Stability Analysis of a Clamped-Pinned Pipeline Conveying Fluid

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Abstract: Increasing advances in materials engineering and cost reduction in their testing have lead to the study of the stability of vibration of pipes conveying fluid an important problem to deal with. Currently, such analysis is done either by means of simulation with costly specialized software or by making laboratory tests of the selected material. One of the main issues with the last process is that if appears any trouble on the material selection, it is necessary to restart all the process, and it is happening each time there is a mistake on the material selection. In order to avoid such costly tests, a general mathematical description of the dynamic behavior of a clamped-pinned pipeline conveying fluid is presented. The system stability has been studied by means of the eigenvalues of a Hamiltonian linear system associated. From this analysis, characteristic expressions dependent on material constants have been developed as inequalities, which ensure the stability of the material if it matches all expressions. Finally, some specific materials are introduced as study cases to compare the mathematical description proposed with the results obtained from specialized software as ANSYS, in order to validate the results.

Key–Words: Stability, eigenvalues analysis, pipe conveying fluid, material selection.

1 Introduction

The dynamics as well as the stability of pipes conveying fluid have been studied thoroughly in the last decades see for example with various analysis techniques for different end conditions and different models of the fluid-conveying pipeline (see for example [8, 9, 10, 11, 13, 14, 17, 18, 19, 21]. These authors analyze stability of pinned-pinned, clamped-clamped and cantilevered fluid-conveying pipes, even in the presence of a tensile force and a harmonically perturbed flow field.

It is well known that the dynamical behavior of pipes of a finite length depends strongly on the type of boundary. The type of supports considered (fixed, one end fixed, etc.) and their position (horizontal, vertical) must be distinguished.

The dynamics of the system can be described by a partial differential equation [20, 23]

$$a_4\frac{\partial^4 y}{\partial x^4}+a_3\frac{\partial^2 y}{\partial x^2}+a_2\frac{\partial^2 y}{\partial x\partial t}+a_1\frac{\partial^2 y}{\partial t^2}=0 \qquad (1)$$

with boundary conditions at ends of a clamped-pinned pipe. We find approximate solution of this equation using Galerkin's methode obtaining as a result a linear gyroscopic system possessing the properties of linear Hamiltonian systems. Then, the eigenvalues of this linear Hamiltonian system gives information about stability: a stable Hamiltonian system is characterized by pure imaginary eigenvalues. It is known that the stability of a linear Hamiltonian system is not asymptotic, nevertheless the study provides the necessary stability condition for the original non-linear system.

Different qualitative analysis of multiparameter linear systems as well bifurcation theory of eigenvalues can be found in [4, 6, 7, 20].

The aim of the paper is by means of linear Hamiltonian system to model the clamped-pinned pipeline problem and to analyze the structural stability of the proposed model. This paper refers to a one end fixed horizontal pipeline.

The structure of paper is as follows. Section 2 presents a mathematical statement of the problem, including some preliminary concepts. Section 3 is devoted to analyze the stability of linearized system obtained in subsection 2.1. Section 4 presents and a simulation of the dynamic system using ANSYS for some different materials used in real cases, such as PVC, Polyethylene, Concrete, Steel and Aluminium, in order to validate the results obtained analytically. Finally, in Section 5, some conclusions are summarized.

2 Mathematical problem statement

The system under consideration is a straight, tight and of finite length pipeline, passing through it a fluid. The following assumptions are taken into account in the analysis of the system:

i) Are ignored the effects of gravity, the coefficient damping material, the shear strain and rotational inertia

ii) The pipeline is considered horizontal

iii) The pipe is inextensible

iv) The lateral movement of $y(x,t)$ is small, and with large length wave compared with the diameter of the pipe, so that theory Euler-Bernoulli is applicable for the description of vibration bending of the pipe.

v) It ignores the velocity distribution in the cross section of pipe.

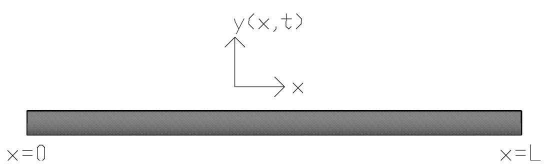

Figure 1: Pipeline

The equation for a single span prestressed pipeline where the fluid is transported is a function of the distance x and time t and is based on the beam theory, [1, 15]:

$$EI\frac{\partial^4 y}{\partial x^4} + m_p\frac{\partial^2 y}{\partial t^2} = f_{int}(x,t) \tag{2}$$

where EI is the bending stiffness of the pipe (Nm^2), m_p is the pipe mass per unit length ($\frac{kg}{m}$) and f_{int} is an inside force acting on the pipe.

The internal fluid flow is approximated as a plug flow, so all points of the fluid have the same velocity U relative to the pipe. This is a reasonable approximation for a turbulent flow profile. Because of that the inside force can be written as:

$$f_{int} = -m_f\frac{d^2 y}{dt^2}\bigg|_{x=Ut} \tag{3}$$

where m_f is the fluid mass per unit length ($\frac{kg}{m}$) and U is the fluid velocity ($\frac{m}{s}$).

