Autotuning Principles for Time-delay Systems

ROMAN PROKOP, JIŘÍ KORBEL, RADEK MATUŠŮ
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nám. TGM 5555, 760 01 Zlín
CZECH REPUBLIC
prokop@fai.utb.cz http://www.fai.utb.cz

Abstract: This paper presents a set of single input – output (SISO) principles for tuning of continuous-time controllers used in autotuning schemes. The emphasis of designed autotuners is laid to SISO systems with time delay. Autotuners represent a combination of relay feedback identification and some control design method. In this contribution, models with up to three parameters are estimated by means of a single asymmetrical relay experiment. Then a stable low order transfer function with a time delay term is identified by a relay experiment. Controller parameters are analytically derived from general solutions of Diophantine equations in the ring of proper and stable rational functions R_{PS}. The generalization for a two degree of freedom (2DOF) control structure is performed. This approach covers a generalization of PID controllers and enables to define a scalar positive parameter for further tuning of the control performance. The analytical simple rule is derived for aperiodic control response and the scalar tuning parameter $m>0$. Autotuning principles of this contribution are applied to SISO systems with delays. Moreover, the Smith predictor scheme is applied for systems with a time delay term. The simulations are performed in the Matlab environment and a toolbox for automatic design and simulation was developed.

Key-Words: Algebraic control design, Relay experiment, Autotuning, Pole-placement problem, Smith predictor

1 Introduction

Proportional-Integral-Derivative (PID) controllers have survived changes in technology and they have been the most common way of using feedback in engineering systems [1], [2]. Yu in [3] refers that more than 97 % of control loops are of this type and most of them are actually under PI control. The practical advantages of PID controllers can be seen in a simple structure, in an understandable principle and in control capabilities. It is widely known that PID controllers are quite resistant to changes in the controlled process without meaningful deterioration of the loop behavior. The Ziegler – Nichols tuning rule has been glorified and vilified as well. However, there are many limitations, drawbacks and infirmities in the behavior of the Ziegler–Nichols setting. A solution for qualified choice of controller parameters can be seen in more sophisticated, proper and automatic tuning of PID controllers. Besides, the PID-based control loops are easy to simulate so no complex methods have to be used [4].

The development of various autotuning principles was started by a simple symmetrical relay feedback experiment proposed by Åström and Hägglund in [5] in the year 1984. The ultimate gain and ultimate frequency are then used for adjusting of parameters by original Ziegler-Nichols rules. During the period of more than two decades, many studies have been reported to extend and improve autotuners principles; see e.g. [6], [7], [11], [12]. The extension in relay utilization was performed in [3], [8], [10], [17] by an asymmetry and hysteresis of a relay. Over time, the direct estimation of transfer function parameters instead of critical values began to appear. Experiments with asymmetrical and dead-zone relay feedback are reported in [13]. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

In this paper, a new combination for autotunig method of PI and PID controllers with an aperiodic control rule is proposed and developed. The basic autotuning principle combines an asymmetrical relay identification experiment and a control design performed in the ring of proper and stable rational functions R_{PS}. The factorization approach proposed in [14] was generalized to a wide spectrum of control problems in [15], [18] - [23]. The pole placement problem in R_{PS} ring is formulated through a Diophantine equation and the pole is analytically tuned according to aperiodic response of the closed loop. The proposed method is compared by an equalization setting proposed in [16]. Naturally, there exist also different principles of control design syntheses which can be used for autotuning methods, e.g. [25], [31], [33].

This contribution deals with two simplest SISO linear dynamic systems with a delay term. The first model of the first order (stable) plus dead time (FOPDT) is supposed in the form:

$$G(s) = \frac{K}{Ts+1} \cdot e^{-\Theta s} \quad (1)$$

Similarly, the second order model plus dead time (SOPDT) is assumed in the form:

$$G(s) = \frac{K}{(Ts+1)^2} \cdot e^{-\Theta s} \quad (2)$$

The contribution is organized as follows. Section 2 represents a background of algebraic control design and the derivation for first and second order systems is derived. Section 3 deals with aperiodic tuning for a PI controller. Then the principle of the Smith predictor is introduced. Section 5 presents some facts about relay identification for autotuning principles. Then a Matlab program environment for design and simulations is described. Finally, section 7 presents a simulation results for three types of SISO systems.

2 Algebraic Control Design

The control design is based on the fractional approach; see e.g. [14], [15], [18]. Any transfer function $G(s)$ of a (continuous-time) linear system is expressed as a ratio of two elements of R_{PS}. The set R_{PS} means the ring of (Hurwitz) stable and proper rational functions. Traditional transfer functions as a ratio of two polynomials can be easily transformed into the fractional form simply by dividing, both the polynomial denominator and numerator by the same stable polynomial of the appropriate order.

Then all transfer functions can be expressed by the ratio:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \quad (3)$$

$$n = \max(\deg(a), \deg(b)), \quad m > 0 \quad (4)$$

Then, all feedback stabilizing controllers for the feedback system depicted in Fig. 1 are given by a general solution of the Diophantine equation:

$$AP + BQ = 1 \quad (5)$$

which can be expressed with Z free in R_{PS}:

$$\frac{Q}{P} = \frac{Q_0 - AZ}{P_0 + BZ} \quad (6)$$

In contrast of polynomial design, all controllers are proper and can be utilized.

