{h_1\lambda}{2P_1}E_3 \end{aligned} \tag{16}$$

$$a_5 = \frac{(h_2 - h)}{2}\begin{bmatrix} 1 - \dfrac{2\lambda}{P} - \dfrac{2\lambda E(x)}{P_1} + \dfrac{2\lambda^2 E(x)}{P P_1} \\ + \dfrac{\lambda^2}{P^2} + \dfrac{\lambda^2}{P_1^2}E_3 \end{bmatrix} \tag{17}$$

Thus, Eq. (11) becomes

$$E\left[TCU(Q, n+1)\right] = a_1 + a_2\left(nQ^{-1}\right) + a_3\left(Q^{-1}\right) + a_4(Q) + a_5\left(Qn^{-1}\right) \tag{18}$$

Further rearrangement, Eq. (18) becomes

$$\begin{aligned} E\left[TCU(Q, n+1)\right] = {} & a_1 + Q^{-1}\left[a_3 + a_4 Q^2\right] \\ & + Qn^{-1}\left[a_2\left(Q^{-1}n\right)^2 + a_5\right] \end{aligned} \tag{19}$$

$$\begin{aligned} E\left[TCU(Q, n+1)\right] = {} & a_1 \\ & + Q^{-1}\begin{bmatrix} \left(\sqrt{a_3}\right)^2 + \left(Q\sqrt{a_4}\right)^2 - 2\left(\sqrt{a_3}\right)\left(Q\sqrt{a_4}\right) \\ +2\left(\sqrt{a_3}\right)\left(Q\sqrt{a_4}\right) \end{bmatrix} \\ & + Qn^{-1}\begin{bmatrix} \left[\sqrt{a_2}\left(Q^{-1}n\right)\right]^2 + \left(\sqrt{a_5}\right)^2 - 2\left[\sqrt{a_2}\left(Q^{-1}n\right)\right]\left(\sqrt{a_5}\right) \\ +2\left[\sqrt{a_2}\left(Q^{-1}n\right)\right]\left(\sqrt{a_5}\right) \end{bmatrix} \end{aligned} \tag{20}$$

Therefore, one obtains

$$\begin{aligned} E\left[TCU(Q, n+1)\right] = {} & a_1 + Q^{-1}\left[\left(\sqrt{a_3}\right) - \left(Q\sqrt{a_4}\right)\right]^2 \\ & + Qn^{-1}\left[\sqrt{a_5} - \sqrt{a_2}\left(Q^{-1}n\right)\right]^2 + 2\sqrt{a_3 a_4} + 2\sqrt{a_2 a_5} \end{aligned} \tag{21}$$

Thus, E[$TCU(Q,n$+1)] is minimized, if the second and the third square terms in Eq. (21) equal zeros. That is