Total acceleration can be decomposed into local acceleration, Coriolis and centrifugal.

$$m_f\frac{d^2y}{dt^2}\bigg|_{x=Ut} = m_f\left[\frac{d}{dt}\left(\frac{\partial y}{\partial t}+\frac{\partial y}{\partial x}\frac{dx}{dt}\right)\bigg|_{x=Ut}\right] = $$
$$= m_f\left[\frac{d}{dt}\left(\frac{\partial y}{\partial t}+U\frac{\partial y}{\partial x}\right)\bigg|_{x=Ut}\right] = $$
$$= m_f\left[\frac{\partial^2 y}{\partial t^2}+2U\frac{\partial^2 y}{\partial x\partial t}+U^2\frac{\partial^2 y}{\partial x^2}\right] \tag{4}$$

The internal fluid causes an hydrostatic pressure on the pipe wall.

$$T = -A_iP_i \tag{5}$$

where A_i is the internal cross sectional area of the pipe (measured in m^2) and P_i is the hydrostatic pressure inside the pipe (measured in Pa).

Finally if by considering that the total acceleration is equal to the composition of local, coriolis and centrifugal acceleration. The resulting equation describing the oscillations of the pipe is (1):

$$EI\frac{\partial^4 y}{\partial x^4}+(m_fU^2-T)\frac{\partial^2 y}{\partial x^2}+2m_fU\frac{\partial^2 y}{\partial x\partial t}+$$
$$(m_p+m_f)\frac{\partial^2 y}{\partial t^2}=0 \tag{6}$$

2.1 Linear approximation

To find approximate solution to equation (6), the method used is the Galerkin's method with two coordinate function, that is to say, taking $n=2$ with respect $\left\{\sin\frac{i\pi}{L}x\right\}_{i=1,2,\ldots}$ basis defined over a open set $\Omega\subset\mathbb{R}^n$ and the inner product $<f,g>=\int_0^L fg$ the approximate solution is:

$$y(x,t)=\varphi_1(t)\text{sen}\,\frac{\pi}{L}x+\varphi_2(t)\text{sen}\,\frac{2\pi}{L}x$$

Replacing the solution in the equation (6), it is obtained that:

$$EI\varphi_1(t)\left(\frac{\pi^4}{L^4}\text{sen}\,\frac{\pi}{L}x+\varphi_2(t)\frac{16\pi^4}{L^4}\text{sen}\,\frac{2\pi}{L}x\right)+$$

$$(m_fU^2-T)\cdot$$
$$\left(-\varphi_1(t)\frac{\pi^2}{L^2}\text{sen}\,\frac{\pi}{L}x-\varphi_2(t)\frac{4\pi^2}{L^2}\text{sen}\,\frac{2\pi}{L}x\right)+$$

$$2m_fU\left(\dot\varphi_1(t)\frac{\pi}{L}\cos\frac{\pi}{L}x+\dot\varphi_2(t)\frac{2\pi}{L}\cos\frac{2\pi}{L}x\right)+$$

$$(m_p+m_f)\left(\ddot\varphi_1(t)\text{sen}\,\frac{\pi}{L}x+\ddot\varphi_2(t)\text{sen}\,\frac{2\pi}{L}x\right)=0 \tag{7}$$

Making the scalar product by $\text{sen}\,\frac{\pi}{L}\xi$ and $\text{sen}\,\frac{2\pi}{L}\xi$, respectively, it can be obtain:

$$\begin{aligned}&\frac{L}{2}(m_p+m_f)\ddot\varphi_1(t)-\frac{8}{3}m_fU\dot\varphi_2(t)+\\&\quad\left(EI\frac{\pi^4}{2L^3}-\frac{(m_fU^2-T)\pi^2}{2L}\right)\varphi_1(t)=0\\&\frac{L}{2}(m_p+m_f)\ddot q_2(t)-\frac{8}{3}m_fU\dot\varphi_1(t)+\\&\quad\left(EI\frac{8\pi^4}{L^3}-(m_fU^2-T)\frac{4\pi^2}{L^2}\right)\varphi_2(t)=0\end{aligned} \tag{8}$$

The previous equation system can be written as matrix form like:

$$M\ddot\varphi+B\dot\varphi+C\varphi=0$$

that corresponds to lineal system:

$$\ddot x+G\dot x+Kx=0 \tag{9}$$

with $M^{-1/2}\varphi=x$ (we write the variable as x if confusion it is not possible),

$$G=M^{-1/2}BM^{-1/2}=\frac{16m_f}{L(m_f+m_p)}\begin{pmatrix}0&-1\\1&0\end{pmatrix}$$

$$K=M^{-1/2}CM^{-1/2}=\frac{2}{L(m_f+m_p)}\begin{pmatrix}K_1&0\\0&K_2\end{pmatrix}$$

and

$$K_1=EI\frac{\pi^4}{2L^3}-\frac{(m_fU^2-T)\pi^2}{2L}$$

$$K_2=EI\frac{8\pi^4}{L^3}-(m_fU^2-T)\frac{4\pi^2}{L^2}$$

Introducing the vector:

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\\dot x+Gx/2\end{pmatrix}$$

and calculating the derivatives of x and y it can be found $\dot x=y-Gx/2$, $\dot y=\ddot x+G\dot x/2$ and considering that $\ddot x=-G\dot x-Kx$ and linearizing the system a linear equation is obtained:

$$\begin{pmatrix}\dot x\\\dot y\end{pmatrix}=\begin{pmatrix}-G/2&I_2\\G^2/4-K&-G/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=A\begin{pmatrix}x\\y\end{pmatrix}.$$

Notice that the matrix A is a Hamiltonian matrix because QA is symmetrical, where Q is the antisymmetrical matrix:

$$Q=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}.$$

So, the properties of linear Hamiltonian systems can be used for analysis of the equation (9).

In order to simplify, the following parameters considered

$$\begin{aligned}\Lambda&=\frac{EI\pi^4}{L^3}\\\delta&=(m_fU^2-T)\frac{\pi^2}{L}\\\beta&=\frac{1}{L(m_f+m_p)}\end{aligned} \tag{10}$$

and the matrices G and K are written as:

$$G=16m_f\beta\begin{pmatrix}0&-1\\1&0\end{pmatrix}$$
$$K=2\beta\begin{pmatrix}\frac{1}{2}\Lambda-\frac{1}{2}\delta&0\\0&8\Lambda-\frac{4}{L}\delta\end{pmatrix} \tag{11}$$

Therefore, matrix A is:

$$A=\begin{pmatrix}0&a&1&0\\-a&0&0&1\\b&0&0&a\\0&c&-a&0\end{pmatrix} \tag{12}$$

where:

$$\begin{aligned}a&=8m_f\beta\\b&=-64m_f^2\beta^2-\beta\Lambda+\beta\delta\\c&=-64m_f^2\beta^2-16\beta\Lambda+\frac{8}{L}\beta\delta.\end{aligned} \tag{13}$$

Removing the variable change it is known that:

$$
\begin{aligned}
a &= \frac{8m_f}{L(m_f+m_p)} \\
b &= \frac{-64m_f^2}{L^2(m_f+m_p)^2} - \frac{EI\pi^4}{L^4(m_f+m_p)} + \\
&\quad \frac{(m_fU^2+A_iP_i)\pi^2}{L^2(m_f+m_p)} \\
c &= \frac{-64m_f^2}{L^2(m_f+m_p)^2} - \frac{16EI\pi^4}{L^4(m_f+m_p)} + \\
&\quad \frac{8(m_fU^2+A_iP_i)\pi^2}{L^3(m_f+m_p)}.
\end{aligned}
$$

So, the pipeline has been modeled as a linear system.

3 Bifurcation analysis

In this section the stability properties and bifurcation analysis at the critical points of linear dynamic systems representing the pipeline are studied. A detailed discussion of the effect of the stabilization in terms of the bifurcation theory of eigenvalues is presented.

A stable hamiltonian system $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$ with A as in (12), is characterized by eigenvalues lying on the imaginary axis. The characteristic equation of the matrix is the following biquadratic equation:

$$\lambda^4 + (2a^2 - b - c)\lambda^2 + (a^2+c)(a^2+b) = 0 \quad (14)$$

then, the eigenvalues are the roots of this equation that in terms of the parameters Λ, δ, β are

$$\lambda = \pm\sqrt{\frac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}} \quad (15)$$

with

$$
\begin{aligned}
\lambda_1 &= -256m_f^2\beta^2 - 17\beta\Lambda + \left(1+\frac{8}{L}\right)\beta\delta \\
\lambda_2 &= 65536m_f^4\beta^2 + 8704m_f^2\beta\Lambda - \\
&\quad \left(512+\frac{4096}{L}\right)m_f^2\beta\delta + 225\Lambda^2 + \\
&\quad \left(1+\frac{64}{L^2}-\frac{16}{L}\right)\delta^2 + \left(30-\frac{240}{L}\right)\Lambda\delta
\end{aligned}
\quad (16)
$$

As it has been said, the system is stable in Lyapunov's sense, if the eigenvalues lie on the imaginary axe and they are smple or semi-simple.

The data in the system are know only approximately, the matrix A in the system can be considered as a family of matrices depending differentiably on parameters a, b, c in a neighborhood of a fixed point p_0. Using this family will try to study the stability border. The point p_0, in which correspond only simple pure imaginary eigenvalues, is always an interior point of the stability domain, while the points on the boundary of the stability domain are characterized by the existence of multiple pure imaginary or zero eigenvalues, (when the other eigenvalues are simple and pure imaginary).