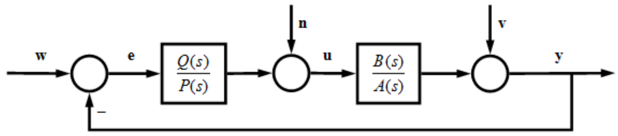

Fig. 1: One-degree of freedom (1DOF) control loop

The Diophantine equation for designing the feedforward controller depicted in Fig. 2 is:

$$F_w S + BR = 1 \quad (7)$$

with parametric solution:

$$\frac{R}{P} = \frac{R_0 - F_w Z}{P_0 + BZ} \quad (8)$$

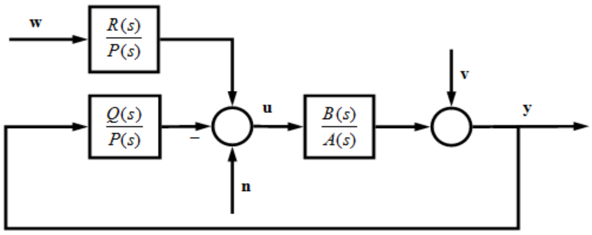

Fig. 2: Two-degree of freedom (2DOF) control loop

Asymptotic tracking is then ensured by the divisibility of the denominator P in (6) by the denominator of the reference $w = G_w / F_w$. The most frequent case is a stepwise reference with the denominator in the form:

$$F_w = \frac{s}{s+m}; \quad m > 0 \quad (9)$$

The similar conclusion is valid also for the load disturbance $d = G_d / F_d$. The load disturbance attenuation is then achieved by divisibility of P by F_d. More precisely, for tracking and attenuation in the closed loop according to Fig. 2 the multiple of AP must be divisible by the least common multiple of denominators of all input signals. The divisibility in R_{PS} is defined through unstable zeros and it can be achieved by a suitable choice of rational function Z in (6), see [14], [18] for details.

2.1 First order systems

Diophantine equation (5) for the first order systems (1) without the time delay term can be easily transformed into polynomial equation:

$$\frac{(Ts+1)}{s+m}p_0+\frac{K}{s+m}q_0=1 \tag{10}$$

with general solution:

$$P=\frac{1}{T}+\frac{K}{s+m}\cdot Z$$
$$Q=\frac{Tm-1}{TK}-\frac{Ts+1}{s+m}\cdot Z \tag{11}$$

where Z is free in the ring R_{PS}. Asymptotic tracking is achieved by the choice:

$$Z=-\frac{m}{TK} \tag{12}$$

and the resulting PI controller is in the form:

$$C(s)=\frac{Q}{P}=\frac{q_1s+q_0}{s} \tag{13}$$

where parameters q_1 a q_0 are given by:

$$q_1=\frac{2Tm-1}{K} \qquad q_0=\frac{Tm^2}{K} \tag{14}$$

The feedforward part of the 2DOF controller follows from (7):

$$\frac{s}{s+m}s_0+\frac{K}{s+m}r_0=1 \tag{15}$$

with general solution:

$$P=\frac{1}{T}+\frac{K}{s+m}\cdot Z$$
$$R=\frac{m}{K}-\frac{s}{s+m}\cdot Z \tag{16}$$

The final PI like controller is given:

$$C_1(s)=\frac{R}{P}=\frac{r_1s+r_0}{s} \tag{17}$$

with parameters

$$r_1=\frac{Tm+m}{K} \qquad r_0=\frac{Tm^2}{K} \tag{18}$$

2.2 Second order systems

The control synthesis for the SOPDT is based on stabilizing Diophantine equation (8) applied for the transfer function (4) without a time delay term. The Diophantine equation (5) takes the form:

$$\frac{(Ts+1)^2}{(s+m)^2}\cdot\frac{p_1s+p_0}{s+m}+\frac{K}{(s+m)^2}\cdot\frac{q_1s+q_0}{s+m}=1 \tag{19}$$

and after equating the coefficients at like powers of s in (22) it is possible to obtain explicit formulas for *pi*, q_i:

$$p_1=\frac{1}{T^2}; \qquad p_0=\frac{3Tm-2}{T}$$
$$q_1=\frac{1}{K}\left[3m^2-\frac{1}{T^2}(1+3m-\frac{2}{T})\right]; \tag{20}$$
$$q_0=\frac{1}{K}\left[m^3-\frac{1}{T^2}(3m-\frac{2}{T})\right]$$

The rational function $P(s)$ has its parametric form (similar as in (14) for FOPDT):

$$P=\frac{p_1s+p_0}{(s+m)}+\frac{K}{(s+m)^2}\cdot Z \tag{21}$$

with Z free in R_{PS}. Now, the function Z must be chosen so that P is divisible by the denominator of the reference which is (12). The required divisibility is achieved by $z_0=-\frac{p_0m}{K}$. Then, the particular solution for P, Q is

$$P=\frac{s\left[p_1s+(p_1m+p_0)\right]}{(s+m)^2}$$
$$Q=\frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{(s+m)^2}, \tag{22}$$

where

$$\tilde{q}_0=q_0+p_0m, \quad \tilde{q}_1=q_0+q_1m+2Tp_0m,$$
$$\tilde{q}_2=q_1+T^2p_0m. \tag{23}$$

