The attitude control of the four-rotor unmanned helicopter based on feedback linearization control

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Abstract: - This work is a contribution to stable attitudes control of the small scale four-rotor helicopter. The small scale four-rotor helicopter is a multi-variable strong coupling nonlinear system, especially its complicated attitude control. In order to control the attitude of the helicopter, the deliberate dynamics and kinematics of four-rotor helicopter attitude model was established. Based on the model derived, the feedback linearization method was utilized to decouple model and control the system. According to the characteristics of the attitude in the helicopter's whole motion process, two control tasks were numerically performed, and the corresponding simulation results show that the proposed control strategy behaves remarkably well. This work also built the foundation for the position control of the helicopter.

Key-Words: four-rotor helicopter, feedback linearization method, attitude control

1 Introduction

Recently, a growing interest in small scale unmanned aerial vehicles (UAVs), especially vertical takeoff and landing (VTOL)-UAVs, has been shown among the research community [1-5]. These unmanned vehicles, which are highly maneuverable, are expected urgently when it comes to performing desired tasks in dangerous and/or inaccessible environment, such as observation woods and building fires, meteorological observation, patrol, and military purpose such as reconnaissance, monitoring, and communication etc. [6-8]. Moreover, according to the rapid progress of electronic and computer technology, the technology development of the UAVs is getting faster and faster [9-10].

There are many different types of small scale unmanned VTOL-UAVs, such as traditional helicopter[11-13], the twin-rotor[14-15] or tandem-rotor helicopter[16], the coaxial rotor helicopter[17] and four-rotor helicopters[9]. Although the traditional helicopter, single main rotor and one tail rotor small-scale helicopter, is popular and has been studied widely, yet its characters of complex dynamics and structure, high price and hard to control, instability and easily to crash make it difficult to use by common users. The four–rotor helicopter shares all the merits of the traditional helicopters; moreover, it has four fixed-pitch rotors mounted at the four ends of a simple cross frame. Owing to the symmetry, this vehicle is dynamically elegant, inexpensive, and simple to design and manufacture personally. It is an omni-directional vehicle, and has almost no constraints on its motion. It has exclusive maneuverability advantages, and it can be flown in tight spaces. These characteristics make the four-rotor helicopter a good candidate to be utilized in the real life [18, 19].

Unfortunately the exclusive maneuverability characteristics of the four-rotor helicopter give a big complexity and instability in its dynamics, hence making it hard to control. The development of four-rotor helicopter is challenging, and it brings out an emerging area in nonlinear control and model study among researchers. There are two methods in modeling the four-rotor helicopter, one is based on the first principle [20–23], and the other is built on the LaGrange method [24, 25]. These two models all treat the four-rotor helicopter as a rigid body with force and torques acted on. Based on these two models, many different kinds of attitude and trajectory tracking controller have been designed.

Lozano et al. [26] approached global stabilization of a PVTOL vehicle using Lyapunov analysis, designing a simple nonlinear controller by analyzing boundedness and convergence of each state. Bouabdallah et al. [27] control the attitude of three DOF model of the four-rotor helicopter based on the Lyapunov theorem, and the PID and linear quadratic were used to control the helicopter at near-hover condition. In reference [28], it decoupled the system into sets of double integrators. The control structure was divided into four inner loops, two position loops. An integral sliding mode control was used to handle the altitude. The position control was using integral LQR control, with states added for the lag in the inner loop resulting in an eight state model. However, this modeling and simplifying methods lost much information of the quad-copters. In references [29], the proportional derivative derivative (PDD) and proportional derivative (PD) controller had been used in the quad-copter control, and the experiment showed that these two controllers could keep the asymptotic stability of the system. In reference [30], the flatness-based controller has been proposed to solve the trajectory tracking problem for an unmanned four-rotor aircraft, it can make the simplest model of the quad copter tracking desired trajectory. Although these methods could fulfill the control of the position of the system, yet they all, more or less, have to adjust the control coefficient with many tries online, and these will bring out a problem, that a bad control parameter might lead to the crash of helicopter.

In order to control the four-rotor helicopter more easily, based on the research foundation described above and the fact that the attitude control is the fundamental of the whole control of the four-rotor helicopter [31], the author is trying to solve the attitude control problem of the four-rotor helicopter as the first step in four-rotor helicopter study. In the first place, derive the nonlinear attitude model of the four-rotor helicopter, and use the nonlinear feedback theory to decouple the nonlinear attitude model and utilize Hurwitz polynomials to derive the coefficients of the controller.Accroding to the theorem that the roots located in the left portion of the complex plane in the Hurwitz polynomials, the system can be stable. This method can make the control parameters adjust safer online. And it can adjust the response speed of the system by locate the roots of the Hurwitz polynomials. The simulation and experiment show that this control method can fulfill the control of the attitude of the four-rotor helicopter. This paper is arranged as follows. Section 2 describes the principle and structure of the four-rotor helicopter. A detail description of kinematics and dynamics of the four-rotor helicopter is provided in section 3. And the kernel of the system controller, feedback linearization theory, is described in section 4.In section 5, the nonlinear feedback attitude controller is designed in detail. In section 6, the attitude controller proposed is tested by simulation with Mat-lab. Finally, the conclusion is drawn.

