Composite Control for Nonlinear Singularly Perturbed Systems Based on Feedback Linearization Method

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Abstract: This article is devoted to the synthesis of composite control for nonlinear singularly perturbed system using feedback linearization (FL). The idea of this method consists in converting the original nonlinear system into a linear one by means of state feedback and coordinate transformation. Then, methods of control theory for linear systems are used for system design. If the original nonlinear system cannot be linearized exactly by state feedback, the method of approximate feedback linearization (AFL) is used. The essence of approximate feedback linearization methods lies in the feedback linearization only of the some part of the original nonlinear system (not the entire system). In this paper, we propose a method of approximate feedback linearization control of nonlinear singularly perturbed (SP) systems. Proposed method is based on the decomposition of the original SP system and the construction of control input in the form of asymptotic composition of FL controls for slow and fast subsystems. The resulting AFL control is obtained in the form of composite control. The application of the proposed approach is illustrated through the speed control of a DC series motor.

Key-Words: approximate feedback linearization, singularly perturbed system, composite control, decomposition.

1 Introduction

Objective complexity of real problems, high requirements to modern control systems make it necessary to use complex mathematical apparatus, and information technology. For a closer approximation to reality in the mathematical description of processes and systems it is necessary to take into account various factors (non-linearity, small parameters, uncertainties and other). In place of the one-dimensional linear models come multidimensional nonlinear models. The methods of modeling, analysis and synthesis of nonlinear control systems are developed [1-7].

One of the most common methods of nonlinear control systems synthesis is the method of feedback (external) linearization [1-4]. The idea of this method consists in converting the original nonlinear system into a linear one by means of feedback. Then, methods of control theory for linear systems are used for system design.

The applicability of the method of external linearization guaranteed under strict conditions of controllability and involutivity for the nonlinear system, that is not always true. In this situation, the method of AFL is used [8-12].

The most widely used approach to approximate linearization was based on the expansion of the original system in a Taylor series [8,9]. Another approach is presented in [10,11] is based on the use of the dynamic feedback concept and of the relative degree d-wherein d - an integer. Here the basic idea is to transform the dynamics equation of the system to linear form, in which additional terms of higher order than a given integer d are added. The main drawback of this solution is associated with an increase in the order of the system and the order of dynamic feedback. From this point of view in [12] presented the opposite way to approximate linearization, based on the idea of a Singularly Perturbed representation of the system and fast and slow motion separation.

Multiple scales can be caused by different physical factors, such as the presence of small masses and moments of inertia, the high gain feedback etc (for example see [13-21]). A mathematical description of such systems uses small parameters that multiply the time derivative of some state variables in the equation of the system state.

There are numerous publications devoted to the analysis and design of SP systems, which are reviewed in [21-23]. A recent review [23] includes more than 500 references and demonstrates a growing interest of researchers in nonlinear SP systems and their applications.

The problem of nonlinear SP systems control using FL is studied in [24-27]. In [24] a diffeomorphism is proposed that is independent of the small singular perturbation parameter. Moreover, this diffeomorphism should satisfy the slow-fast dynamics separation condition. In [25] a new diffeomorphism is proposed, which does not require compliance with the dynamics separation conditions. Both of the above mentioned papers can be attributed to the same direct approach, which considers the linearization of the entire SP system .

Articles [26, 27] introduce another approach that is based on FL of the so-called "slow" subsystem, not the entire SP system. According to this indirect approach the original FL problem of nonlinear SP system is reduced to the simpler problem of FL of unperturbed slow subsystem. Restriction of the result in [26,27] is the class of nonlinear SP systems that are linear in control input and fast state variables.

In this paper the problem of FL control is considered for the class of nonlinear SP system with nonlinear equation for slow state variables in general form and equation for fast state variables that is nonlinear only in slow state variables. We propose a method of AFL control based on the decomposition of the original nonlinear SP system and the construction of control input in the form of asymptotic composition of FL controls for slow and fast subsystems. The resulting AFL control is obtained in the form of composite control.

The paper is organized as follows: Section 2 contains the formulation of FL composite control problem for the class of SP systems described above; Section 3 discusses the decomposition process of nonlinear SP system using standard singular perturbation technique; Section 4 is devoted to the problem of synthesis of FL control for slow and fast subsystems; an example of the speed control system development for a DC series motor using composite FL control is shown in Section 5; conclusion with the main findings and acknowledgements are shown in Section 6 and Section 7 respectively.