The final (asymptotic tracking) controller has the transfer function:

$$C(s)=\frac{Q}{P}=\frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{s(p_1s+(p_1m+p_0))} \tag{24}$$

Also the feedforward part for the 2DOF structure can be derived for the second order system. For asymptotic tracking Diophantine equation takes the form:

$$\frac{s}{s+m}\frac{s_1s+s_0}{(s+m)}+\frac{K}{(s+m)^2}r_0=1 \tag{25}$$

The 2DOF control law is only dependent upon the rational function R with general expression

$$R=\frac{m^2}{K}-\frac{s}{s+m}Z \tag{26}$$

also with Z free in R_{PS}. The final feedforward controller

$$C_1(s) = \frac{R}{P} = \frac{\frac{m^2}{K}(s+m)^2}{s\left[p_1 s + (p_1 m + p_0)\right]} \tag{27}$$

It is obvious that parameters of both parts of the controller (feedback and/or feedforward) depend on the tuning parameter $m > 0$ in a nonlinear way. For both systems FOPDT and SOPDT the scalar parameter $m > 0$ seems to be a suitable „tuning knob" influencing control performance as well as robustness properties of the closed loop system. Naturally, both derived controllers correspond to classical PI and PID ones. Equation (13) represents a PI controller:

$$u(t) = K_P \cdot \left(e(t) + \frac{1}{T_I} \cdot \int e(\tau) d\tau \right) \tag{28}$$

and the conversion of parameters is trivial. Relation (20) represents a PID in the standard four-parameter form [6]:

$$\begin{aligned} &u(t) = K_P \cdot \left(e(t) + \frac{1}{T_I} \cdot \int e(\tau) d\tau + T_D y_f(t) \right) \\ &\tau y_f{}'(t) + y_f(t) = y\ (t) \end{aligned} \tag{29}$$

3 Aperiodic Tuning

There are many tuning principles and modifications of the Ziegler – Nichols rule developed from 1940s, see [6], [16], [25], [32]. Only in [25], more than 240 tuning rules are referred for PID and more than 100 rules for PI controllers.

A simple and attractive choice for the tuning parameter $m > 0$ can be easily obtained analytically. In the R_{PS} expression, the closed-loop transfer function K_{wy} is for (1) and PI controller (13) given in a very simple form:

$$K_{wy} = \frac{BQ}{AP + BQ} = BQ = \frac{(2Tm - 1)s + Tm^2}{(s+m)^2} \tag{30}$$

The step response of (30) can be expressed by Laplace transform:

$$\begin{aligned} h(t) &= L^{-1}\left\{ \frac{K_{wy}}{s} \right\} = L^{-1}\left\{ \frac{k_1 s + k_0}{s(s+m)^2} \right\} = \\ &= L^{-1}\left\{ \frac{A}{s} + \frac{B}{(s+m)} + \frac{C}{(s+m)^2} \right\}, \end{aligned} \tag{31}$$

where A, B, C are calculated by comparing appropriate fractions in (31) and $k_1 = 2mT - 1$, $k_0 = Tm^2$.

The response $h(t)$ in time domain is then

$$h(t) = A + Be^{-mt} + Cte^{-mt} \tag{32}$$

The overshoot or undershoot of this response is characterized by the first derivative condition

$$h'(t) = -mBe^{-mt} + C(e^{-mt} - tme^{-mt}) = 0 \tag{33}$$

From (33) time of the extreme of response $h(t)$ is then easily calculated by the relation:

$$t_e = \frac{C - mB}{mC} = \frac{1}{m} - \frac{B}{C} \tag{34}$$

Since the aperiodic response means that the extreme does not exist for positive t_e, it implies $t_e < 0$ and after substitutions of A, B, C, k_1, k_0 relation (34) takes the simple form

$$1 < m\frac{B}{C} = \frac{1}{\frac{1}{Tm} - 1} \tag{35}$$

The denominator of (35) must be positive and less than 1 and $m > 0$ which implies the inequality:

$$\frac{1}{2T} < m < \frac{1}{T} \tag{36}$$

Any positive parameter m from (36) ensures aperiodic response. It is a question for further investigation and simulation what choice from interval (36) is the best. The time constant is always an estimation in the autotuning philosophy and then the middle value of (36) would be a reasonable choice in the form

$$m = \frac{3}{4 \cdot T} \tag{37}$$

Also other tuning principles for aperiodic tuning certainly exist. For the mentioned algebraic synthesis, the equalization method developed by Gorez and Klán in [16]. The idea goes out from PI controller in the form (24). The tuning rule is very simple and it leads in relations:

$$K_P = \frac{1}{2K} \quad T_I = 0.4 \cdot T_u \tag{38}$$

where K is a process gain and T_u is the ultimate period obtained from the Ziegler-Nichols experiment. However, the fulfillment of (38) by unique value of m>0 is impossible, see [19]. The exact fulfillment of both relations in (38) could be obtained in the case of two distinct roots in denominator (30), so $(s+m_1)(s+m_2)$ instead of $(s+m)^2$.