Stability conditions requires that the roots obtained in (15), $\lambda^2 = \frac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}$ are real and negative. Imposing these conditions the stability zone in the parameter space can be determined.

It can be observed that the points $p = (a, b, c)$ such that

$$\left.\begin{aligned} 2a^2 - b - c &= 0 \\ (a^2+c)(a^2+b) &= 0 \end{aligned}\right\}, \quad (17)$$

the characteristic polynomial is λ^4,

The set (17) corresponds to the union of parameterized curves $\psi(t) = (t, 3t^2, -t^2)$ and $\psi(t) = (t, -t^2, 3t^2)$. In the intersection can be found the most degenerate case, with respect the algebraic structure of the system as it can be seen below.

If the parameter a has not zero ($a \neq 0$) the matrix A under similarity relation preserving structure can be reduced to the following normal Jordan form

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

A Jordan basis transforming the matrix in its reduced form is

$$S_1 = \begin{pmatrix} 1 & 0 & b-a^2 & 0 \\ 0 & -a & 0 & -a(a^2+b) \\ 0 & b & 0 & c(a^2-b) \\ 0 & 0 & -a(b+c) & 0 \end{pmatrix}$$

if $a^2 + b \neq 0$, and

$$S_2 = \begin{pmatrix} 0 & a & 0 & a(a^2+c) \\ 1 & 0 & c-a^2 & 0 \\ 0 & 0 & a(b+c) & 0 \\ 0 & c & 0 & -b(a^2+c) \end{pmatrix}$$

if $a^2 + c \neq 0$. (Observe that $a^2 + b$ and $a^2 + c$ can not be zero simultaneously because $a \neq 0$).

If $a = 0$ the normal Jordan form of A is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

In this case, a Jordan basis reducing the matrix A can be

$$S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

At the points (17) the system have singularities of the type 0^4 and the more degenerate $0^2 0^2$ on the stability boundary. In both cases eigenvalues lie in imaginary axis but they are not semisimple.

Near of these singularities it is possible to find the lest degenerate matrices, for example

$$A(t) = \begin{pmatrix} 0 & t & 1 & 0 \\ -t & 0 & 0 & 1 \\ 0 & 0 & 0 & t \\ 0 & 0 & -t & 0 \end{pmatrix}$$

where the eigenvalues are $\pm ti$, and the stable case:

$$\widetilde{A} = \begin{pmatrix} 0 & -0.1 & 1 & 0 \\ 0.1 & 0 & 0 & 1 \\ -0.0001 & 0 & 0 & -0.1 \\ 0 & -0.0001 & 0.1 & 0 \end{pmatrix}$$

where the eigenvalues (computed using Matlab) are $0 + 0.1100i$, $0 - 0.1100i$, $0 + 0.0900i$, $0 - 0.0900i$.

Following the analysis of eigenvalues, the eigenvalue 0 can be also obtained at the points (a, b, c) such that

$$\left.\begin{array}{rl} (a^2 + c)(a^2 + b) & = 0 \\ 2a^2 - b - c & \neq 0 \end{array}\right\} \quad (18)$$

At the points $(a, b, -a^2)$ there are two possibilities depending on b if it is equal or not to $-a^2$

For $b \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{-3a^2 + b} & 0 \\ 0 & 0 & 0 & -\sqrt{-3a^2 + b} \end{pmatrix}$$

For $b = -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2ai & 0 \\ 0 & 0 & 0 & -2ai \end{pmatrix}.$$

This case corresponds to a stability point because of all eigenvalues are semisimple and lie in the imaginary axe. It is important to note that in this case the reduced form is not structurally stable (a small perturbation makes that the double eigenvalue bifurcates into two distinct eigenvalues or into a double nonderogatory eigenvalue of type 0^2.

By symmetry, at the points $(a, -a^2, c)$ there are two cases depending on c be equal or not to $-a^2$

For $c \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{-3a^2 + c} & 0 \\ 0 & 0 & 0 & -\sqrt{-3a^2 + c} \end{pmatrix}$$

For $c = -a^2$ the Jordan form coincides with the case $b = -a^2$.

Analogously, the case $c = a^2$ is out of the stability space

For the case $b \neq -a^2$ and $c \neq -a^2$ the system have singularities of the type 0^2 in the boundary of stability.

It remains to study the case that no eigenvalue is zero

The roots of $\mu^2 + (2a^2 - b - c)\mu + (a^2 + c)(a^2 + b) = 0$, are real and negative when

$$\left.\begin{array}{l} 2a^2 - b - c > 0 \\ (a^2 + c)(a^2 + b) > 0 \\ (2a^2 - b - c)^2 \geq 4(a^2 + c)(a^2 + b) \end{array}\right\} \quad (19)$$

In the case $(2a^2 - b - c)^2 = 4(a^2 + c)(a^2 + b)$ the eigenvalues are $\lambda = \pm i\sqrt{2a^2 - b - c} = \pm i\omega$ double.