2 Problem formulation

Consider a nonlinear SP system of the type

$$\begin{gathered} \dot{x}_1 = f_1(x_1, x_2, u), \\ \varepsilon \cdot \dot{x}_2 = f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)u, \\ x_1(0) = x_1^0,\ x_2(0) = x_2^0, \end{gathered} \tag{1}$$

where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$ are the state variables vectors, $u \in R^1$ is the scalar control input, $\varepsilon > 0$ is a small parameter (singular perturbation).

It is assumed that the system (1) satisfies the following assumptions:

Assumption 1. The functions $f_1(x_1, x_2, u)$, $f_{21}(x_1)$, $f_{22}(x_1)$ and $g_2(x_1)$ are uniformly continuous and bounded, with a sufficient number of derivatives in their arguments.

Assumption 2. The function $f_{22}(x_1)^{-1}$ exists for all $x_1 \in D \subset R^{n_1}$.

It is necessary to transform SP system (1) to the "block triangular" form

$$\begin{gathered} \dot{z}_1(t) = F_1(z_1, u),\ z_1(0) = z_1^0, \\ \dot{z}_2(\tau) = F_2(z_1, z_2, u),\ z_2(0) = z_2^0,\ \tau = t/\varepsilon, \end{gathered} \tag{2}$$

where the first equation is independent of the fast state variables vector z_2. The form of the functions $F_1(z_1, u)$ and $F_2(z_1, z_2, u)$ will be determined later.

After the conversion of system (1) to form (2) the feedback linearization problem should be solved separately for the slow subsystem that is described by the first equation of system (2) and for the fast subsystem that is described by the second equation of system (2). The resulting feedback linearizing composite control for the original SP system (1) is defined as the sum of slow and fast controls

$$u = u_s + u_f, \tag{3}$$

where u_s and u_f – controls, obtained by solving the feedback linearization problem for slow and fast subsystems respectively.

3 Decomposition of the system

Conversion of SP system (1) to the "block triangular" form (2) is carried out using standard singular perturbation technique [28, 29]. Letting $\varepsilon = 0$ in system (1) and solving the second equation of obtained reduced system with respect to x_2, by assumption 1 and 2 we have

$$\begin{gathered} x_2 = h_0(x_1, u_s) = \\ = -f_{22}(x_1)^{-1}(f_{21}(x_1) + g_2(x_1)u_s). \end{gathered} \tag{4}$$

Substituting (4) into the first equation of system (1), we obtain an expression for the slow subsystem (via the change of variables $x_1 = z_1$)

$$\dot{z}_1(t) = f_1(z_1, h_0(z_1, u_s), u_s) = \\ = F_1(z_1, u_s),\ z_1(0) = z_1^0 = x_1^0. \quad (5)$$

To obtain the equation of the fast subsystem we represent the second equation of system (1) in fast time scale, replacing $\tau = t/\varepsilon$:

$$\dot{x}_2(\tau) = f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)u, \\ x_2(0) = x_2^0, \quad (6)$$

Substituting into (6) the expression for the composite control (3) we get the fast subsystem

$$\dot{x}_2(\tau) = f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)(u_s + u_f).$$

Then applying the change of variables

$$x_1 = z_1,$$

$$z_2 = x_2 + f_{22}(z_1)^{-1}(f_{21}(z_1) + g_2(z_1)u_s),$$

we finally have

$$\dot{z}_2(\tau) = f_{22}(z_1)z_2 + g_2(z_1)u_f, \\ z_2(0) = z_2^0 = x_2^0 - h_0(z_1^0, u_s(0)). \quad (7)$$

In equation (7) slow state variables vector z_1 is treated as a fixed parameter [28, 29].

4 Feedback linearization

Before considering the problem of feedback linearization, we give the following basic notation [3, 4].

Let $\varphi(x)$ be a smooth function and $f(x)$ be a smooth vector field defined on $X \subset R^n$. The scalar function introduced as

$$L_f\varphi(x) = \frac{\partial\varphi}{\partial x} f(x)$$

is so-called a (scalar) Lie derivative of scalar function $\varphi(x)$ along $f(x)$.