4 Smith Predictors

The Smith predictor was designed in the late 1950s for systems with time delay, see e.g. [31], [32]. The basic classical interpretation of the Smith predictor is depicted in Fig. 3. The time delay term $e^{-\Theta s}$ has a negative influence to feedback stability which follows from frequency analysis. The feedback signal for the main controller $C(s)$ in Fig. 3 is a predicted value of the output. It means that the signal $y(t)$ inputs into the control error instead of the delayed $y(t-\Theta)$, it explains the name predictor. The Smith predictor launched the high development of *Internal Model Controllers* (*IMC*), where the plant model is present in the feedback loop (see [31], [33]). When the transfer function $G(s)$ is stable then the feedback systems in Fig. 3 is equivalent to the IMC version depicted in Fig. 4.

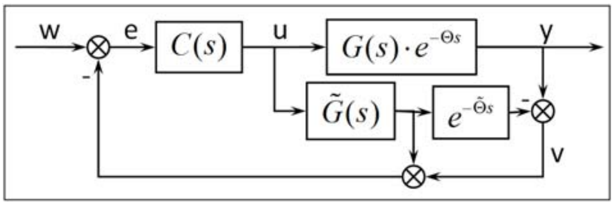

Fig. 3: Smith predictor – classical version

The main advantage of the Smith predictor is that the controller $C(s)$ can be designed according to delay-free part $G(s)$ of the plant. However, there are two main weak points in this sophisticated scheme. The first one is that the signal $v(t)$ is zero only in the case when the transfer function $G(s)$ is the same in the outer and inner loops in Fig. 3. The second weakness is that the transfer function must be stable. In the case of autotuning, always the approximated transfer function of the plant can be incorporated into the feedback.

Then the signal $v(t)$ in Fig. 3 and Fig. 4 is:

$$V(s) = G(s)e^{-\Theta s} - \tilde{G}(s)e^{-\tilde{\Theta} s} \tag{39}$$

In the case of discrepancy, this non-zero signal indeed negatively influences the control performance. Note that the nominal transfer function for control design is $\tilde{G}(s)$.

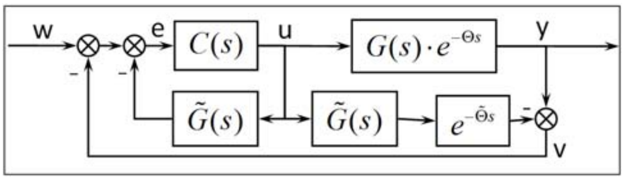

Fig. 4: Smith predictor – IMC version

5 Relay Feedback Estimation

The estimation of the process or ultimate parameters is a crucial point in all autotuning principles. The relay feedback test can utilize various types of relay for the parameter estimation procedure. The classical relay feedback test [5] was proposed for stable processes by symmetrical relay without hysteresis. Following sustained oscillation are then used for determining the critical (ultimate) values. The control parameters (PI or PID) are then generated in standard manner.

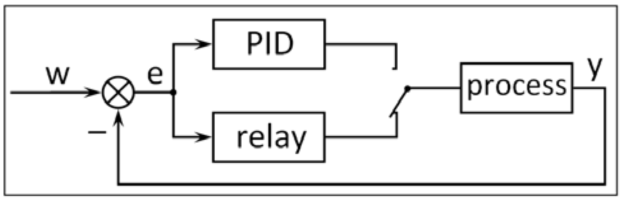

Fig. 5: Block diagram of an autotuning principle

Asymmetrical relays with or without hysteresis bring further progress [3], [17]. After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such relay experiment is depicted in Fig. 6. The relay asymmetry is required for the process gain estimation (40) while a symmetrical relay would cause the zero division in the appropriate formula.

In this paper, an asymmetrical relay with hysteresis is used. This relay enables to estimate transfer function parameters as well as a time delay term. For the purpose of the aperiodic tuning the time delay is not exploited.

The process gain can be computed by the relation (see [13]):

$$K = \frac{\int_0^{iT_y} y(t)dt}{\int_0^{iT_y} u(t)dt}; \quad i = 1,2,3,.. \tag{40}$$

The time constant and time delay terms are then given by:

$$\begin{aligned} T &= \frac{T_y}{2\pi}\cdot\sqrt{\frac{16\cdot K^2\cdot u_0^2}{\pi^2\cdot a_y^2}-1} \\ \Theta &= \frac{T_y}{2\pi}\cdot\left[\pi - arctg\frac{2\pi T}{T_y} - arctg\frac{\varepsilon}{\sqrt{a_y^2-\varepsilon^2}}\right] \end{aligned} \tag{41}$$

where a_y and T_y are depicted in Fig. 6 and ε is the hysteresis.

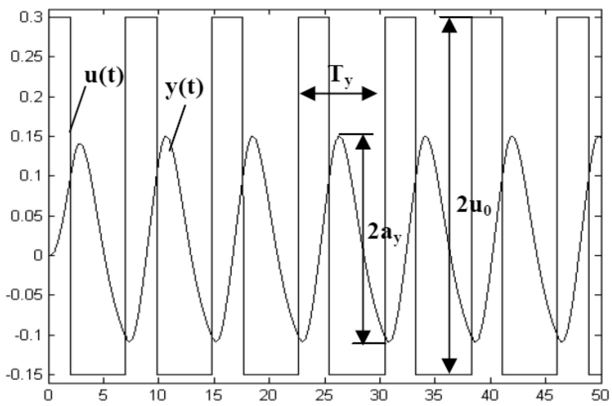

Fig. 6: Asymmetrical relay oscillation

The gain is given by (40), the time constant and time delay term can be estimated according to [13] by the relation:

$$
\begin{aligned}
T &= \frac{T_y}{2\pi} \cdot \sqrt{\frac{4 \cdot K \cdot u_0}{\pi \cdot a_y} - 1} \\
\Theta &= \frac{T_y}{2\pi} \cdot \left[\pi - 2arctg \frac{2\pi T}{T_y} - arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right]
\end{aligned}
\tag{42}
$$

