Multiple observer design for a nonlinear Takagi-Sugeno system submitted to unknown inputs and outputs

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Abstract: - In this paper we focus on the state estimation of a nonlinear system described by a Takagi-Sugeno multiple model submitted to unknown inputs and outputs. The proposed approach consists on a mathematical transformation which enables to consider the unknown outputs as unknown inputs that can be eliminated by a designed multiple observer. To evaluate the efficiency of the proposed approach, the convergence conditions of the state estimation error are formulated as linear matrix inequalities (LMI). Simulation Examples are given to illustrate the proposed methods.

Key-Words: - Multiple model approach, multiple observer, state estimation, unknown inputs and outputs.

1 Introduction

The state estimation of a system is an analytical source of redundancy which makes it possible to generate failure symptoms of the system by making a comparison between the real signals of the system and the estimated signals. A not desired variation between these signals indicates the possible presence of defects affecting the operation of the system. The generation of the residuals is based on the use of the state observers.

An observer is generally a dynamical system allowing the state reconstruction from the system model and the measurements of its inputs and outputs [28].

A physical process is often subject to disturbances, which can result from fault of actuators and/or sensors and which may have harmful effects on the normal behavior of the process. The disturbances are called unknown inputs when they affect the input of the system and their presence can complicate the state estimation. Various studies dealing with the presence of unknown inputs acting on the system have been proposed in the literature [3, 12, 19, 34].

Over the past decades, many researchers have paid attention to the problem of state estimation of dynamic linear systems subjected to both known and unknown inputs [11, 8, 33, 36]. They can be classified in two categories [4]. The first one supposes an a priori knowledge of information on these non measurable inputs. The second category proceeds either by estimation of the unknown inputs, or by their complete elimination from the system equations [16].

However, the physical systems are often nonlinear. As it is difficult to synthesize an observer for an unspecified nonlinear system [1], the multiple model approach constitutes a tool which is largely used in the modeling of nonlinear systems [10, 29]. The multiple model approach consists in representing the nonlinear system by an interpolation of different local linear models. Each one is a linear time invariant system valid in a particular zone of operation. The global model is the sum of the local models weighted by respective weighting functions [24]. These weighting functions quantify the relative contribution of each local model to the global model according to the corresponding operating point of the system [30]. The Takagi-Sugeno (T-S) structure is the most used in the multiple model approach [18, 20, 22, 31]. The main advantage of T-S structure is its simplicity as it results from the interpolation between linear systems and therefore the analysis and the design methods developed for linear systems can be generalized to nonlinear systems [17].

The state estimate of the multiple model is carried out in general by using a proportional multiple observer. However, other types of multiple observers were developed (for example with unknown inputs [6, 7, 21]).

In this paper, the proposed technique for state estimation consists in associating to each local model a local unknown input observer. The considered observer is then a convex interpolation of these local observers. This interpolation is obtained throughout the same activation functions as the T-S model [2]. Our contribution lies in the design of unknown input multiple observers for Takagi-Sugeno multiple model subject to unknown inputs and outputs.

In the case of a nonlinear system affected by unknown inputs and described by a multiple model, a technique for multiple model state estimation by using a multiple observer has already been proposed [6, 26, 27]. In order to extend the use of this unknown input multiple observer in the case of systems subjected to unknown inputs and outputs, a mathematical transformation, which is proposed in the linear system case in [12] is used allowing us to consider unknown outputs as unknown inputs.

The paper is organized as follows. Section 2 presents an overview of the multiple model approach. In section 3, the multiple observer of a system with unknown inputs is presented. Section 4 presents the main results concerning the synthesis of an observer to estimate the state of linear system in the first part and the state of nonlinear system submitted to unknown outputs in the second part. Numerical example is given in section 5 to illustrate the result. The design of a multiple observer with unknown inputs and outputs is studied is section 6. Simulation example is given in section 7 to show the validity of the proposed method.

2 On the multiple model approach

The principle of the multiple model approach is based on the reduction of the system complexity by the decomposition of its operation space in a finite number of operation zones, each one is characterized by a local model or sub-model that describes the system behavior around an operation point. The behavior of the nonlinear system is obtained by the sum of those of the local models each weighted by its respective weighting function.

Two main structures of multiple models, uncoupled structure and coupled one can be distinguished according to nature of the coupling between local models [31, 14]. The coupled structure or the Takagi-Sugeno structure provides a useful tool to represent with a good precision a large class of nonlinear systems [35].

The multiple model representation is given by:

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[A_i\ x_m(t) + B_i\ u(t) + D_i\right] \\ y_m(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[C_i\ x_m(t) + E_i\ u(t) + N_i\right] \end{cases} \quad (1)$$

where $\mu_i(\xi(t))$ are the activation functions and $\xi(t)$ is the decision vector which may depend on the known input and/or the measured state variables.

If the output $y_m(t)$ is linear, i.e. ($C_1 = C_2 = \cdots = C_M = C$), the structure of the Takagi-Sugeno multiple model becomes:

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[A_i\ x(t) + B_i\ u(t)\right] \\ y(t) = C\, x(t) \end{cases} \quad (2)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^p$ represents the measured output. $A_i(t) \in R^{n*n}$ is the state matrix, $B_i \in R^{n*m}$ is the input matrix and $C \in R^{p*n}$ is the output matrix of the system. x(t) represents the vector of decision depending on the input. M is the number of local models, it depends on the precision of desired modelling, the complexity of the nonlinear system and the choice of the structure of the weighting functions.

The normalized weighting functions are nonlinear and depend on the decision variable x (t). They satisfy the convexity property:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \le \mu_i(\xi(t)) \le 1 \quad (3)$$

3 On multiple observer with unknown inputs.

In this part, one considers the state estimation of a nonlinear system perturbed by unknown inputs. The structure of that observer results of the aggregation of local observers [10]. The design of this multiple observer is based on the elimination of these unknown inputs.

3.1 Principle of the state reconstruction

Consider a nonlinear system represented by the following Takagi-Sugeno multiple model with unknown inputs:

$$\begin{cases} \dot{x}_m(t)= \sum_{i=1}^{M} \mu_i(\xi(t)) \left[A_i\ x(t) + \right. \\ \qquad\qquad \left. + B_i\ u(t) + R\,\bar{u}(t)\right] \\ y(t)= C\,x(t) \end{cases} \tag{4}$$

where $\bar{u}(t) \in R^q$, $q < n$ is the vector of unknown inputs and R is the matrix summarizing the influence of the unknown inputs.