2 Problem Formulation

Different from the classical helicopter, which uses a single main-rotor to lift the helicopter and one auxiliary tail-rotor to adjust the helicopter's attitude, the four-rotor helicopter is a special flying helicopter, which is composed of four rotors to lift the helicopter and adjust its attitude. This model was often designed with four arms in a cross configuration and one motor at the end of each arm. The four rotors are all with fixed collective angles propelled by the non-brush DC motor. The thrust force of each rotor is changed along with the rotation velocity of the rotor, while the velocity of the rotor is controlled by the control input Pulse Width Modulation (PWM) signals produced by onboard processor. The diagonal motors are running in the same direction (anti-clockwise); whereas the other diagonal motors running in the clockwise direction to eliminate the anti-torque, gyroscopic effects, and aerodynamic torques tend to cancel [28].

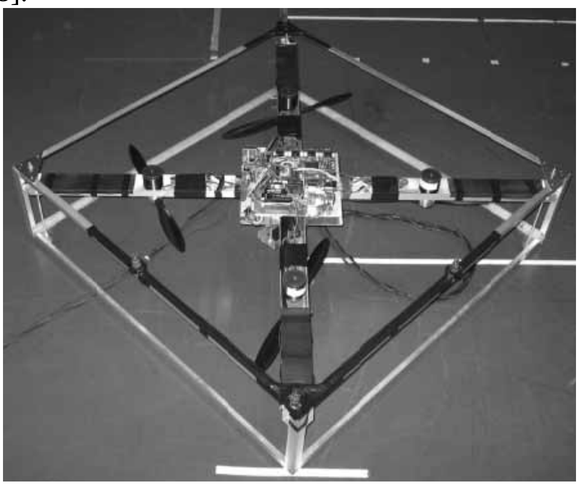


Fig.1 the four-rotor helicopter

The operation theory of the four-rotor helicopter can be explained below. Firstly, when the rotor speeds vary with the same quantity, the lift forces change accordingly, in the case, the altitude of system will change and enable the helicopter take-off or landing. The forward and backward, the left and right, and the yaw motions are achieved through a differential control strategy of the thrust force generated by each motor. The rotation directions of all these four rotors will not be changed in the system. When the angle velocities of these two sets are the same in the magnitude but rotate in the opposite direction, the copter can fulfill the movement of hover. In order to change the roll angle of the vehicle (its rotation about the forward-back axis), the thrust on left motor can be raised and thrust on the right motor can be decreased, without affecting the balance of torques being applied to the vehicle. The same can be done to change the pitch of the vehicle using the front and back motors. To cause a change in the yaw angle (direction the front motor is pointing), front and back motor thrusts are increased while left and right thrusts are decreased, meanwhile, maintaining the same total thrust. The structure of the four rotor helicopter is shown in Figure 1. The change of the attitude of the four rotor helicopter will change the translational acceleration and velocity in three directions and then make the copter track the predefined trajectory. Then the attitude and trajectory control of the four-rotor helicopter can be realized. It can be noticed that the attitude control is the fundamental of the velocity and position control. Compared to the traditional helicopter control mode, the control structure is more clearly and more maneuverability, but they have a lot in common in control methods. Thus it becomes a best research object for the researchers on unmanned helicopter to test the controller and realize the research aim.

3 Attitude dynamics and kinematics of the helicopter

According to the description in last session, the attitude of the four-rotor helicopter is mainly controlled by the thrust forces exerted by the four rotors. In order to derive the physical model of the copter, the first assumption is to consider the whole helicopter as a rigid body with six Degrees of Freedom (DOF). Generally, to change the movement of the rigid body in the space, there should be three forces and three torques acted upon it and the motion of a rigid body should be defined relative to a Cartesian inertial frame. Thus the theory in inertial system can be utilized to derive the model.

3.1 Attitude dynamics

Many coefficients of the helicopter can be described easily in body itself while difficult to narrate on the ground Cartesian inertial frame. Therefore, in order to describe the motion of the system clearly, there are at least two frames should be established. The first one is the inertial frame (Earth-fixed frame) defined as $N=\{o,n_1,n_2,n_3\}$,which is one typical conventional Earth-fixed frame where n_1 points East, n_2 points North and n_3 points opposite the center of the Earth. The second frame is the body-fixed reference frame defined as $F_B=\{o_f, f_1, f_2, f_3\}$ where the center o_f is located at the Center of Gravity (CG) of the four-rotor helicopter. The vector f_1 points at the right side of the fuselage, f_2 points forward through the helicopter nose, f_3 points upwards. Although the theory used on the helicopter should be used in inertia system, yet the moments and products of the inertia of the helicopter changes with time, which make it almost impractical to analyze. While in the body-fixed frame, the moments and products of inertia are constant. Therefore, the probably analysis method could be derived equations of motion with respect to the body-fixed frame.