6 Simulation and Program System

A Matlab program system was developed for engineering applications of auto-tuning principles. This program enables a choice for the identification of the controlled system of arbitrary order. The estimated model is of a first or second order transfer function with time delay. The user can choose three cases for the time delay term. In the first case the time term is neglected, in the second one the term is approximated by the Pade expansion and the third case utilizes the Smith predictor control structure. The program is developed with the support of the Polynomial Toolbox. The Main menu window of the program system can be seen in Fig. 7.

In the first phase of the program routine, the controlled transfer function is defined and parameters for the relay experiment can be adjusted. Then, the experiment is performed and it can be repeated with modified parameters if necessary. After the experiment, an estimated transfer function in the form of (1) or (2) is performed automatically and controller parameters are generated after pushing of the appropriate button. Parameters for experimental adjustment are defined in the upper part of the window.

The second phase begins with the "Design controller parameters" button and the actual control design is performed. According to above mentioned methodology and identified parameters, the controller is derived and displayed. The control scheme depends on the choice for the 1DOF or 2DOF structure and on the choice of the treatment with the time delay term.

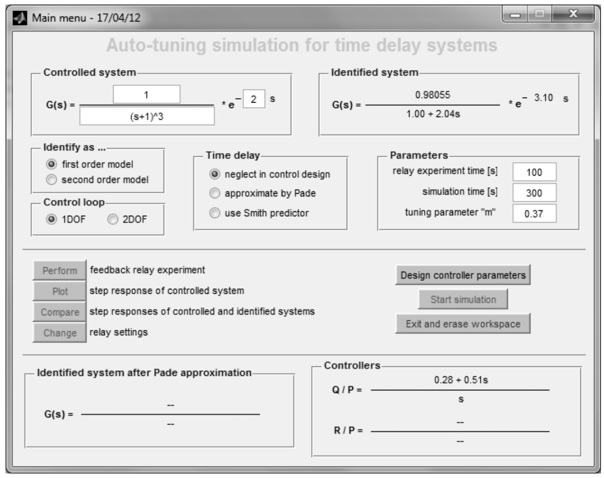

Fig. 7: Main Menu

During the third phase, after pushing the "Start simulation" button, the simulation routine is performed and required outputs are displayed. The simulation horizon can be prescribed as well as tuning parameter *m*, other simulation parameters can be specified in the Simulink environment. In all simulation a change of the step reference is performed in the second third of the simulation horizon and a step change in the load is injected in the last third. A typical control loop of the case with the Smith predictor in Simulink is depicted in Fig. 8.

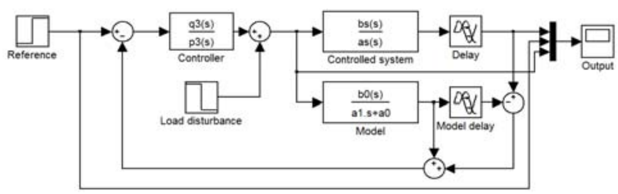

Fig. 8: Control loop in Simulink

Also the step responses can be displayed and the comparison of the controlled and estimated systems can be depicted. Another versions of the similar program systems were developed and they are referred in e.g. [19], [20].

7 Examples and Simulations

The following examples illustrate the situation where the estimated model is in the form (1) or (2) with a time delay term. The controllers are designed according to Part 2 with neglecting of the time delays.

Example 1: A second order controlled system with time delay with the transfer function:

$$G(s)=\frac{1}{(2s+1)^2}\cdot e^{-2s} \tag{43}$$

was identified by the relay experiments as a first and second order system. The results give the following transfer functions:

$$\tilde{G}(s)=\frac{0.98}{3.46s+1}\cdot e^{-2.77s},\quad \tilde{\tilde{G}}(s)=\frac{0.98}{3.41s^2+3.69s+1}\cdot e^{-2.49s} \tag{44}$$

The first controller was designed for the identified system with neglecting of the time delay term and the tuning parameter m = 0.22 was derived from the aperiodic condition (36). The PID for the second order estimation (44) was designed for the tuning parameter m = 0.41. The final controllers are governed by the transfer functions:

$$C_1(s)=\frac{\tilde{Q}(s)}{\tilde{P}(s)}=\frac{0.51s+0.17}{s},\quad C_2(s)=\frac{\tilde{\tilde{Q}}(s)}{\tilde{\tilde{P}}(s)}=\frac{0.71s^2+0.70s+0.18}{1.85s^2+s},\quad C_3(s)=\frac{\tilde{\tilde{R}}(s)}{\tilde{\tilde{P}}(s)}=\frac{1.06s^2+0.86s+0.18}{1.85s^2+s} \tag{45}$$

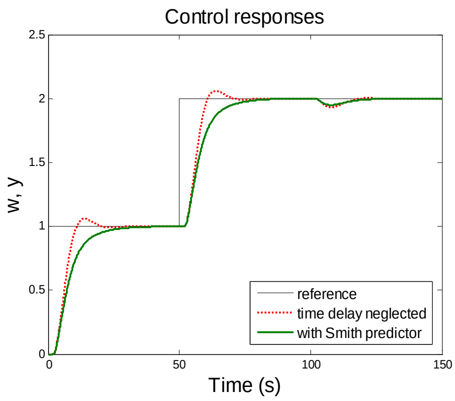

Fig. 9: Control responses 1DOF first order

The original system $G(s)$ from (43) was controlled by (44) in two different control ways. The simple control response in the sense of 1DOF is depicted in Fig. 9 by dashed line while the Smith predictor scheme represents an aperiodic response in the same figure. Fig. 10 displays the same simulation for the second order controller C_2 in (45).