$$\left(\sqrt{a_3}\right) - \left(Q\sqrt{a_4}\right) = 0 \tag{22}$$

and

$$\left(\sqrt{a_5}\right) - \sqrt{a_2}\left(Q^{-1}n\right) = 0 \tag{23}$$

or

$$Q^* = \sqrt{\frac{a_3}{a_4}} \tag{24}$$

and

$$n^* = \sqrt{\frac{a_5}{a_2}} \cdot Q^* \tag{25}$$

or

$$n^* = \sqrt{\frac{a_5}{a_2}} \cdot \sqrt{\frac{a_3}{a_4}} \tag{26}$$

4 Results and Discussion

Substituting Eqs. (13) to (17) in Eq. (25) and with further derivations, one has

$$n^* = \sqrt{\frac{(K_1 + K)(h_2 - h)\begin{bmatrix} 1 - \dfrac{2\lambda}{P} - \dfrac{2\lambda E(x)}{P_1} \\ + \dfrac{2\lambda^2 E(x)}{P P_1} + \dfrac{\lambda^2}{P^2} + \dfrac{\lambda^2 E_3}{P_1^2} \end{bmatrix}}{K_1\left\{\begin{matrix} h\left[-\dfrac{\lambda E_3}{P_1} + 1 - \dfrac{\lambda}{P}\right] + h_2\left[\dfrac{2\lambda^2 E_0}{P^2} + \dfrac{2\lambda^2 E_1}{P P_1}\right] + \dfrac{h_1\lambda E_3}{P_1} \\ +(h_2 - h)\begin{bmatrix} \dfrac{-2\lambda^3 E_0}{P^3} - \dfrac{4\lambda^3 E_1}{P^2 P_1} + \dfrac{2\lambda^3 E_2}{P P_1^2} + \dfrac{\lambda^2}{P^2} \\ + \dfrac{2\lambda^2 E(x)}{P P_1} + \dfrac{\lambda^2 E_3}{P_1^2} \end{bmatrix} \end{matrix}\right\}}} \tag{27}$$

Because n only takes on integer value, let n^+ denote the smallest integer greater than or equal to n (derived from Eq. (27)) and n^- denote the largest integer less than or equal to n. Therefore, n^* is determined to be either n^+ or n^-, a known constant. Therefore, one can now retreat E[$TCU(Q,n$+1)] as a cost function with single decision variable Q. Similarly, considering n as constant, Eq. (11) can be rearranged as

$$E\left[TCU(Q, n+1)\right] = a_1 + Q^{-1}z_1 + Qz_2 \tag{28}$$

where

$$z_1 = na_2 + a_3;\ \ z_2 = a_4 + n^{-1}a_5 \tag{29}$$

$$E\left[TCU(Q, n+1)\right] = a_1 + Q\left[\left[Q^{-1}\sqrt{z_1}\right]^2 + \left(\sqrt{z_2}\right)^2\right] \tag{30}$$

or

$$\begin{aligned} E\left[TCU(Q, n+1)\right] = {} & a_1 + Q\left[\left(Q^{-1}\sqrt{z_1}\right) - \left(\sqrt{z_2}\right)\right]^2 \\ & + 2\sqrt{z_1}\sqrt{z_2} \end{aligned} \tag{31}$$

Thus, E[$TCU(Q,n$+1)] is minimized if the second square term in Eq. (31) equals zero. Or

$$Q = \frac{\sqrt{z_1}}{\sqrt{z_2}} \tag{32}$$

Substituting Eqs. (13) to (17) in Eq. (32), and with further derivations, one has

$$Q^* = \sqrt{\frac{n(\lambda K_1)+\lambda(K_1+K)}{\begin{array}{l}\frac{h}{2}\left[\frac{2\lambda^3 E_0}{P^3}+\frac{4\lambda^3 E_1}{P^2 P_1}+\frac{2\lambda^3 E_2}{P P_1^2}-\frac{\lambda E_3}{P_1}+1-\frac{\lambda}{P}\right]\\ +h_2\left[\frac{\lambda^2}{P^2}E_0-\frac{\lambda^3}{P^3}E_0-\frac{2\lambda^3}{P^2 P_1}\cdot E_1+\frac{\lambda^3}{P P_1^2}\cdot E_2+\frac{\lambda^2}{P P_1}E_1\right]\\ +\frac{(h_2-h)}{2}\left[\frac{\lambda^2}{P^2}+\frac{2\lambda^2 E(x)}{P P_1}+\frac{\lambda^2}{P_1^2}\cdot E_3\right]+\frac{h_1\lambda}{2P_1}E_3\\ +\frac{(h_2-h)Q}{2n}\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2 E(x)}{P P_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2}{P_1^2}E_3\right]\end{array}}} \tag{33}$$

or

$$Q^* = \sqrt{\frac{2\lambda\left[(n+1)K_1+K\right]}{\begin{array}{l}h\left\{\frac{2\lambda^3 E_0}{P^3}+\frac{4\lambda^3 E_1}{P^2 P_1}+\frac{2\lambda^3 E_2}{P P_1^2}-\frac{\lambda(1-\theta)E_3}{P_1}+1-\frac{\lambda}{P}\right\}\\ +(h_2-h)\left[\frac{\lambda^2}{P^2}+\frac{2\lambda^2 E(x)}{P P_1}+\frac{\lambda^2 E_3}{P_1^2}\right]+\frac{h_1\lambda E_3}{P_1}\\ +\frac{(h_2-h)}{n}\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2 E(x)}{P P_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2 E_3}{P_1^2}\right]\\ +2h_2\lambda^2\left[\frac{E_0}{P^2}-\frac{\lambda E_0}{P^3}-\frac{2\lambda E_1}{P^2 P_1}+\frac{\lambda E_2}{P P_1^2}+\frac{E_1}{P P_1}\right]\end{array}}} \tag{34}$$