To derivate the equations of motion, Newton's second law are applied and the four assumptions are proposed as follows:

• The body is rigid and symmetrical;
• The centre of mass and the body fixed frame origin coincide; this is to ensure the resulting motions are pure rotations;
• The propellers are rigid, i.e. no blade flapping occurs.
• The operating range for pitch and roll are $\pm 50^\circ$ and $\pm 180^\circ$ for the yaw.

Let τ^{B} denote the vector torque acted on the helicopter and the angular velocity ω^B of the fuselage, is represented in the body-fixed frame, $\omega^{\mathrm{B}}=\begin{bmatrix} p & q & r\end{bmatrix}^T$ and $\tau^{\mathrm{B}}=\begin{bmatrix}\tau_x^B & \tau_y^B & \tau_z^B\end{bmatrix}$ are the coordinates with respect to the body-fixed frame. From [28, 29], the angular momentum of the body-fixed reference frame are given by $H^B=I\omega^B$, where I denotes the inertia matrix:

$$I=\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz}\end{bmatrix} \tag{1}$$

According to the assumption, the principal axes coincide with the axes of the body-fixed frame, therefore:

$$\begin{aligned} I_{xy} &= I_{yx} = 0 \\ I_{yz} &= I_{zy} = 0 \\ I_{xz} &= I_{zx} = 0 \end{aligned} \tag{2}$$

Based on the Newton's second law for the rotational motion i.e. the external moments acting on the rigid body are equal to the time rate of change of the angular momentum with respect to the inertial reference frame.

$$\vec{\tau}=\left.\frac{d\vec{H}}{dt}\right|_I \tag{3}$$

Equation (3) can be describe in body frame as

$$\left.\frac{d\vec{H}}{dt}\right|_I=\left.\frac{d\vec{H}}{dt}\right|_B+\vec{\omega}\times\vec{H} \tag{4}$$

The analytic expression of Newton's second law for the rotational motion of the four-rotor helicopter is:

$$\begin{aligned}\tau_x &= I_{xx}\dot{p}+qr(I_{zz}-I_{yy})\\ \tau_y &= I_{yy}\dot{q}+pr(I_{xx}-I_{zz})\\ \tau_z &= I_{zz}\dot{r}+pq(I_{yy}-I_{xx})\end{aligned} \tag{5}$$

i.e

$$\begin{aligned}\dot{p}&=-qr(I_{zz}-I_{yy})/I_{xx}+\tau_x/I_{xx}\\ \dot{q}&=-pr(I_{xx}-I_{zz})/I_{yy}+\tau_y I_{yy}\\ \dot{r}&=-pq(I_{yy}-I_{xx})/I_{zz}+\tau_z/I_{zz}\end{aligned} \tag{6}$$

Remarks: The dynamics of the equation can be got from above derivation. Although there is not much couple in inputs and outputs directly, there still many nonlinear states couple parts can be seen in the equation.

3.2 kinematics of the helicopter

The orientation motion of the four-rotor helicopter is that of the body-frame relative to the inertial frame. The angular velocity has been provided in equation (6), while the helicopter's orientation can be reach by the commutation of the frames.

Let $F_1=\left\{O_B \quad \vec{i}_1 \quad \vec{j}_1 \quad \vec{k}_1\right\}$ is an intermediate frame that is aligned with N frame and centered on the CG of the four-rotor helicopter. The helicopter orientation may be obtained by performing three consecutive rotations of F_1 until it is aligned with F_B.There are several kinds of three consecutive rotation method. Here the $Z-Y-X$ Euler angles, denoted $\Theta=\begin{bmatrix}\phi & \theta & \psi\end{bmatrix}^T$ are adopted. Firstly, rotate an angle ψ about $\vec{k}_1$ in F_1 frame, the rotation moves the helicopter to the direction defined by $F_2=\left\{O_B \quad \vec{i}_2 \quad \vec{j}_2 \quad \vec{k}_2\right\}$,where k_2 is the same that k_1.Secondly, rotate of an angle θ about $\vec{j}_2$.This rotation moves the helicopter to the direction described by $F_3=\left\{o_B \quad \vec{i}_3 \quad \vec{j}_3 \quad \vec{k}_3\right\}$,where $\vec{i}_3$ is aligning with i_B.Rotate an angle ϕ can make F_3 the frame the final orientation frame F_B.