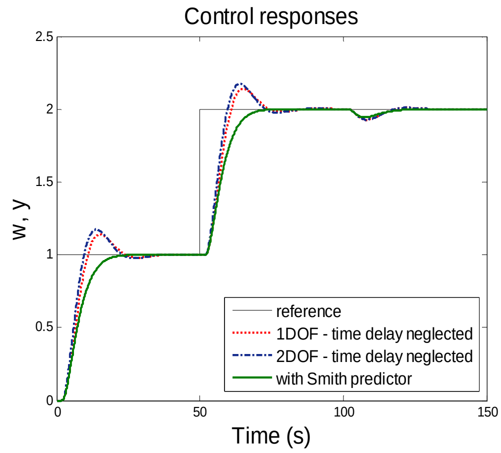

Fig. 10: Control responses second order

Example 2: A fifth order system with time delay $G(s)$ was identified in the form of a first order transfer function with time delay:

$$G(s)=\frac{3}{(2s+1)^5}\cdot e^{-5s} \tag{46}$$

The first and second order estimation results in the following transfer functions:

$$\tilde{G}(s)=\frac{2.99}{5.88s+1}\cdot e^{-10.35s},\quad \tilde{\tilde{G}}(s)=\frac{2.99}{11.19s^2+6.69s+1}\cdot e^{-8.49s} \tag{47}$$

Then controllers were designed for the identified models (47) with time delay terms neglected. The PI controller was derived for the value of m = 0.13 and the PID one was derived for m = 0.22. Both controllers in the 1DOF structure have the transfer functions:

$$C_1(s)=\frac{0.17s+0.03}{s},\quad C_2(s)=\frac{0.42s^2+0.23s+0.03}{3.35s^2+s} \tag{48}$$

The control responses for the first order approximation and design are depicted in Fig. 11. In this case the difference of responses between neglecting the time delay term and with the use of the Smith predictor is remarkably stronger. While standard feedback control response is quite poor and oscillating then the response with Smith predictor in the loop is smooth and aperiodic.

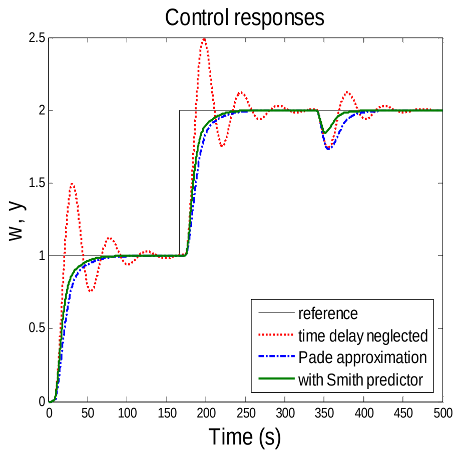

Fig. 11: Control responses 1DOF - first order

Almost the same situation is illustrated in Fig. 12 where the second order approximation and synthesis were utilized. However, comparison of Fig. 11 and Fig. 12 shows that the first order synthesis is sufficient and the second order is redundant.

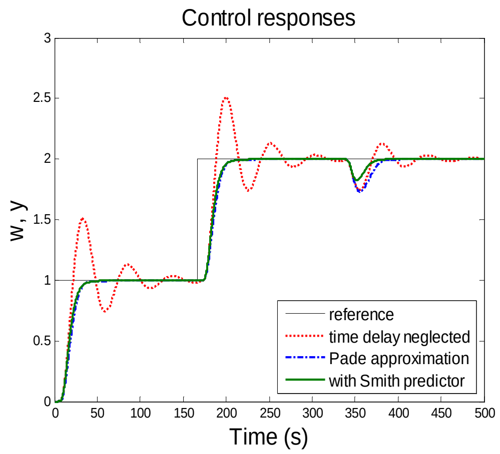

Fig. 12: Control responses 1DOF - second order

Example 3: This example represents a case of higher order system without delay approximated by a law order system with a time delay term. A higher order system (8th order) with transfer function $G(s)$ is supposed:

$$G(s) = \frac{3}{(s+1)^8} \tag{49}$$

After the relay experiment, a first order and second estimation gives the following transfer functions:

$$\begin{aligned} \tilde{G}(s) &= \frac{2.96}{4.22s+1} \cdot e^{-4.96s} \\ \tilde{\tilde{G}}(s) &= \frac{2.96}{4.83s^2+4.40s+1} \cdot e^{-4s} \end{aligned} \tag{50}$$

The step responses of systems (49) and (50) are shown in Fig. 13.