The proposed observer for the multiple model (4) is a linear combination of local observers. It is described as follows [5]:

$$\begin{cases} \dot{z}(t)= \sum_{i=1}^{M} \mu_i(\xi(t)) \left[N_i\ z(t) + \right. \\ \qquad \left. + G_{i1}\ u(t) + G_{i2} + L_i\ y(t)\right] \\ \hat{x}(t)= z(t) - E\,y(t) \end{cases} \tag{5}$$

$N_i \in R^{n*n}$, $G_{i1} \in R^{n*m}$, $L_i \in R^{n*p}$ is the gain of the ith local observer, $G_{i2} \in R^n$ is a constant vector and E is a matrix transformation. All these matrices or vectors have to be defined so that the reconstructed state converges asymptotically to the actual state x(t).

The state estimation error, e(t), is given as follows:

$$e(t) = x(t) - \hat{x}(t) = (I + EC)\,x(t) - z(t) \tag{6}$$

The dynamic evolution of e(t) is given by:

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[P\,(A_i\ x(t) + B_i\ u(t) + R\,\bar{u}(t)) \right. \\ \left. + P\,D_i - N_i\ z(t) - G_{i1}\ u(t) - G_{i2} - L_i\ y(t)\right] \tag{7}$$

where P = I + E C

Replacing y(t) by its expression, expression (7) can be written as:

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[N_i\ e(t) + (P\,A_i - N_i - K_i\ C)\,x(t) \right. \\ \left. + (P\,B_i - G_{i1})\,u(t) + (P\,D_{i1} - G_{i2}) + PR\,\bar{u}(t)\right] \tag{8}$$

If the following conditions are fulfilled [5, 13]:

$$\begin{cases} P\,R = 0 \\ P = I+E\,C \\ N_i = P\,A_i - K_i\ C \\ L_i = K_i - N_i\ E \\ G_{i1} = P\,B_i \\ G_{i2} = P\,D_i \\ \sum_{i=1}^{M} \mu_i(\xi(t))\ N_i \text{stable} \end{cases} \tag{9}$$

Equation (7) reduces to :

$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\ N_i e(t) \tag{10}$$

It is important to note that the stability of matrices $\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\ N_i e(t)$ does not guarantee the stability of the matrix $\sum_{i=1}^{M} \mu_i(\xi(t))\ N_i$ [5].

3.2 Global convergence of the multiple observer

The state estimation error between the multiple model (4) and the unknown input multiple observer (5) converges towards zero, if all the pairs (A_i, C) are observable and if the following conditions are checked $\forall\ i \in \{1, ..., M\}$ [5]:

$$\begin{cases} N_i^T\ X + X\,N_i \langle 0 \\ N_i = P\,A_i - K_i\ C \\ P = I + E\,C \\ P\,R = 0 \\ L_i = K_i - N_i\ E \\ G_{i1} = P\,B_i \\ G_{i2} = P\,D_i \end{cases} \tag{11}$$

where $X \in R^{n*n}$ is a positive definite symmetric matrix.

4 Multiple observer of a system with unknown outputs

This section is devoted to the synthesis of a multiple observer with unknown outputs. A mathematical transformation is used to consider unknown outputs as unknown inputs of an augmented system. In so doing, a multiple observer based on the elimination of these unknown inputs is designed.

4.1 Linear system case

Consider the linear model affected by a sensor fault described by:

$$\begin{cases} \dot{x}(t)=A\,x(t)+B\,u(t) \\ y(t)=C\,x(t)+D\,\bar{u}(t) \end{cases} \qquad (12)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ represents the input vector, $y(t) \in R^p$ is the measured output, $\bar{u}(t) \in R^r$ represents the sensor fault. A, B, C and D are known constant matrices with appropriate dimensions. D is of full column rank.

Consider a new state z(t) [12, 35] which satisfies

$$\dot{z}(t) = -A\,z(t) + \bar{A}\,C\,x(t) + \bar{A}\,D\,\bar{u}(t) \qquad (13)$$

where $\bar{A}$ is a p-dimensional stable matrix.

Let's consider the augmented system

$$X(t) = \begin{bmatrix} x(t)^T & z(t)^T \end{bmatrix}^T \qquad (14)$$

Which can be modelled as:

$$\begin{cases} \dot{X}(t)= A_a\,X(t) + B_a\,u(t) + D_a\,\bar{u}(t) \\ Y(t) = C_a X(t) \end{cases} \qquad (15)$$

where $A_a \in R^{(n+p)*(n+p)}$, $B_a \in R^{(n+p)*m}$, $D_a \in R^{(n+p)*r}$ and $C_a \in R^{p*(n+p)}$.

These matrices are described as follows:

$$A_a = \begin{bmatrix} A & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_a = \begin{bmatrix} C & 0 \end{bmatrix} \text{ and } D_a = \begin{bmatrix} 0 \\ \bar{A}D \end{bmatrix}$$

Sensor fault of (12) appears as an actuator fault of the augmented system (15).

4.2 Nonlinear system case

The objective is to extend the method described in section (IV.1) to a nonlinear system represented by multiple model as:

$$\begin{cases} \dot{x}(t)= \sum_{i=1}^{M} \mu_i(\xi(t)) \left[A_i\,x(t) + B_i\,u(t)\right] \\ y(t)= C\,x(t) + B_i\,u(t) \end{cases} \qquad (16)$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$ denote respectively the state, the input and the output vector, $\bar{u} \in R^r$ represents the sensor fault. A_i, B_i, C et D are known constant matrices with appropriate dimensions. The matrix D is of full column rank and M represents the number of local models.