With these rotations, thus the angular velocity can be expressed as follows:

$$\vec{\omega}=\frac{d\hat{n}}{dt}=\dot{\phi}\vec{i}_B+\dot{\theta}\vec{j}_2+\dot{\psi}\vec{k}_1 \tag{7}$$

On the other hand, the angular velocity can be represented by coordinates:

$$\vec{\omega}=p\vec{i}_B+q\vec{j}_B+r\vec{k}_B \tag{8}$$

Expand the equation (7), and equate the right hand sides of (7) and (8), the following equation can be got:

$$\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}\dot{\phi}\\0\\0\end{bmatrix}+\begin{bmatrix}1&0&0\\0&\cos\phi&\sin\phi\\0&-\sin\phi&\cos\phi\end{bmatrix}\begin{bmatrix}0\\\dot{\theta}\\0\end{bmatrix}+ \\ \begin{bmatrix}1&0&0\\0&\cos\phi&\sin\phi\\0&-\sin\phi&\cos\phi\end{bmatrix}\begin{bmatrix}\cos\theta&0&-\sin\theta\\0&1&0\\\sin\theta&0&\cos\theta\end{bmatrix}\begin{bmatrix}0\\0\\\dot{\psi}\end{bmatrix} \tag{9}$$

Then

$$\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}1&0&-\sin\theta\\0&\cos\phi&\sin\phi\cos\theta\\0&-\sin\phi&\cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}\dot{\phi}\\\dot{\theta}\\\dot{\psi}\end{bmatrix} \tag{10}$$

Then, the kinematics equation of the four-rotor helicopter can be represented as:

$$\begin{bmatrix}\dot{\phi}\\\dot{\theta}\\\dot{\psi}\end{bmatrix}=\begin{bmatrix}1&\sin\phi\tan\theta&\cos\phi\tan\theta\\0&\cos\phi&-\sin\phi\\0&\sin\phi/\cos\theta&\cos\phi/\cos\theta\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix} \tag{11}$$

From equation (6) and (11), the dynamics and kinematics equation can be derived.

Remarks: From the dynamics and kinematics model of the four-rotor helicopter described above, it can be seen that it is a strong nonlinear and coupling system, thus it is difficult to control it directly. In order to control this nonlinear model, nonlinear control method, the linear feedback controller, could be the most suitable way to be adopted.

4 Feedback linearization control theory

Consider a non-affine nonlinear system [34]:

$$\begin{cases} \dot{x} = a(x) + b_1(x)u_1 + \cdots + b_m u_m \\ y_1 = h_1(x) \\ \vdots \\ y_m = h_m(x) \end{cases} \quad (12)$$

In which, $x = [x_1 \quad x_2 \quad \cdots \quad x_n]^{\mathrm{T}} \in R^n$ is the state vector of the system; $a(x) = \begin{bmatrix} a_1(x) & a_2(x) & \cdots & a_n(x) \end{bmatrix}^T$ and $b(x) = \begin{bmatrix} b_{j1} & b_{j2} & \cdots & b_{jn} \end{bmatrix}^T$ are the function vectors of the state variables; $u, y \in R^m$ is the input and output vector respectively, and $h(x)$ is the smooth function vector.

Define $L_{bj} h_i(x) = \dfrac{\partial h_i}{\partial x} \bullet b_j$ is Lie derivative of h_i relative to the vector b_j.

Then

$$L_{\mathrm{j}}^{v} \bullet h_i(x) = L_a \left[L_{\mathrm{a}}^{v-1} \bullet h_i(x) \right] \quad (13)$$

Theorem [34-36]: Given $x_0 \in R^n$, if there exists the domain V of x_0 and the integer vector $\begin{bmatrix} r_1 & r_2 & \cdots & r_n \end{bmatrix}$, where

(1) $L_{bj} \bullet L_a^k h_i(x) = 0, \forall x \in V, 1 \le j \le m, 1 \le i \le m,$ $0 \le k \le r_i - 2$

(2) The matrix

$$g(x) = \begin{bmatrix} L_{b1} L_a^{r_1 - 1} h_1(x) & \cdots & L_{bm} L_a^{r_1 - 1} h_1(x) \\ L_{b1} L_a^{r_2 - 1} h_2(x) & \cdots & L_{bm} L_a^{r_2 - 1} h_2(x) \\ \vdots & & \vdots \\ L_{b1} L_a^{r_m - 1} h_m(x) & \cdots & L_{bm} L_a^{r_m - 1} h_m(x) \end{bmatrix}$$

is nonsingular, $\forall x \in V$, the relative degree vector of the system is $\begin{bmatrix} r_1 & r_2 & \cdots & r_m \end{bmatrix}$.

If the system has the relative degree vector $\begin{bmatrix} r_1 & r_2 & \cdots & r_m \end{bmatrix}$, differentiate y_i.