One notes that Eqs. (34) and (27) are identical to that in Chiu et al. [5]. It follows that the long-run average cost E[$TCU(Q,n+1)$] is

$$E\left[TCU(Q,n+1)\right]=a_1+2\sqrt{z_1}\sqrt{z_2} \tag{35}$$

5 Numerical Example

This section verifies the results by using the same numerical example in [5]. Recall the following system parameters:

λ = 3400 units per year,
P = 60,000 units per year,
x = random defective rate which follows a uniform distribution over interval [0, 0.3],
P_1 = 2,200 units per year,
C = \$100 per item,
K = \$20,000 per production run,
C_R = \$60, repaired cost for each item reworked,
h = \$20 per item per year,
h_1 = \$40 per item reworked per unit time,
h_2 = \$80 per item kept at the customer's end,
K_1 = \$4,350 per shipment, a fixed cost,
C_T = \$0.1 per item delivered.

Applying Eq. (27) one obtains n*=2.44, since n* only take on integer value one can use two adjacent integers, plugging them in Eq. (34) and then into Eq. (35). The resulting costs are E[TCU(2265,3)] =\$470159 and E[TCU(2562,4)]=\$470200.

Therefore, the optimal production-shipment policy is (Q,n+1)= (2265,3), and the long-run average cost is \$470,159. Total cost for the proposed model results a reduction of \$17,458 or 13.41% savings of total other related costs. Figure 4 illustrates the effect of the expected defective rate E[x] on the long-run average cost function E[$TCU(Q,n+1)$]. It is noted that the aforementioned solution procedure can be applied to any given system of the same characteristics.

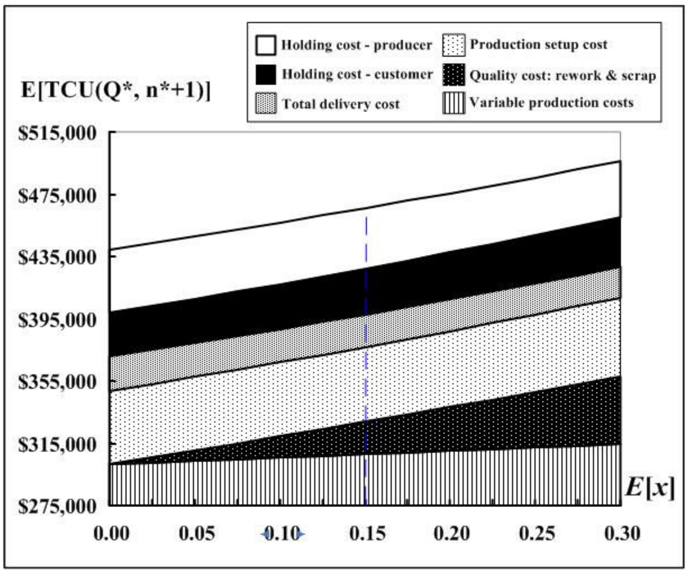

Fig. 4 Effects of random defective rate on the long-run average cost function E[$TCU(Q,n+1)$]

One notes that as x increases, cost E[$TCU(Q,n+1)$] increases significantly due to the quality assurance cost (i.e. the cost for reworking the nonconforming products).

6 Conclusions

Chiu et al. [5] used the conventional method, the differential calculus along the Hessian matrix equations to derive the optimal production-shipment policy a manufacturing system with rework process and a specific cost saving delivery plan in a vendor-buyer supply chains environments. This paper presents an alternative approach to reexamines the solution procedure of their problem without have to refer differential calculus. Such a straightforward algebraic approach assist practitioners who may not have sufficient knowledge of differential calculus in understand with ease such an integrated production-shipment system.

For future research, one interesting direction will be to examine the effect of random demand for the same model.

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