Using the property given by (3), z(t) defined in (13) can be rewritten:

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[-\bar{A}\,z(t) + \bar{A}\,C\,x(t) + \bar{A}\,D\,\bar{u}(t)\right] \qquad (17)$$

Then the augmented system X(t) given by (14) can be expressed as:

$$\begin{cases} \dot{X}(t)= \sum_{i=1}^{M} \mu_i(\xi(t)) \left[A_{ai}\,X(t) + B_{ai}\,u(t) + D_a\,\bar{u}(t)\right] \\ Y(t) = C_a\,X(t) \end{cases} \qquad (18)$$

where

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix}, \quad B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$

$$C_a = \begin{bmatrix} C & I \end{bmatrix} \text{ and } D_a = \begin{bmatrix} 0 \\ \bar{A}D \end{bmatrix}.$$

From the obtained results, sensor fault of the system (16) appears as an actuator fault of the augmented system (18). In so doing, fault estimation strategy is similar to the method of conception of unknown inputs.

The structure of the multiple observer is chosen as follows:

$$\begin{cases} \dot{Z}(t)= \sum_{i=1}^{M} \mu_i(\xi(t)) \left[N_i\ Z(t) + \right. \\ \qquad \left. + u(t) + G_{i2} + L_i\ Y(t)\right] \\ \hat{X}(t) = Z(t) - E\ Y(t) \end{cases} \tag{19}$$

where $\hat{X}(t) \in R^n$ is the state vector, $Y(t) \in R^p$ is the measured output. N_i, G_{i1}, L_i is the gain of the local observer, $G_{i2} \in R^n$ is a constant vector and E a matrix of transformation.

The augmented state estimation error is given by:

$$\tilde{X}(t) = X(t) - \hat{X}(t) \tag{20}$$

Using (19), we have:

$$\begin{aligned} \tilde{X}(t) &= (I + EC_a)\ X(t) - Z(t) \\ &= P\ X(t) - Z(t) \end{aligned} \tag{21}$$

with: $P = I + E\ C_a$.

The dynamic of the augmented state estimation error is given as follows:

$$\dot{\tilde{X}}(t) = \dot{X}(t) - \dot{\hat{X}}(t) \tag{22}$$

that can be expressed as:

$$\begin{aligned} \dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) &\left[P\ (A_{ai}\ X(t) + \right. \\ &+ B_{ai}\ u(t) + D_a\ \bar{u}(t) - N_i\ Z(t) - \\ &\left. - G_{i1}\ u(t) - G_{i2} - L_i\ Y(t)\right] \end{aligned} \tag{23}$$

Using the expressions of Z(t) and Y(t) given respectively by (19) and (18), the equation (23) becomes:

$$\begin{aligned} \dot{\tilde{X}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) &\left[N_i\ \tilde{X}(t) + \right. \\ &+ (P\ A_{ai} - N_i)\ X(t) - K_i\ C_a\ X(t) + \\ &\left. + (P\ B_{ai} - G_{i1})\ u(t) - G_{i2} + P\ D_a\ \bar{u}(t)\right] \end{aligned} \tag{24}$$

with: $K_i = N_i\ E + L_i$.

If the following conditions are satisfied:

$$\begin{cases} P\ D_a = 0 \\ P = I + E\ C_a \\ N_i = P\ A_{ai} - K_i\ C_a \\ L_i = K_i - N_i\ E \\ G_{i1} = P\ B_{ai} \\ G_{i2} = 0 \\ \sum_{i=1}^{M} \mu_i(\xi(t))\ N_i \text{ stable} \end{cases} \tag{25}$$

The reconstruction error of the augmented state tends asymptotically towards zero and (23) is reduced to:

$$\dot{\tilde{X}} = \sum_{i=1}^{M} \mu_i(\xi(t))\ N_i\ \tilde{X}(t)$$

4.3 Global convergence of the multiple observer

The augmented state estimate error converges towards zero, if all the pairs (A_{ai}, C_a) are observable, and if the following conditions are checked $\forall\ i \in \{1, ..., M\}$:

$$\begin{cases} N_i^T\ X + X\ N_i \langle 0 & \text{(26a)} \\ N_i = P\ A_{ai} - K_i\ C_a & \text{(26b)} \\ P = I + E\ C_a & \text{(26c)} \\ P\ D_a = 0 & \text{(26d)} \\ L_i = K_i - N_i\ E & \text{(26e)} \\ G_{i1} = P\ B_{ai} & \text{(26f)} \\ G_{i2} = 0 & \text{(26g)} \end{cases} \tag{26}$$

where $X \in R^{n*n}$ is a positive definite symmetric matrix. Using the expression (26b), the inequality (26a) can be written as:

$$(P\ A_{ai} - K_i\ C_a)^T\ X + X\ (P\ A_{ai} - K_i\ C_a) \langle 0,\ \forall\ i \in \{1, ..., M\} \tag{27}$$

The inequalities (27) are nonlinear with respect to the variables X and K_i. A numerical procedure of resolution by linearization is presented in the section below.

4.4 Method of resolution

Three steps are needed to resolve the system (26):

1. (26.c) and (26d) determines the matrix E of the multiple observer. One notes that $(C_a\ D_a)^{(-)}$ is the pseudo-inverse of $(C_a\ D_a)$:

$$E = -D_a\,(C_a\ D_a)^{(-)} \qquad (28)$$

The matrix P may be deduced from (26c):

$$P = I - D_a\,(C_a\ D_a)^{(-)}\,C_a \qquad (29)$$

2. Considering the following change of variable:

$$W_i = X\,K_i \qquad (30)$$

(27) is rewritten:

$$(P\,A_{ai})^T + X\,(P\,A_{ai}) - C_a^T\,W_i^T - W_i\,C_a \langle 0,\quad \forall\, i \in \{1, \ldots, M\} \qquad (31)$$

The inequalities (31) are of LMI type and the LMI Matlab Tool-box may be used for it resolution. Then, one deduces:

$$K_i = X^{-1}\,W \qquad (32)$$

3. The other matrices defining the observer are deduced knowing E, P and K_i:

$$\begin{cases} N_i = P\,A_{ai} - K_i\,C_a & (33a) \\ L_i = K_i - N_i\,E & (33b) \\ G_{i1} = P\,B_{ai} & (33c) \end{cases} \qquad (33)$$

5 Simulation example

Let's consider the multiple model, made up of two local models and involving two states and two outputs