Then

$$\dot{y}_i = \frac{\partial h_i}{\partial x} \left[a(x) + b(x)u \right] = L_a h_i(x) + \sum_{j=1}^{m} L_{bj} h_i(x) u_j \qquad 1 \le i \le m \quad (14)$$

If $r_i > 0$, then $L_{bj} h_i(x) = 0$, $\forall j (1 \le j \le m)$, then the following equation can be got:

$$\begin{aligned} \dot{y}_i &= L_a h_i(x) \\ y_i^{(2)} &= \frac{\partial}{\partial x} [L_a h_i(x)][a(x) + b(x)u] \\ &= L_a^2 h_i(x) + \sum_{j=1}^{m} L_{bj} L_a h_i(x) u_j \end{aligned} \quad (15)$$

If $r_i > 2$, then $L_{bj} L_a h_i(x) = 0$, $\forall j (1 \le j \le m)$, Then

$$y_i^{(2)} = L_a^2 h_i(x) \quad (16)$$

Then

$$\begin{aligned} y_i^{(r_i - 1)} &= L_a^{r_i - 1} h_i(x) \\ y_i^{r_i} &= L_a^{r_i} h_i(x) + \sum_{j=1}^{m} L_{bj} L_a^{r_i - 1} h_i(x) u_j \end{aligned} \quad (17)$$

Then the system can be rearranged as follows:

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_a^{r_1} h_1 \\ L_a^{r_2} h_2 \\ \vdots \\ L_a^{r_m} h_m \end{bmatrix} + \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_m(x) \end{bmatrix} u = f(x) + g(x)u \quad (18)$$

For the relative degree vector has been decided, $g(x)$ is nonsingular.

Let

$$u = g(x)^{-1} \left[-f(x) + v \right] \quad (19)$$

Then the system can be simplified as

$$y_i^{(r_i)} = v_i, \quad i = 1, 2, \cdots, m \quad (20)$$

If consider v as a new input, the original nonlinear system can be linearized into a linear system, with the characteristics of Hurwitz polynomials, the system can be easily controlled.

5 Feedback linearization attitude controller design of the four-rotor helicopter

According to the attitude kinematics and dynamics model of the four-rotor helicopter derived in section 3 and the control method described in section 4, the controller of the four-rotor helicopter can be designed.

Firstly, the system model (6) and (11) can be re-described as:

$$\dot{x}_m = f_m(x) + B_m u_m \quad (21)$$

$$y_m = h_m(x_m) = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T \quad (22)$$

Where $u_m = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$.

$$f_m(x) = \begin{bmatrix} p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\cos\phi - r\sin\phi \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \\ qr(I_{yy} - I_{zz})/I_{xx} \\ pr(I_{zz} - I_{xx})/I_{yy} \\ pq(I_{xx} - I_{yy})/I_{zz} \end{bmatrix} \quad (23)$$

$$B_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/I_{xx} & 0 & 0 \\ 0 & 1/I_{yy} & 0 \\ 0 & 0 & 1/I_{zz} \end{bmatrix} \quad (24)$$

According to the theory in last section, the system can be reached as:

$$\ddot{y} = L_{fm}^2 h_m(x) + L_{Bm} L_{fm} h_m(x) u_m \quad (25)$$

The complicated nonlinear system can be decoupled, and then the common PD controller can be designed as:

$$u_m = [L_{Bm} L_{fm} h_m(x)]^{-1} [\ddot{y}_d - L_{fm}^2 h_m(x) + k_1 e_m + k_2 \dot{e}_m] \quad (26)$$

The control parameters in equation (26) can be designed by using the characteristics of Hurwitz polynomials. The whole control structure can be shown in figure 2.

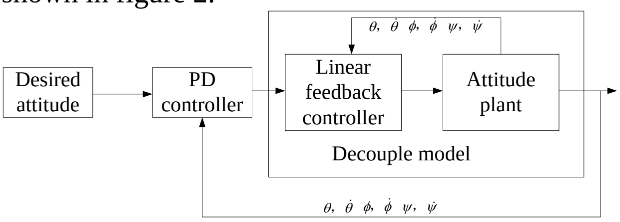


Fig. 2 The control structure

6 The simulation and experiment

Before the simulation, let assume that a 3-DOF model must fully replicate the rotational motion of the four-rotor helicopter. The translational degree of freedoms (DOFs) is removed. To illustrate the performance of the obtained control law and assess its effectiveness, we carried out two numerical experiments described below. The parameters of the corresponding controller, u_m, were fixed as:

$$k_1 = 100, k_2 = 2 \quad (27)$$

6.1 Point-to-point motion

The four-rotor helicopter must reach a desired goal configuration starting from a given initial configuration. The attitude of the four-rotor helicopter must reach and follow an attitude starting from a given initial configuration, and this can ensure the helicopter stay or change to the desire attitude. The angles were initialized as $\left[0.2, 0.1, 0.2\right]$ .Therefore the rotorcraft was not completely horizontal.