Taking into account that rank $(A - (\pm i\omega)I) = 3$ the equivalent normal Jordan form is

$$\begin{pmatrix} i\omega & 1 & 0 & 0 \\ 0 & i\omega & 0 & 0 \\ 0 & 0 & -i\omega & 1 \\ 0 & 0 & 0 & -i\omega \end{pmatrix}$$

At the points (a, b, c) with $2a^2 - b - c > 0$, $(a^2 + c)(a^2 + b) > 0$ and $(2a^2 - b - c)^2 = 4(a^2 + c)(a^2 + b)$ the system have singularities of the type $\pm i\omega^2$.

The last case

$$\left.\begin{array}{l} 2a^2 - b - c > 0 \\ (a^2 + c)(a^2 + b) > 0 \\ (2a^2 - b - c)^2 > 4(a^2 + c)(a^2 + b) \end{array}\right\} \quad (20)$$

determined the open set of stable points (a, b, c) remaining within the area bounded by the above singularities.

4 Application to analysis of the case of study

Taking as a constant parameters $L = 1000mm$, $I = 2,185 \cdot 10^6$, $A_i = 2500\pi$ due to the geometry of the pipe and $m_f = 2,5\pi \cdot 10^{-6}\dfrac{Tn}{mm}$ assuming the fluid is water. It is also supposed that the study is applied to the inside wall of the pipe so U at these points are zero.

Therefore the values a, b y c are:

$$a = \frac{2\pi \cdot 10^{-8}}{(2,5\pi \cdot 10^{-6} + m_p)}$$
$$b = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6} + m_p)^2} - \frac{2,185 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6} + m_p)} + \frac{2,5 \cdot 10^{-3}P_i\pi^3}{(2,5\pi \cdot 10^{-6} + m_p)}$$
$$c = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6} + m_p)^2} - \frac{34,96 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6} + m_p)} + \frac{2 \cdot 10^{-5}P_i\pi^3}{(2,5\pi \cdot 10^{-6} + m_p)}$$

That permit to obtain the following relation depending only on m_p, E and P_i:

$$\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi^2 E - 2.52\pi P_i > 0$$
$$15.27752 \cdot 10^{-4}\pi^2E^2 + P_i^2 - 17.48874 \cdot 10^{-1}\pi E P_i > 0$$
$$\left(\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi E - 2.52P_i\right)^2 > 4\pi^2(76.3877 \cdot 10^{-6}\pi^2E^2 - 87.4437 \cdot 10^{-3}\pi E P_i + 5 \cdot 10^{-2}P_i^2) \quad (21)$$

This study is done to show the stability of pipes with different materials assuming in all of them that the fluid transported is water and causes a constant pressure on its walls of 4 bar. The geometrical conditions of the pipe are the inside diameter equal to 50 mm and the thickness of the pipe which is 6 mm. The materials chosen are PVC, Polyethylene, Concrete, Steel, and Aluminum.

The values of E and m_p of the PVC pipe are:

$$E = 30,581\frac{N}{mm^2}$$
$$m_p = 2,76 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (21) it is found that the solution is unstable.

The values of E and m_p of the PE pipe are:

$$E = 9,174\frac{N}{mm^2}$$
$$m_p = 1,91 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (21) it is found that the solution is unstable.

The values of E and m_p of the Concrete pipe are:

$$E = 221,203\frac{N}{mm^2}$$
$$m_p = 4,40 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (21) it is found that the solution is stable.

The values of E and m_p of the Steel pipe are:

$$E = 210000\frac{N}{mm^2}$$
$$m_p = 15,7 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (21) it is found that the solution is stable.

The values of E and m_p of the Aluminum pipe are:

$$E = 70000\frac{N}{mm^2}$$
$$m_p = 5,4 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (21) it is found that the solution is stable.

In fact, the eigenvalues can be computing by using Matlab software. The eigenvalues are presented in Table 1 for the materials considered in the analysis.

Figure 2 shows the distribution of these values in the complex plane, where it can be seen that exist some unstable values (the ones with positive real part in Table 1).

It is worth to say that the pipe case considering PVC is the furthest away from stability zone.

Material	Eigenvalues			
PVC	-48.042	48.042	$-98.936i$	$98.936i$
PE	-54.549	54.549	$-56.341i$	$56.341i$
Concrete	$-0.362i$	$0.362i$	$-2.479i$	$2.479i$
Steel	$-1.377i$	$1.377i$	$-5.51i$	$5.51i$
Aluminium	$-1.059i$	$1.059i$	$-4.241i$	-4.241

Table 1: Eigenvalues obtained for different materials

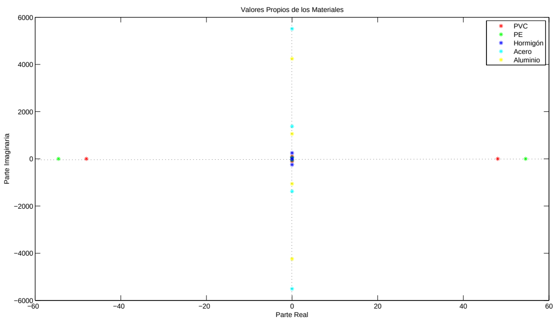

Figure 2: Representation of the eigenvalues obtained for different materials

4.1 Simulation for some specific materials

In this section the equation (6) is solved and the structural stabilities found in the previous section with the stability of the solution is compared. Moreover, vibration characteristics of a pipe conveying fluid is calculated using a Finite Element package called ANSYS.