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\left[A_i\,x(t) + B_i\,u(t)\right] \\ y(t) = C\,x(t) + D\,\bar{u}(t) \end{cases} \qquad (34)$$

The numerical values of matrices are defined as below:

$$A_1 = \begin{bmatrix} -0{,}4 & -1 \\ 0{,}4 & -0{,}1 \end{bmatrix},\ A_2 = \begin{bmatrix} -0{,}6 & -0{,}1 \\ 2 & -0{,}2 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0{,}1 \\ 0{,}2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0{,}15 \\ 0{,}35 \end{bmatrix}.$$

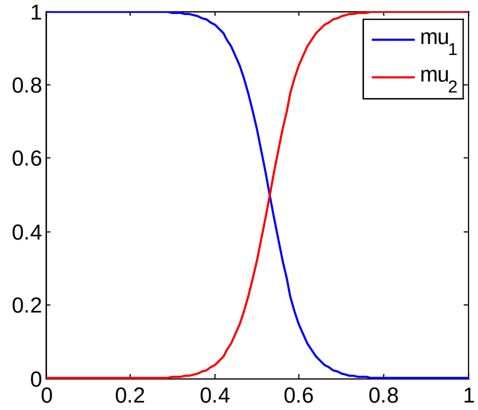

Figure 1: The weighting functions.

The decision vector depends on the system input. The system (34) was simulated using gaussian functions for the weighting functions μ_i. These functions are given in Figure (1).

The new state z(t) is:

$$\dot{z}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\left[-\bar{A}\,z(t) + \bar{A}\,C\,x(t) + \bar{A}\,D\,\bar{u}(t)\right] \qquad (35)$$

with $\bar{A} = 30 * I$ and the augmented system satisfies:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\left[A_{ai}\,X(t) + B_{ai}\,u(t) + D_a\,\bar{u}(t)\right] \\ Y(t) = C_a\,X(t) \end{cases} \qquad (36)$$

with:

$$A_{a1} = \begin{bmatrix} -0{,}4 & -1 & 0 & 0 \\ 0{,}4 & -0{,}1 & 0 & 0 \\ 30 & 30 & -30 & 0 \\ 0 & 30 & 0 & -30 \end{bmatrix},\ B_{a1} = B_{a2} = \begin{bmatrix} 0{,}1 \\ 0{,}2 \\ 0 \\ 0 \end{bmatrix},$$

$$A_{a2} = \begin{bmatrix} -0,6 & -0,1 & 0 & 0 \\ 2 & -0,2 & 0 & 0 \\ 30 & 30 & -30 & 0 \\ 0 & 30 & 0 & -30 \end{bmatrix}, \; D_a = \begin{bmatrix} 0 \\ 0 \\ 4,5 \\ 10,5 \end{bmatrix}$$

and $C_a = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

The structure of the multiple observer is:

$$\begin{cases} \dot{Z}(t) = \sum_{i=1}^{2} \mu_i(\xi(t)) \Big[N_i \, Z(t) + \\ \qquad + G_{i1} \, u(t) + L_i \, Y(t) \Big] \\ \hat{X}(t) = Z(t) - E\, Y(t) \end{cases} \tag{37}$$

The computation of the matrices of the multiple observer (37) yields:

$$N_1 = \begin{bmatrix} -38,67 & 165,65 & -38,27 & 204,92 \\ -12,15 & 156,53 & -12,55 & 169,18 \\ -74,80 & 168,65 & 0,20 & -96,32 \\ -325,15 & -390,35 & -10,15 & -130,88 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -39,40 & 168,31 & -38,80 & 207,21 \\ -11,54 & 158,88 & -13,54 & 172,62 \\ -76,77 & -171,47 & 1,38 & -98,52 \\ -331,01 & -396,23 & -8,66 & -134,14 \end{bmatrix}$$

$$L_1 = 10^3 * \begin{bmatrix} 1,03 & -0,21 \\ 0,87 & -0,17 \\ -0,47 & 0,10 \\ -0,54 & 0,10 \end{bmatrix}, \; G_{11} = G_{21} = \begin{bmatrix} 0,10 \\ 0,20 \\ -0,675 \\ 1,575 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 1,04 & -0,21 \\ 0,89 & -0,17 \\ -0,48 & 0,10 \\ -0,56 & 0,10 \end{bmatrix} \text{ and } E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -2,25 & 0 \\ -5,25 & 0 \end{bmatrix}.$$

The known input of the system (34) is given in Figure (2). The sensor fault is defined by $\overline{u}(t) = 0,1 \sin(2\pi t)$. The simulation results are represented in Figures (3) and (4). As for the previous linear case, the proposed method provides good estimates of the system state.

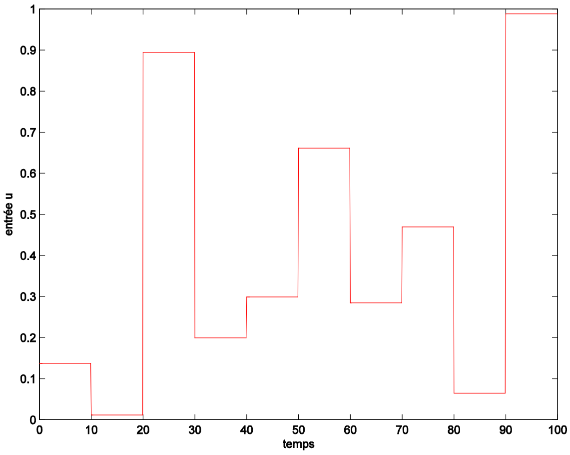

Figure 2: Known input u(t).

6 Multiple observer of a system with unknown inputs and outputs

In this section, one considers the estimation of the state vector of a nonlinear system represented by a Takagi-Sugeno multiple model and subject to the influence of unknown inputs and outputs, by using a multiple observer.

In the case of linear systems affected by unknown outputs, a mathematical transformation is used to consider these unknown outputs in the form of unknown inputs of an augmented system (described in section 6.1.). This result is extended to nonlinear systems represented by multiple model.