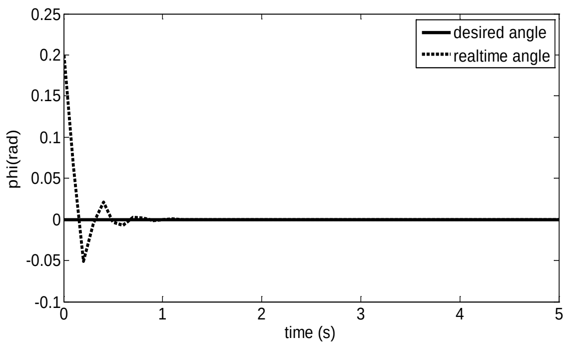
The mass of the helicopter is:

$$m = 0.4995\,\mathrm{Kg} \quad (28)$$

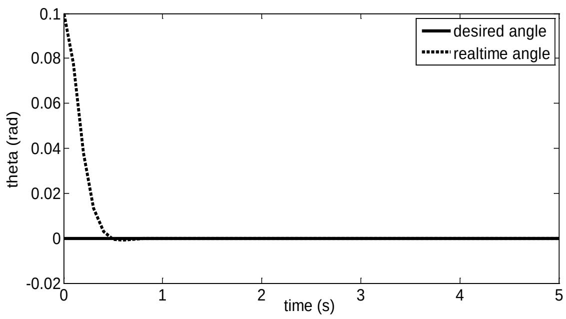
The moment of inertia is

$$I_{xx} = 0.254 kg \bullet m^2$$
$$I_{yy} = 0.254 kg \bullet m^2 \quad (29)$$
$$I_{zz} = 2 * 0.254 kg \bullet m^2$$

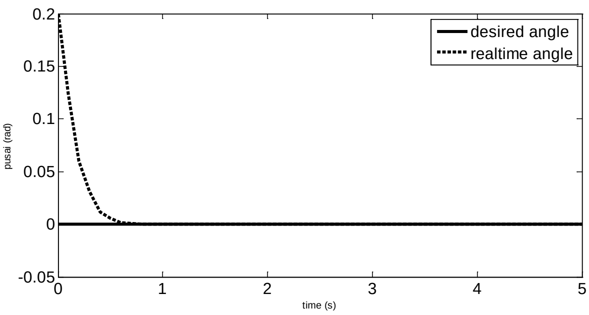
The tracking position coordinates are always stayed in the original position with mechanical equipment. The behavior of angle θ , ϕ and ψ are shown in figure 3 , the errors between the real-time angles and desired angles are shown in figure 4 and their corresponding torques can be seen in figure 5.From these figures we can see that the controller can effectively make the system reach the desired attitude. The angular attitude error set to zero about 1s elapsed, while the input torque errors are stabilized a few second before. Afterwards, the control action reaches its steady state solution.



a. the change process of pitch angle

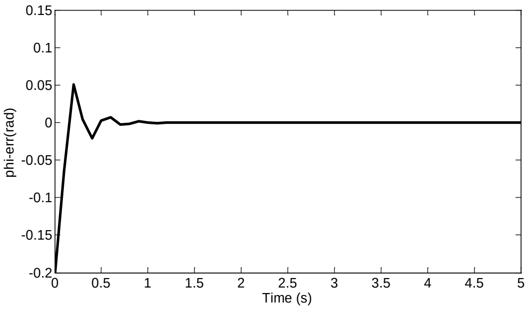


b. the change process of roll angle

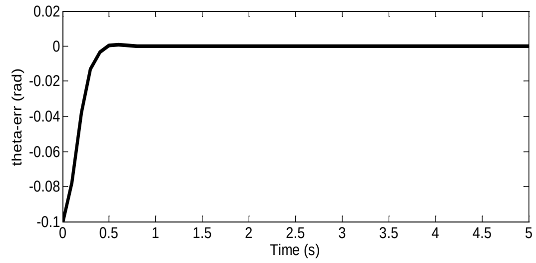


c. the change process of yaw angle

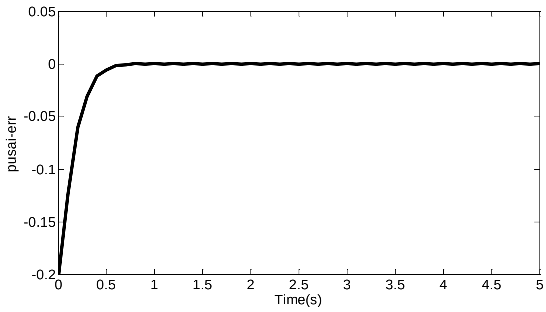
Fig.3 The change process of the attitude of the four-rotor helicopter