To determine the vibration characteristics modal analysis have been used, with this analysis you find natural frequencies and mode shapes which are important parameters in the design of a structure for dynamic studies.

The simulation of the problem varies depending on the boundary conditions. In this case the boundary conditions considered are both sides of the pipe are rigid support. So, the boundary conditions at ends of a clamped-pinned pipe are given as:

$$\begin{aligned} y(0,t) &= 0, & y(L,t) &= 0 \\ \frac{\partial^2 y(0,t)}{\partial t} &= 0, & EI\frac{\partial^2 y(L,t)}{\partial x^2} &= 0 \end{aligned} \qquad (22)$$

where K_{rs} is the stiffness of the rotational spring at the right end.

The following table lists the values of natural frequency and displacement in both cases:

Remark that the frequencies obtained solving linear Hamiltonian system does not coincides with frequencies that can be obtained solving the second order differential equation (6) with initial conditions (23), in fact with Hamiltonian equation is studied structural stability giving information about the qualitative changes that can be in the behavior of systems when the systems are known only approximately.

In order to compare the stability of different types of structures with the previous section, there has been realized a buckling analysis that gives the following table of frequencies:

It is observed that the values are practically the same as in the study of the effect on stability boundary conditions is minimal.

In the following pictures it is shown the perfor-

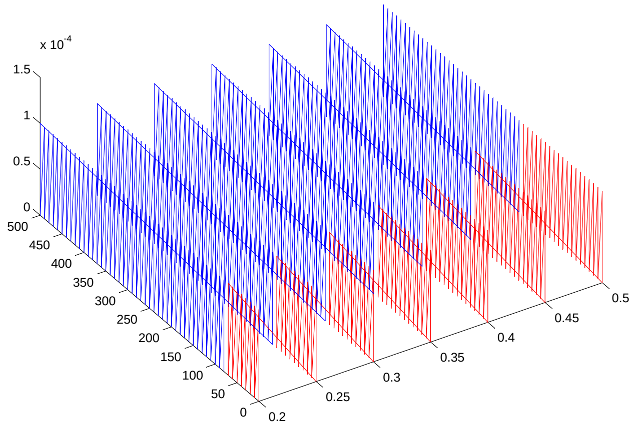

Figure 3: Representation for different values of E

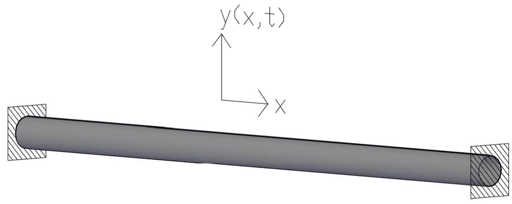

Figure 4: Boundary conditions

	First shape		Second shape	
	f	dx	f	dx
Concrete	0.033927	0.023548	0.033927	0.023548
PE	4.916	29.75	4.916	29.75
PVC	18.205	29.171	18.207	29.171
Steel	424.28	8.767	424.256	8.767
Aluminium	417.58	14.957	417.598	14.957

Table 2: Natural frequencies and displacement obtained from Ansys software analysis

	First shape		Second shape	
	f	dx	f	dx
Concrete	0.219495	0.025123	0.219495	0.025124
PE	0.010997	0.0253	0.010997	0.025305
PVC	0.033822	0.025258	0.033822	0.025264
Steel	219.042	0.025152	219.042	0.025153
Aluminium	74.477	0.025184	74.477	0.025185

Table 3: Frequencies and displacement obtained from Ansys software after a buckling analysis

mance of the first and the second shapes and the natural frequencies of them.

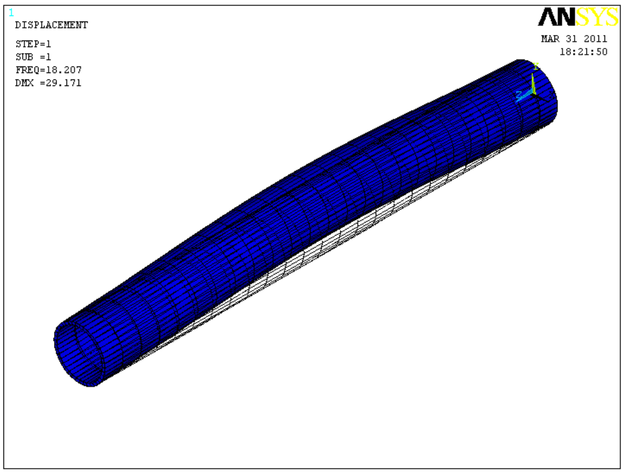

Figure 5: First shape of PVC

As seen in picture 5, 6, 7, 8 and 9 the lowest natural frequency is the Concrete pipe (0, 033927 Hz) and the biggest one is the Steel pipe (424, 28 Hz) but the greater displacement of x axis is the Polyethylene pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete Steel and Aluminium pipes are stable.