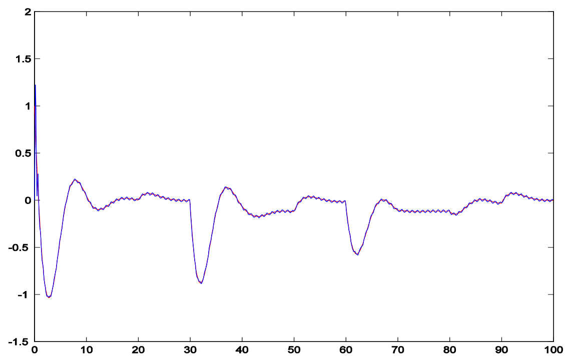

Figure 3: The state x_1 and its estimate.

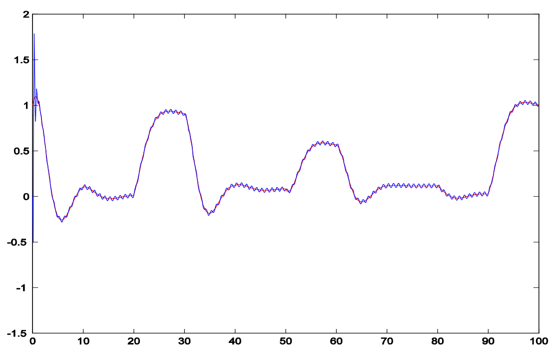

Figure 4: The state x_2 and its estimate.

6.1 Problem formulation

Consider the Takagi-Sugeno multiple model affected by an actuator fault and a sensor fault described by:

$$\begin{cases}\dot{x}(t)= \sum\limits_{i=1}^{M} \mu_i(\xi(t)) \left[A_i\ x(t) + B_i\ u(t) + R\ v(t) + d_i\right] \\ y(t)= C\ x(t) + D\ \bar{u}(t)\end{cases} \qquad (38)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ represents the input vector, $v(t) \in R^q$, $q < n$ is the vector of unknown inputs, $y(t)\, i \in R^p$ represents the measured output, $\bar{u}(t) \in R^r$ is the sensor fault. $A_i \in R^{n*n}$, $B_i \in R^{n*m}$, $R \in R^{n*q}$, $C \in R^{p*n}$, $D \in R^{p*r}$ are known constant matrices. The weighting functions $\mu_i(\xi(t))$ must satisfy the convexity conditions given by (3).

The state z(t) given in (13) becomes, using the property given by (3):

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \left[-\bar{A}\ z(t) + \bar{A}\ C\ x(t) + \bar{A}\ D\ \bar{u}(t)\right] \qquad (39)$$

The augmented system X(t) is given by:

$$\begin{cases}\dot{X}(t)= \sum\limits_{i=1}^{M} \mu_i(\xi(t)) \left[A_{ai}\ X(t) + B_{ai}\ u(t) + D_r\ \gamma(t) + D_i\right] \\ Y(t)= C_a\ X(t)\end{cases} \qquad (40)$$

Where

$$A_{a1} = \begin{bmatrix} A & 0 \\ \bar{A}\,C & -\bar{A}\end{bmatrix},\ B_{ai} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$C_a = \begin{bmatrix} C & 0 \end{bmatrix},\ D_r = \begin{bmatrix} R & 0 \\ 0 & \bar{A}\,D \end{bmatrix},$$

$$D_i = \begin{bmatrix} d_i \\ 0 \end{bmatrix},\ \text{and}\ \gamma(t) = \begin{bmatrix} v(t) \\ \bar{u}(t) \end{bmatrix}.$$

The sensor fault affected the system (38) appears as an actuator fault of the augmented system (40).

The multiple observer that estimates the state vector of the multiple model is described by:

$$\begin{cases}\dot{Z}(t)= \sum\limits_{i=1}^{M} \mu_i(\xi(t)) \left[N_i\ Z(t) + G_{i1}\ u(t) + G_{i2} + L_i\ Y(t)\right] \\ \hat{X}(t)= Z(t) - E\ Y(t)\end{cases} \qquad (41)$$

where $\hat{X} \in R^n$ represents the state estimate vector, $Y(t) \in R^p$ is the measured output. N_i, G_{i1}, L_i is the gain of the local observer, $G_{i2} \in R^n$ is a constant vector and E is a transformation matrix.

Let us consider the augmented state estimation error:

$$e_a(t) = X(t) - \hat{X}(t) \qquad (42)$$

The state estimation error dynamics is given by:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \Big[P\left[A_{ai}\ X(t) + B_{ai}\ u(t) + D_r\ \gamma(t) + D_i\right] - N_i\ Z(t) - G_{i1}\ u(t) - G_{i2} + L_i\ Y(t)\Big] \qquad (43)$$

Replacing Z(t) and Y(t) by their expressions given respectively by (41) and (40), the dynamic of the state estimation error becomes:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) \Big[N_i\ e(t) + (P\,B_{ai} - G_{i1})\ u(t) + \left(P_{ai}\ A_{ai} - N_i - K_i\ C_a\right) X(t) + P\,D_i - G_{i2} + P\,D_r\ \gamma(t)\Big] \qquad (44)$$

with : $K_i = N_i E + L_i$.

If the following conditions are satisfied:

$$\begin{cases} P D_r = 0 \\ P = I + E C_a \\ N_i = P A_{ai} - K_i C_a \\ L_i = K_i - N_i E \\ G_{i1} = P B_{ai} \\ G_{i2} = P D_i \\ \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e(t) \text{ stable} \end{cases} \qquad (45)$$

The equation (43) is reduced to:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e(t) \qquad (46)$$

Thus, the constraints (45) allow to synthesise the multiple observer of a system with unknown inputs and outputs.

6.2 Global convergence of the multiple observer

In this section, we will develop the sufficient conditions of the asymptotic global convergence of the state reconstruction.