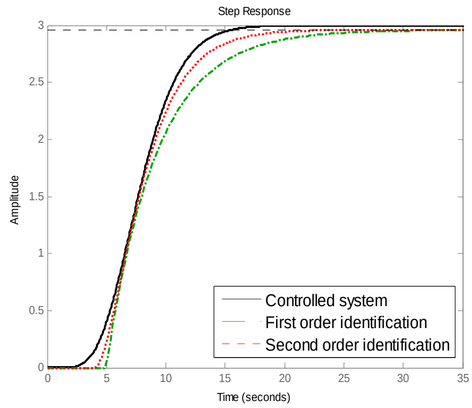

Fig. 13: Step responses of systems (49)

Naturally, both step responses of the estimated systems are quite different from the original system $G(s)$.

Again, PI controllers are derived from (10), (11) and the tuning parameter $m>0$ can influence the control behaviour. Since the difference of controlled and estimated systems is considerable, it can be expected that not all values of and some of $m>0$ represent acceptable behaviour.

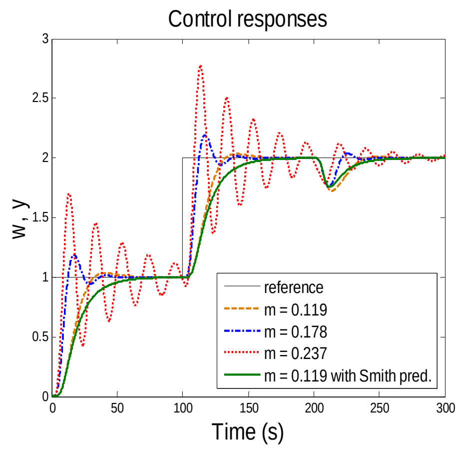

Fig. 14: Control responses 1DOF first order

With respect of (36), three responses are shown in Fig. 14. Generally, larger values of $m>0$ implicate larger overshoots and oscillations. As a consequence, for inaccurate relay identifications, lower values of $m>0$ in interval (36) can be recommended. The PI controller for m = 0.18 has the form

$$C(s) = \frac{0.17s + 0.05}{s} \qquad (51)$$

The control responses for (49) and (51) with and without the Smith predictor are shown in Fig. 14.

The second order identification and synthesis of example 3 for m = 0.34 gives the PID controller:

$$C(s) = \frac{0.28s^2 + 0.23s + 0.05}{2.20s^2 + s} \qquad (52)$$

The higher order system (49) was controlled by (52) and two responses are depicted in Fig. 15. The first one represents neglecting of a time delay term in (50) while the second one utilizes the Smith predictor structure. It is obvious that the Smith predictor brings a significant improvement of overshoots.

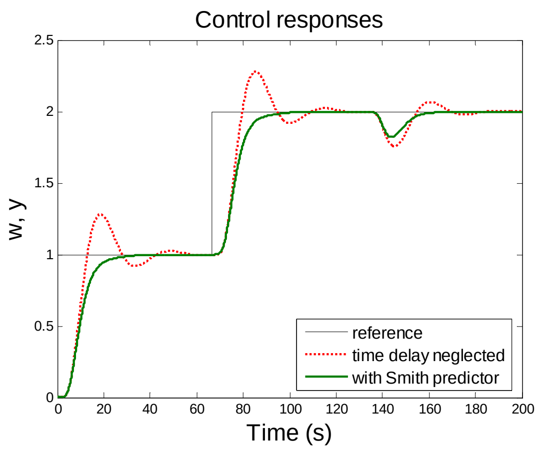

Fig. 15: Control responses 1DOF second order

8 Conclusion

This contribution gives some rules for autotuning principles with a combination of relay feedback identification and a control design method.

The estimation of a low order transfer function parameters is performed from asymmetric limit cycle data, see [13]. The control synthesis is carried out through the solution of a linear Diophantine equation according to [14], [15], [18]. This approach brings a scalar tuning parameter which can be adjusted by various strategies. A first order estimated model generates PI-like controllers while a second order model generates a class of PID ones. The aperiodic tuning through the parameter m>0 is proposed by the analytic derivation, more details in [20]. In both cases also the Smith predictor influence was compared with neglecting of time delay terms. The methodology is illustrated by several examples of various orders and dynamics. The results of all simulations prove that the Smith predictor structure brings a significant improvement of the aperiodic responses. The price for the improvement is a more complex structure of the feedback control system.

Acknowledgements

This work was supported by the European Regional Development Fund under the project CEBIA-Tech No. CZ.11.05./2.1.00 /03.0089.

References:

[1] D. H. Kim, J. H. Cho, Robust Tuning of PID Controller with Disturbance Rejection Using Bacterial Foraging Based Optimization, *WSEAS Transactions on Systems*, Vol. 3, No. 11, 2004, pp. 2834-2840.

[2] P. C. Patic, L. Duta, Comparative Study Regarding the Optimization and Analysis of Automatic Systems using PID, LQR and PI-MIMO, *WSEAS Transactions on Systems*, Vol. 9, No. 7, 2010, pp. 754-763.

[3] Ch.Ch. Yu, *Autotuning of PID Controllers. Springer,* London, 1999.

[4] F. Neri, Software Agents as a Versatile Simulation Tool to Model Complex Systems. *WSEAS Transactions on Information Science and Applications*, WSEAS Press (Wisconsin, USA), Vol. 7, No. 5, 2010, pp. 609-618.