Let $f(x)$ and $g(x)$ be smooth vector fields defined on X. The vector field introduced as

$$L_f g(x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x),$$

is a vector Lie derivative, often called a Lie bracket

$$[f(x), g(x)] = L_f g(x).$$

In addition, the following notation is used to define the Lie derivative of order k

$$L_f^0\varphi(x) = \varphi(x),\ \ L_f^0 g(x) = g(x),$$

$$L_g L_f = L_g(L_f),$$

$$L_f^k = L_f(L_f^{k-1}),\ k = 2,3,\ldots\ .$$

4.1 Feedback linearization of slow subsystem

Consider the slow subsystem

$$\dot{z}_1(t) = F_1(z_1, u_s),\ z_1(0) = z_1^0. \quad (8)$$

Let there exist a scalar function $\varphi(z_1)$. We define the Lie derivative of the function $\varphi(z_1)$ along the vector field $F_1(z_1, u_s)$ as

$$L_{F_1}\varphi(z_1, u_s) = \frac{\partial\varphi(z_1)}{\partial z_1} F_1(z_1, u_s).$$

For the Lie derivatives of higher orders we have

$$L_{F_1}^k\varphi(z_1, u_s) = \frac{\partial L_{F_1}^{k-1}\varphi(z_1, u_s)}{\partial z_1} F_1(z_1, u_s),$$

where $L_{F_1}^0\varphi(z_1, u_s) = \varphi(z_1)$.

Assumption 3. The system (8) has a relative degree $r = n_1$ at the point (z_1^0, u_s^0), that is the next conditions hold:

1) $\frac{\partial}{\partial u_s} L_{F_1}^k\varphi(z_1, u_s) = 0$ for all z_1 in a neighborhood of z_1^0, all u_s in a neighborhood u_s^0 and all $k < r$.

2) $\frac{\partial}{\partial u_s} L_{F_1}^r\varphi(z_1^0, u_s^0) \neq 0$.

Due to the above conditions the first r Lie derivatives do not depend explicitly on the input:

$$L_{F_1}^k\varphi(z_1, u_s) = L_{F_1}^k\varphi(z_1),\ 0 \le k \le r-1.$$

Under the above assumptions, there exists a diffeomorphism [30]

$$T_s(z_1) = \{T_s^k(z_1)\},$$

$$T_s^k(z_1) = L_{F_1}^{k-1}\varphi(z_1),\ 1 \le k \le n_1.$$

We define the vector ξ with elements expressed in terms of the transformation

$$\xi_1 = T_s^1(z_1) = L_{F_1}^0\varphi(z_1) = \varphi(z_1),$$

$$\xi_k = \dot{\xi}_{k-1} = T_s^k(z_1) = L_{F_1}^{k-1}\varphi(z_1),\ 2 \le k \le n_1.$$

Differentiating with respect to time t the variable ξ_{n_1}, we obtain

$$\dot{\xi}_{n_1} = \vartheta(\xi, u_s) = L_{F_1}^{n_1}\varphi(T_s^{-1}(\xi), u_s).$$

The feedback linearizing control law for the slow subsystem (8) is obtained by solving the following nonlinear algebraic equation with respect to u_s

$$\vartheta(\xi, u_s) = \upsilon_s. \quad (9)$$

If the equation (9) has an analytic solution $u_s = \psi(\xi, \upsilon_s)$, then the diffeomorphism $T_s(z_1)$ and the control law $u_s = \psi(\xi, \upsilon_s)$ transform the system (8) to the linear form

$$\dot{\xi}_1 = \xi_2,\ \xi_1 = \varphi(z_1),$$
$$\dot{\xi}_2 = \xi_3,$$
$$\vdots$$
$$\dot{\xi}_{n_1} = \upsilon_s.$$

4.2 Feedback linearization of fast subsystem

Consider the fast subsystem

$$\begin{aligned} \dot{z}_2(\tau) &= f_{22}(z_1)z_2 + g_2(z_1)u_f, \\ z_2(0) &= z_2^0 = x_2^0 - h_0(z_1^0, u_s(0)). \end{aligned} \qquad (10)$$

In this equation the slow state variables vector z_1 is treated as a fixed parameter.