(46) is globally asymptotically stable if there exists a positive definite symmetric matrix X, such that [7]:

$$N_i^T X + X N_i \langle 0 \qquad (47)$$

The design of the observer carries out to extract the following theorem:

6.2.1 Theorem:

The state estimation error converges towards zero, if all the pairs (A_{ai}, C_a) are observables and if the following conditions hold $\forall i \in \{1, ..., M\}$*:*

$$\begin{cases} N_i^T X + X N_i \langle 0 & (48a) \\ N_i = P A_{ai} - K_i C_a & (48b) \\ P = I + E C_a & (48c) \\ P D_r = 0 & (48d) \\ L_i = K_i - N_i E & (48e) \\ G_{i1} = P B_{ai} & (48f) \\ G_{i2} = P D_i & (48g) \end{cases} \qquad (48)$$

where $X \in R^{n*n}$ *is a positive definite symmetric matrix.*

Using (48b), the expression (48a) can be written as:

$$(P A_{ai} - K_i C_a)^T X + X (P A_{ai} - K_i C_a) \langle 0, \quad \forall i \in \{1, ..., M\} \qquad (49)$$

It is noted that the inequalities (49) are bilinear compared to variables X and Ki. To be reduced to the case of a linear problem, changes of variables are used.

6.3 Method of resolution

In order to solve the system (45), three steps are needed:

1. The matrix E is given, using the expression (48d), as:

$$E = -D_r (C_a D_r)^{(-)} \qquad (50)$$

where $(C_a D_r)^{(-)}$ is the pseudo-inverse of $(C_a D_r)$ and the matrix P is deduced from (48c):

$$P = I - D_r (C_a D_r)^{(-)} C_a \qquad (51)$$

2. By the variable change

$$W_i = X K_i \qquad (52)$$

The inequalities (49) are written as:

$$(P A_{ai})^T X + X (P A_{ai}) - C_a^T W_i^T - W_i C_a \langle 0, \quad \forall i \in \{1, ..., M\} \qquad (53)$$

The inequalities (53) are of LMI type and LMIMatlab Toolbox can be used for that resolution. The controller is

$$K_i = X^{-1} W_i \qquad (54)$$

3. The other matrices defining the observer are deduced knowing E, P and K_i:

$$\begin{cases} N_i = P A_{ai} - K_i C_a & (55a) \\ L_i = K_i - N_i E & (55b) \\ G_{i1} = P B_{ai} & (55c) \end{cases} \qquad (55)$$

7 Numerical example

Consider the Takagi-Sugeno multiple model with two local models, two states and two outputs.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t)) \Big[A_i\, x(t) + \\ \qquad + B_i\, u(t) + R\, v(t) \Big] \\ y(t) = C\, x(t) + D\, \bar{u}(t) \end{cases} \tag{56}$$

The numerical values of all these matrices are:

$$A_1 = \begin{bmatrix} -0,6 & -2 \\ 0,5 & -0,2 \end{bmatrix},\ A_2 = \begin{bmatrix} -0,7 & -0,3 \\ 2 & -0,3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0,1 \\ 0,2 \end{bmatrix},\ B_2 = \begin{bmatrix} 0,3 \\ 0,4 \end{bmatrix},\ R = \begin{bmatrix} 0,2 \\ 0,5 \end{bmatrix},$$

$$D = \begin{bmatrix} 0,15 \\ 0,35 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

The vector of decision is depending on the system input. The weighting functions μ_i are Gaussian and given in figure (1).

The new state z(t) satisfies the following equation:

$$\dot{z}(t) = \sum_{i=1}^{2} \mu_i(u(t)) \Big[-\bar{A}\, z(t) + \bar{A}\, C\, x(t) + \bar{A}\, D\, \bar{u}(t) \Big] \tag{57}$$

where $\bar{A} = 20 * I$.

The augmented system has the following expression:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{2} \mu_i(u(t)) \Big[A_{ai}\, X(t) + \\ \qquad + B_{ai}\, u(t) + D\, \bar{u}(t) \Big] \\ Y(t) = C_a\, X(t) \end{cases} \tag{58}$$

with:

$$A_{a1} = \begin{bmatrix} -0,6 & -2 & 0 & 0 \\ 0,5 & -0,2 & 0 & 0 \\ 20 & 20 & -20 & 0 \\ 0 & 20 & 0 & -20 \end{bmatrix},\ B_{a1} = \begin{bmatrix} 0,1 \\ 0,2 \\ 0 \\ 0 \end{bmatrix},$$

$$A_{a2} = \begin{bmatrix} -0,7 & -0,3 & 0 & 0 \\ 2 & -0,3 & 0 & 0 \\ 20 & 20 & -20 & 0 \\ 0 & 20 & 0 & -20 \end{bmatrix},\ B_{a2} = \begin{bmatrix} 0,3 \\ 0,4 \\ 0 \\ 0 \end{bmatrix},$$

$$D_a = \begin{bmatrix} 0,2 & 0,5 & 0 & 0 \\ 0 & 0 & 3 & 7 \end{bmatrix}^T \text{ and } C_a = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The multiple observer able to estimate the state of the multiple model (56) is as follows :

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{2} \mu_i(u(t)) \Big[N_i\, Z(t) + \\ \qquad + G_{i1}\, u(t) + L_i\, Y(t) \Big] \\ Y(t) = C_a\, X(t) - EY(t) \end{cases} \tag{59}$$

The computation of the matrices of the multiple observer (59) gives:

$$N_1 = \begin{bmatrix} -2,5 & 3 & 0 & 0 \\ -3,2 & -2,5 & 0 & 0 \\ 0 & 0,0002 & -20 & 0 \\ 0 & 0,0001 & 0 & -20 \end{bmatrix},\ G_{11} = \begin{bmatrix} -0,06 \\ -0,20 \\ 0 \\ 0 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2,5 & 4,8 & 0 & 0 \\ -4,8 & -2,5 & 0 & 0 \\ 20 & 0,0002 & -20 & 0 \\ 0 & 0,0001 & 0 & -20 \end{bmatrix},\ G_{21} = \begin{bmatrix} -0,02 \\ -0,40 \\ 0 \\ 0 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 1,50 & -2,14 \\ 2,70 & -7,56 \\ 20 & 0,0002 \\ 0 & 20,0001 \end{bmatrix},\ L_2 = \begin{bmatrix} 0,20 & 2,54 \\ 2,80 & -8,84 \\ 20 & 0,0002 \\ 0 & 20,0001 \end{bmatrix}$$

$$\text{and } E = \begin{bmatrix} 0 & -0,8 \\ 0 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Figure (5) represents the known input and Figure (6) visualizes the unknown input. The sensor fault affecting the system is given by Figure (7).