[5] K.J. Åström and T. Hägglund, Automatic tuning of simple regulators with specification on phase and amplitude margins. *Automatica*, Vol.20, 1984, pp. 645-651.

[6] K.J. Åström and T. Hägglund, *PID Controllers: Theory, Design and Tuning.* Research Triangle Park, NC: Instrumental Society of America, 1995.

[7] R.F. Garcia and F.J.P. Castelo, A complement to autotuning methods on PID controllers, In: *Preprints of IFAC Workshop PID'00*, 2000, pp. 101-106.

[8] R.R. Pecharromán and F.L. Pagola, Control design for PID controllers auto-tuning based on improved identification, In: *Preprints of IFAC Workshop PID'00,* pp. 89-94, 2000.

[9] C.C. Hang, K.J. Åström and Q.C. Wang, Relay feedback auto-tuning of process controllers – a tutorial review, *Journal of Process Control*, Vol. 12, No. 6, 2002.

[10] T. Thyagarajan and Ch.Ch. Yu, Improved autotuning using shape factor from relay feedback, In: *Preprints of IFAC World Congres*, 2002.
[11] S. Majhi and D.P. Atherton, Autotuning and controller design for unstable time delay processes, In*: Preprints of UKACC Conf an Control,* 1998, pp. 769-774.
[12] F. Morilla, A. Gonzáles and N. Duro, Auto-tuning PID controllers in terms of relative damping , In: *Preprints of IFAC Workshop PID'00,* 2000, pp. 161-166.
[13] M. Vítečková, and A. Víteček, Plant identification by relay methods. In: *Engineering the future (edited by L. Dudas)*. Sciyo, Rijeka, 2010, pp. 242-256.
[14] M. Vidyasagar*, Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, M.A., 1987.
[15] V. Kučera, Diophantine equations in control - A survey, Automatica, Vol. 29, No. 6, 1993, pp. 1361-75.
[16] R. Gorez and P. Klán, Nonmodel-based explicit design relations for PID controllers, In: *Preprints of IFAC Workshop PID'00*, 2000, pp. 141-146.
[17] I. Kaya and D.P. Atherton, Parameter estimation from relay autotuning with asymmetric limit cycle data, *Journal of Process Control*, Vol. 11, No4, 2001, pp. 429-439.
[18] R. Prokop and J.P. Corriou, Design and analysis of simple robust controllers*, Int. J. Control,* Vol. 66, 1997, pp. 905-921.
[19] R. Prokop, J. Korbel and Z. Prokopová, Relay feedback autotuning – A polynomial approach, In: *Preprints of 15th IFAC World Congress*, 2010.
[20] R. Prokop, Korbel, J. and Prokopová, Z., Relay based autotuning with algebraic control design, In: *Preprints of the 23rd European Conf. on modelling and Simulation*, Madrid, 2009, pp. 531-536.
[21] R. Matušů and R. Prokop, Robust Stabilization of Interval Plants using Kronecker Summation Method. In: *Last Trends on Systems, 14th WSEAS International Conference on Systems*, Corfu Island, Greece, 2010, pp. 261-265.
[22] L. Pekař and R. Prokop, Non-delay depending stability of a time-delay system. In*: Last Trends on Systems, 14th WSEAS International Conference on Systems*, Corfu Island, Greece, 2010, pp. 271-275.
[23] L. Pekař and R. Prokop, Control of Delayed Integrating Processes Using Two Feedback Controllers: RMS Approach, In*: Proceedings of the 7th WSEAS International Conference on System Science and Simulation in Engineering*, Venice, 2008, pp. 35-40.
[24] R. Prokop, J. Korbel and O.Líška, A novel principle for relay-based autotuning. *International Journal of Mathematical Models and Methods in Applied Science,* 2011, Vol. 5, No. 7, pp. 1180-1188.
[25] A. O´Dwyer, *Handbook of PI and PID controller tuning rules.* London: Imperial College Press, 2003.
[26] L. Pekař and R. Prokop: Algebraic Control of integrating Processes with Dead Time by Two Feedback Controllers in the Ring RMS. *Int. J. of Circuits, Systems and Signal Processing*, Vol. 2, No. 4, 2008, pp. 249-263.
[27] P. Dostálek, J. Dolinay, V. Vašek and L. Pekař. Self-tuning digital PID controller implemented on –bit Freescale microcontroller. *International Journal of Mathematical Models and Methods in Applied Sciences,* Vol. 4, No. 4, 2010, pp. 274-281.
[28] L. Pekař, R. Prokop and R. Matušů. Stability conditions for a retarded quasipolynomial and their applications*. International Journal of Mathematics and Computers in Simulations*, Vol. 4, No. 3, 2010, pp. 90-98
[29] R. Matušů and R. Prokop. Experimental verification of design methods for conventional PI/PID controllers. *WSEAS Trans. on Systems and Control*, Vol. 5, Issue 5, 2010, pp.269-280.
[30] Q. Zhohg. *Robust Control of Time-delay systems.* Springer, London, 2006.
[31] K.J. Åström and R.M. Murray, *Feedback Systems.* Research Triangle Park, NC: Instrumental Society of America, 1995.
[32] O.J.M. Smith, *Feedback Control Systems.* McGraw-Hill Book CompanyInc., 1958.
[33] M. Morari and E. Zafiriou, *Robust Process Control.* Prentice Hall, New Jersey, 1989.