a. the error change process of pitch angle

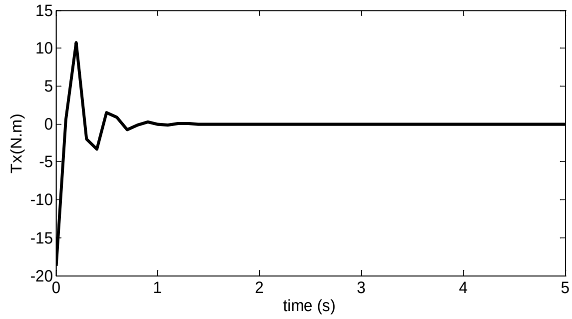


b. the error change process of roll angle

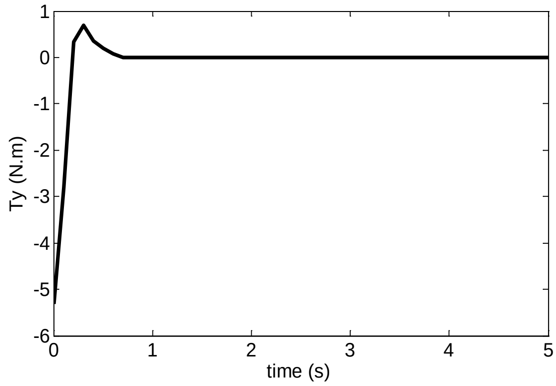


c. the error change process of yaw angle

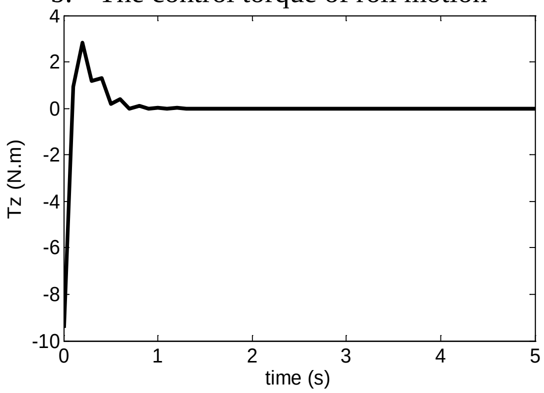
Fig.4 The error change process of the attitude of the four-rotor helicopter



a. The control torque of pitch motion



b. The control torque of roll motion



c. the control torque of yaw angle

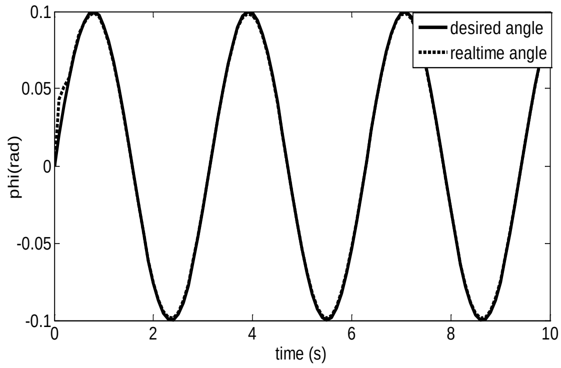
Fig.5 The change torque of the four-rotor helicopter

6.2 Trajectory-tracking:

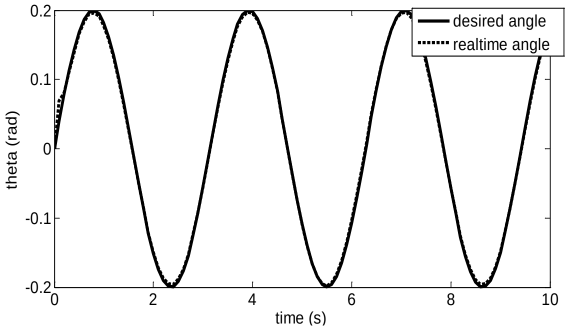
A more challenging task was selected for the second experiment, which was used to simulate the high frequency vibration in the real flight, where a helicoidal motion was selected as the reference attitude. These situations much more coincide with real-time flight, because the attitude will react more quickly than the position. This reference was defined as:

$$\begin{aligned} \phi_d &= 0.1\sin(2t) \\ \theta_d &= 0.2\sin(2t) \\ \psi_d &= 1\sin(t) \end{aligned} \tag{30}$$

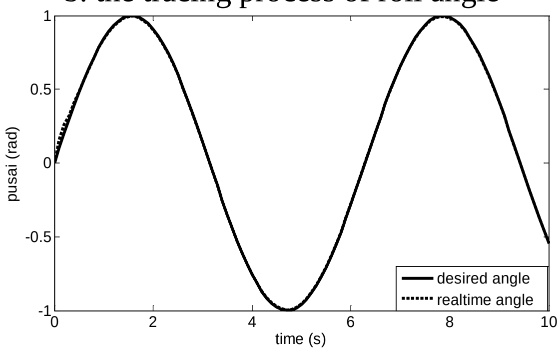
Initially, the four-rotor helicopter was placed in the completely horizontal position, that is $[\phi \quad \theta \quad \psi]=[0 \quad 0 \quad 0]$.Figures 6 shows how the system follows the helicoidal attitude of θ, ϕ and ψ.The corresponding torques τ_x, τ_y and τ_z are shown in figure 8. In figure 6 the solid line is the desired angle while the dash line is real-time angle, it can be seen that the real-time attitude can track the desired one quickly and the error in between is very small, which can be seen in figure 7 . As we can see from these figures, the four-rotor helicopter is able to effectively track the helicoidal attitude reference.