The simulation results are represented Figure (8). The proposed method provides good estimates of the system state. Indeed, the convergence of the state vector of the multiple observer towards those of the Takagi-Sugeno multiple model is quite good.

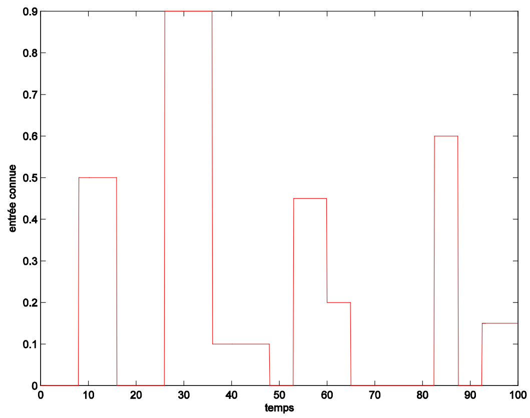

Figure 5: The known input u.

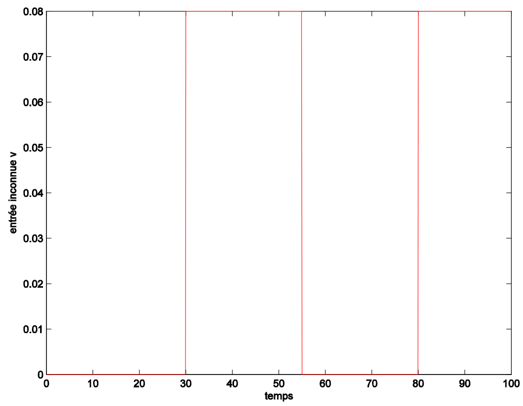

Figure 6: The unknown input.

8 Illustration example: Three column

The chosen system is a hydraulic process composed of three columns [1, 38]. The process is effected by an actuator fault v(t) and a sensor fault $\bar{u}(t)$. The three columns T_1, T_2 and T_3 have equal section A and connected to each others by identical section connectors S_n. The considered output is that of the column T_2; it ensures to empty the columns filled by pumps 1 and 2 with respective flows $Q_1(t)$ and $Q_2(t)$. Some combinaison of three levels are measured. The connexions between columns are assured by certain valves which are tuned manually to activate or not the corresponding pump. The three levels x_1, x_2 and x_3 satisfy $x_1 > x_2 > x_3$. The non linear model describing the process behaviour is given by [38]:

$$\begin{cases} A\dfrac{dx_1}{dt}(t) = -\alpha_1 S_n\left[2g\left(x_1(t)-x_3(t)\right)\right]^{1/2} \\ \qquad + Q_1(t) + Qf_1\,\bar{u}(t) \\ A\dfrac{dx_2}{dt}(t) = -\alpha_3 S_n\left[2g\left(x_3(t)-x_2(t)\right)\right]^{1/2} \\ -\alpha_2 S_n\left[2g\,x_2(t)\right]^{1/2} + Q_2(t) + Qf_2\,\bar{u}(t) \\ A\dfrac{dx_3}{dt}(t) = -\alpha_1 S_n\left[2g\left(x_1(t)-x_3(t)\right)\right]^{1/2} \\ -\alpha_3 S_n\left[2g\left(x_3(t)-x_2(t)\right)\right]^{1/2} + Qf_3\,\bar{u}(t) \end{cases} \tag{60}$$

where $\alpha_1 = \alpha_2 = 0.78$, $\alpha_3 = 0.75$, $g = 9.81\ ms^{-2}$, $S_n = 5\ 10^{-5}\ m^2$, $Qf_i = 10^{-4}$ and $A = 0.0154\ m^2$. $\bar{u}(t)$ is an unknown input. $Qf_i / f_i(t),\ i = 1, 2, 3$ are the additive massic flows in the columns.

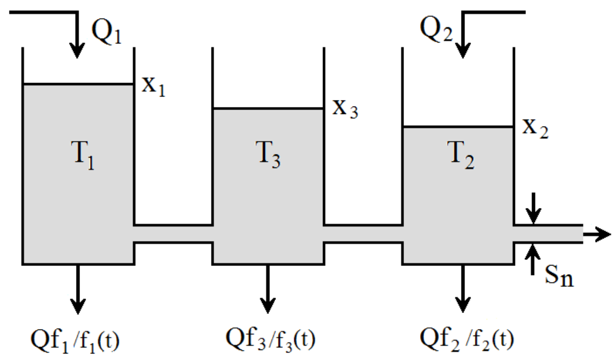

Figure 7. Three columns system.

The multi-model associated to the non linear system (60) is:

$$\dot{x}(t) = \sum_{i=1}^{4} \mu_i(\xi(t)) \left[A_i\ x(t) + B_i\ u(t) + R\ v(t) + d_i\right] \tag{61}$$

$$y(t) = C\ x(t) + D\ \bar{u}(t)$$

The matrices A_i, B_i and d_i are computed by linearizing model (60) around four set points chosen in system functioning zone. The numerical values of these matrices are:

$$A_1 = \begin{bmatrix} -0.0109 & 0 & 0.0109 \\ 0 & -0.0206 & 0.0106 \\ 0.0109 & 0.0106 & -0.0215 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0110 & 0 & 0.0110 \\ 0 & -0.0205 & 0.0104 \\ 0.0110 & 0.0104 & -0.0215 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.0084 & 0 & 0.0084 \\ 0 & -0.0206 & 0.0095 \\ 0.0084 & 0.0095 & -0.0180 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.0205 & 0.0095 \\ 0.0085 & 0.0095 & -0.0180 \end{bmatrix},$$

$$d_1 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.38 \\ 0.11 \end{bmatrix}, \; d_2 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.34 \\ 0.038 \end{bmatrix},$$

$$d_3 = 10^{-3} * \begin{bmatrix} -3.7 \\ -0.14 \\ 0.69 \end{bmatrix}, \; d_4 = 10^{-3} * \begin{bmatrix} -3.67 \\ -0.18 \\ 0.62 \end{bmatrix},$$

$$B_i = 1/A * \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

We choose $\bar{A}_1 = 30 * I$ where I is the 3-identity matrix.