As seen in picture 10, 11, 12, 13 and 14 the lowest natural frequency is the concrete pipe (0,033927 Hz) and the biggest one is the Steel pipe (424, 256 Hz) but the greater displacement of x axis is the PVC pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete, Steel and Aluminium pipes are stable.

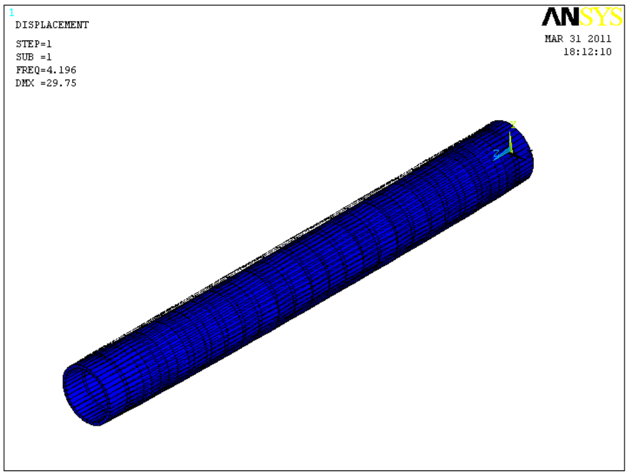

Figure 6: First shape of Polyethylene

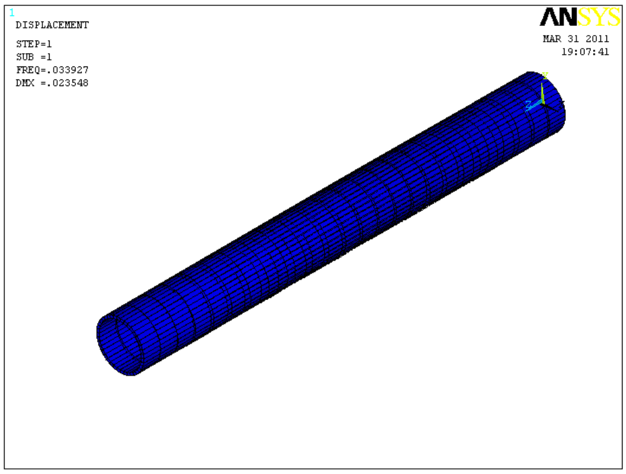

Figure 7: First shape of Concrete

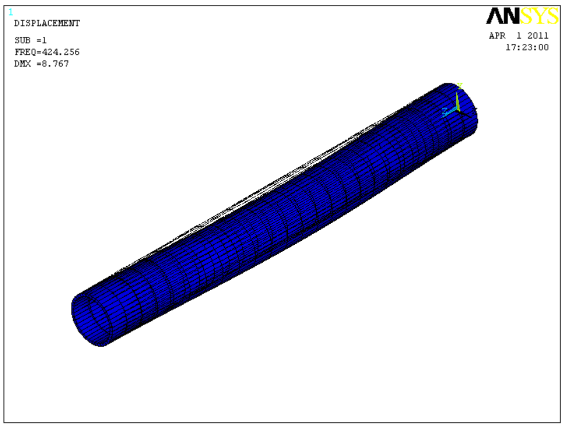

Figure 8: First shape of Steel

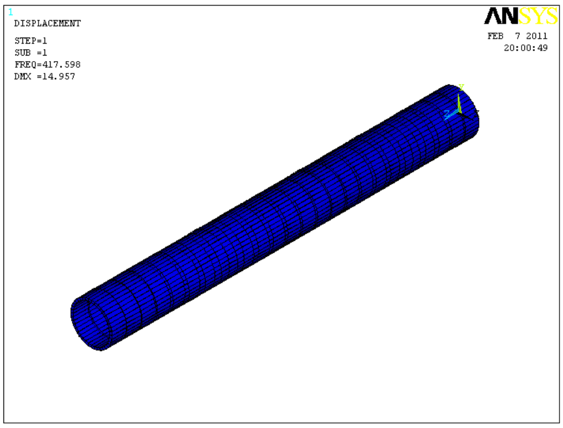

Figure 9: First shape of Aluminium

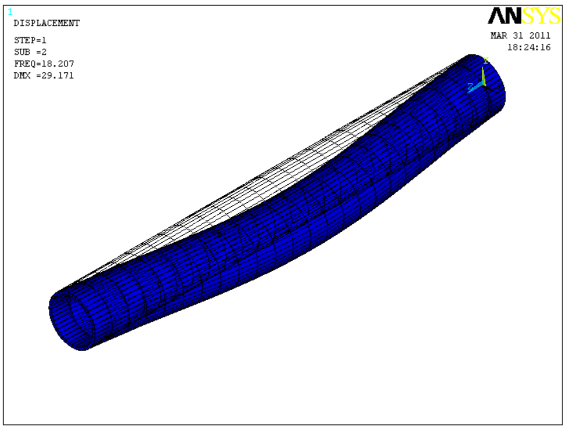

Figure 10: Second shape of PVC

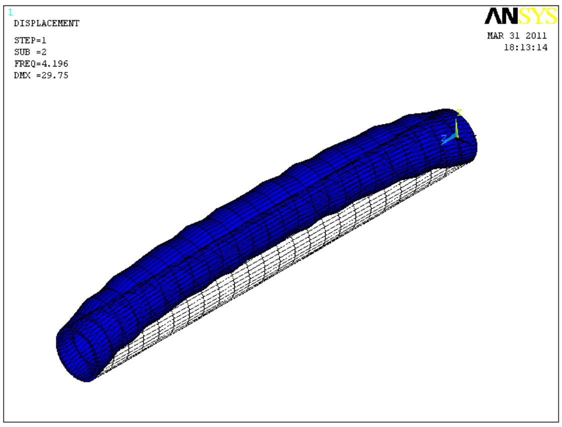

Figure 11: Second shape of Polyethylene

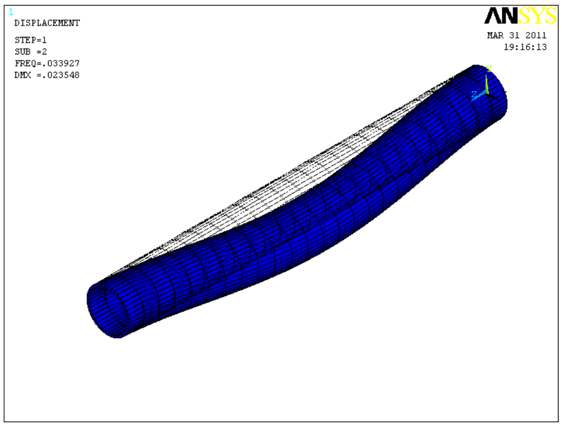

Figure 12: Second shape of Concrete

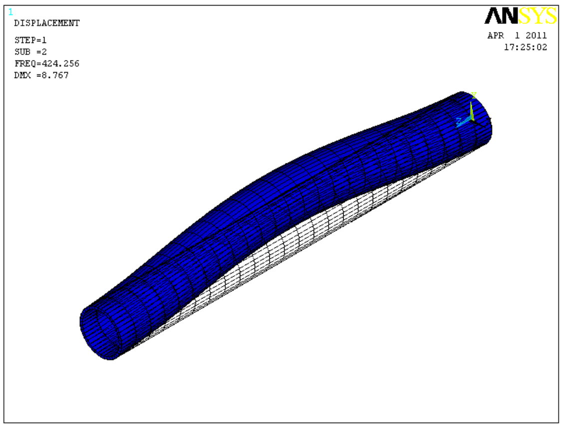

Figure 13: Second shape of Steel

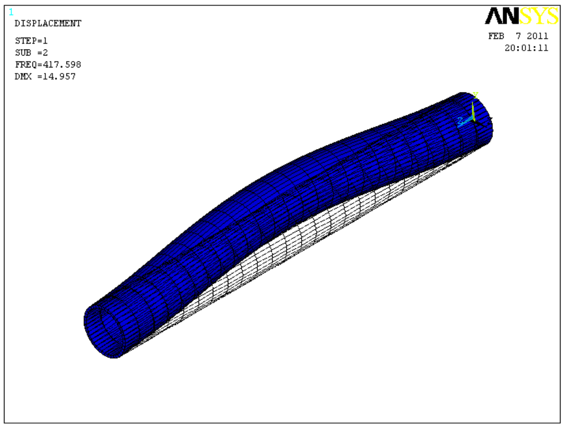

Figure 14: Second shape of Aluminium

5 Conclusion

In this paper a non-linear dynamic model for a pipe conveying fluid have presented. Moreover, a linearization method have been done by approximation of the non-linear system to the linear gyroscopic system. From the linear system, the stability of the pipe is analyzed in a general form by means of the first Lyapunov's methods. The stability generalization of the system have been done obtaining the stability limits as function of the material parameters.

In this paper the calculations and the simulation of typical materials for a pipe used in public works have been compared to verify the results obtained.

It have been shown that the dynamics and stability of pipes conveying fluid not only depends on the boundary conditions but it is also strongly important the material of the pipe and the pressure produced by the fluid.

6 Acknowledgments

The authors would like to thank J.L. Domínguez-García (IREC, Spain) for his discussion and providing us a number of important comments.

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