a. the tracing process of pitch angle

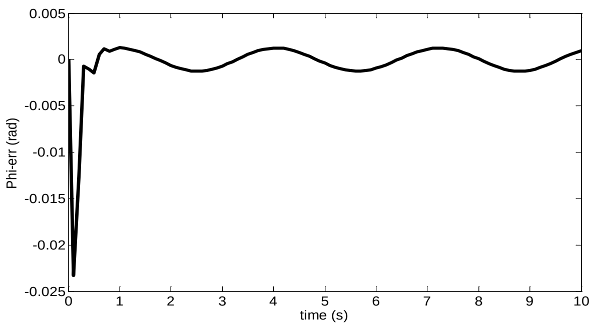


b. the tracing process of roll angle

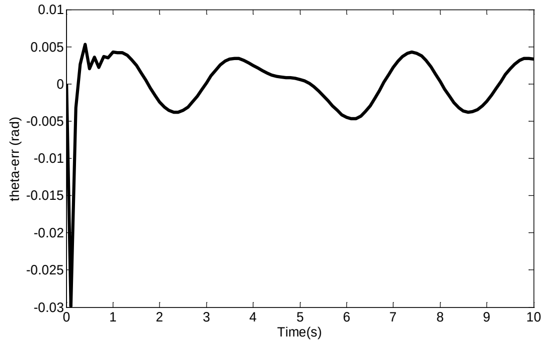


c. the tracing process of yaw angle

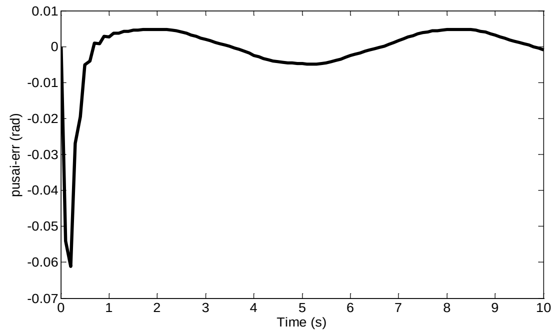
Fig.6 The tracing process of the attitude of the four-rotor helicopter with helicoidal reference.



a. the tracing error of pitch angle

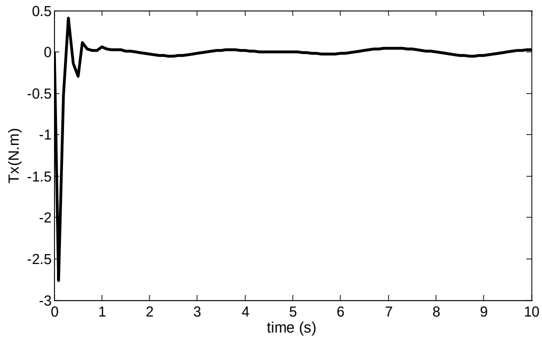


b. the tracing error of roll angle

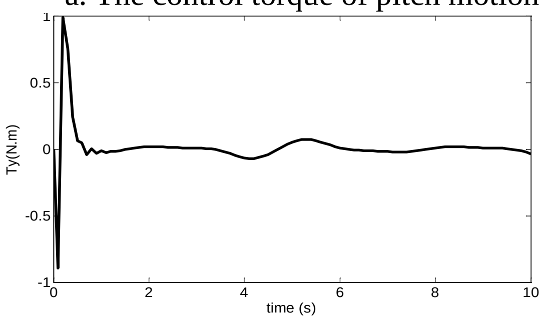


c. the tracing error of yaw angle

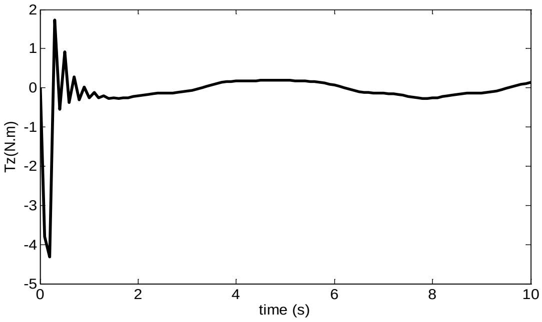
Fig.7 The tracing errors change process of the attitude of the four-rotor helicopter with helicoidal reference.



a. The control torque of pitch motion



b. The control torque of roll motion



c the control torque of yaw angle

Fig.8 The change torque of the four-rotor helicopter when tracing the helicoidal reference

7. Conclusion

The four rotor helicopter is a nonlinear strong coupling system, and it is difficult to control. The attitude dynamics and kinematics model is more complicated when comparing with the translational one and is the key problem in controlling helicopters. In order to solve the attitude control problem of the four-rotor helicopter, in this paper, two reference frames were established to derive the kinematics of the four-rotor helicopter in detail and Newton's second law was used to derive the dynamics of the helicopter. It was found that these two equations were very complicated. In order to control this complex attitude of the four-rotor helicopter, the feedback linearization control theory has been introduced to decouple the attitude of the helicopter. In order to assess the effectiveness of the proposed controller, two control tasks were numerically performed. From the corresponding simulation results we claim that the proposed control strategy behaves remarkably well.

This work also built the foundation for the position control of the whole helicopter. This feedback linearization controller is based on the accurate model of the four-rotor helicopter. When the coefficients of the four-rotor helicopter are unknown, the intelligent controller in reference [37] might be the suitable candidate to indentify the parameters of the four-rotor helicopter when using the controller provided for its fast calculation speed online.

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