The known input u(t) is defined by: $u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T$,

$$\begin{cases} u_1(t) = 0.5 \sin(0.15\,\pi t) \\ u_2(t) = 0.25 \sin(0.25\,\pi t) \end{cases}$$

The unknown input v(t) is defined by:

$$v(t) = \begin{cases} 0.01 * \sin(\pi t), & 0 < t \le 40s \\ 0, & t > 40s \end{cases}$$

The sensor fault $\bar{u}(t)$ is:

$$\bar{u}(t) = \begin{cases} 0, & t \le 10s \\ 0.1*\sin(0.2\pi t), & 10s < t \le 45s \\ 0, & t > 45s \end{cases}$$

The multiple observer gains are:

$$L_1 = \begin{bmatrix} -19.5381 & 73.2584 & 51.0916 \\ 22.0748 & -93.8480 & -69.0445 \\ 31.7600 & -117.6084 & -88.2063 \\ 30.0183 & -0.0683 & -0.0509 \\ -0.0016 & 30.0058 & 0.0049 \\ 0.0004 & -0.0014 & 29.9990 \end{bmatrix},$$

$$L_2 = L_4 = \begin{bmatrix} -19.5180 & 73.1819 & 51.0341 \\ 22.0655 & -93.8111 & -69.0205 \\ 31.7663 & -117.6330 & 88.2247 \\ 30.0183 & -0.0683 & -0.0509 \\ -0.0016 & 30.0058 & 0.0049 \\ 0.0003 & -0.0014 & 29.9990 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} -19.6094 & 73.5038 & 51.2952 \\ 22.1074 & -93.8275 & -68.9971 \\ 32.1424 & -119.0394 & -89.2798 \\ 30.0184 & -0.0684 & -0.0509 \\ -0.0016 & 30.0058 & 0.0049 \\ 0.0004 & -0.0014 & 29.9990 \end{bmatrix}.$$

In Figure 8 we plot the states and their estimations and Figure 9 draws the state estimation error. Figure 10 illustrates the sensor fault and figure 11 shows the evolution of state components (x_1 and x_2) for the Takagi-Sugeno multiple model described by relation (56) and the multiple observer given by relation (59). We note the perfect concordance between the state components and heir estimates by the multiple observer.

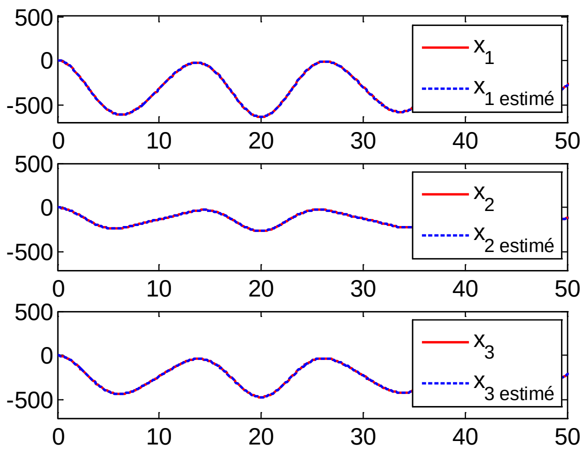

Figure 8: States and their estimations.

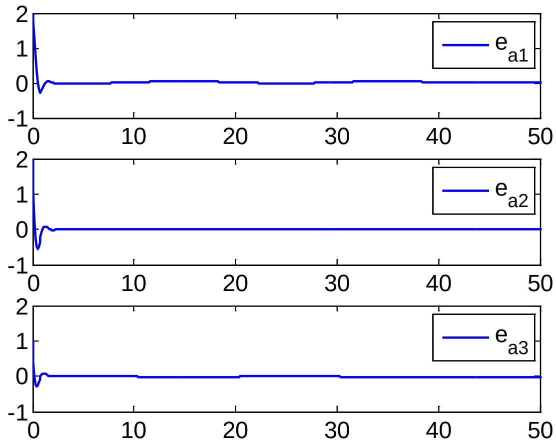

Figure 9: State estimation error.

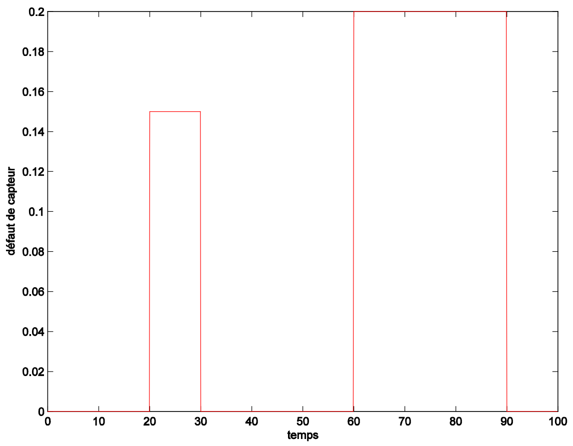

Figure 10: The sensor fault.

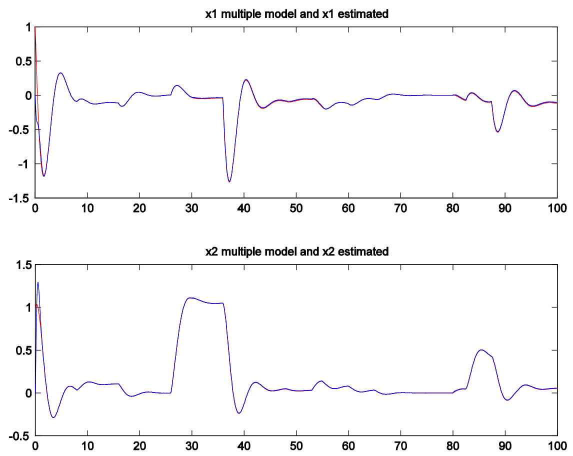

Figure 11: Multiple model (56) and multiple observer (59).

9 Conclusion

Using a multiple model representation, this paper has proposed new methods to design multiple observers for nonlinear systems submitted to unknown inputs and outputs. A mathematical transformation is used in order to formulate unknown outputs as unknown inputs. The proposed method is based on the principle of unknown input multiple observer which used the principle of the interpolation of local observers. The synthesis conditions of that observer are expressed in LMI terms. The simulation results show that we succeeds in making the estimation of state design the observer in spite of the existence of disturbances.

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