As the system (10) is linear in the fast variables state vector z_2 and the control u_f, the external linearization (via feedback) is not required. In this situation, the fast control u_f is selected as a feedback (here $x_1 = z_1$)

$$\begin{aligned} u_f &= -G_f(x_1)z_2 = -G_f(x_1)x_2 - \\ &- G_f(x_1) f_{22}(x_1)^{-1}(f_{21}(x_1) + g_2(x_1)u_s) \end{aligned}, \qquad (11)$$

where $G_f(x_1)$ is designed such that

$$\mathrm{Re}\,\lambda\big(f_{22}(z_1) - g_2(z_1)G_f(z_1)\big) < 0,\ \forall z_1 \in D.$$

The existence of $G_f(x_1)$ satisfying the last inequality is guaranteed if the pair $(f_{22}(z_1), g_2(z_1))$ is controllable uniformly in z_1, i.e. the corresponding controllability Grammian is bounded from below by a positive-definite matrix.

4.3 Feedback linearizing composite control

The resulting composite control is determined from (3) by substituting the relations for the fast and slow control:

$$\begin{aligned} u &= \big(1 - G_f(x_1) f_{22}(x_1)^{-1} g_2(x_1)\big) u_s - \\ &- G_f(x_1)\big(x_2 + f_{22}(x_1)^{-1} f_{21}(x_1)\big). \end{aligned} \qquad (12)$$

It should be noted that the feedback linearizing composite control (12) does not depend on a small parameter ε. The singular perturbation parameter ε can be considered as an uncertainty in the original system (1) [31]. Moreover, there exists some $\varepsilon^* > 0$ such that $\forall \varepsilon:\ 0 < \varepsilon \le \varepsilon^*$ the original SP system (1) is stable, if the slow and the fast subsystems are stable. In this sense, the SP system (1) with the composite control (12) is robustly stable with respect to ε.

5 Illustrative example

As an example, consider the problem of speed control of DC series motor [24]. A mathematical model of the control problem is presented below

$$\begin{aligned} L\frac{di}{dt} &= -R \cdot i - K_T \cdot i \cdot \omega + U, \\ J\frac{d\omega}{dt} &= K_T \cdot i^2 - D \cdot \omega - \tau_L, \end{aligned} \qquad (13)$$

where i is the current in the armature winding, ω is the angular velocity of the motor, U is the voltage in the armature circuit (control input), τ_L is the load torque. Constant parameters of the motor are [24]: $L = 0.0917$ H, $R = 7.2$ Ohm, $J = 7.06 \cdot 10^{-4}$ kg·m^2, $D = 4 \cdot 10^{-4}$, N·m·s/rad, $K_T = 0.1236$, N·m/Wb·A. The load torque is taken so $\tau_L = K_{mL}^2 \omega / (R_{aL} + R_L)$, where $K_{mL} = 0.173$ N·m/A, $R_{aL} = 2.5$ Ohm, R_{aL} – external resistor, $R_{aL} = 5$ Ohm.

Following [24], we formulate the control problem as the problem of tracking control system design for angular velocity ω of the DC motor, the reference signal is ω_{ref}.

The system (13) should be represented in the singularly perturbed form (1):

$$\begin{aligned} \dot{x}_1 &= a_1 x_1 + a_2 x_2^2, \\ \varepsilon \cdot \dot{x}_2 &= (a_3 x_1 + a_4)x_2 + u, \end{aligned} \qquad (14)$$

where

$$x_1 = \omega,\ x_2 = i,\ u = V,\ \varepsilon = L,$$
$$a_1 = -\left(D + \tfrac{K_{mL}^2}{R_{aL} + R_L}\right)/J,\ a_2 = K_T / J,$$
$$a_3 = -K_T,\ a_4 = -R.$$

For SP system (14) the assumptions 1 and 2 are satisfied. The slow subsystem is:

$$\dot{z}_1 = a_1 z_1 + \frac{a_2}{(a_3 z_1 + a_4)^2} u_s^2,\ z_1 = x_1. \qquad (15)$$

Since the problem is formulated as a speed tracking control problem, the function $\varphi(z_1)$ is defined as $\varphi(z_1) = z_1 = \omega$. The system (15) will have a relative degree $r = 1$, if the assumption 3 is satisfied, i.e.

$$\frac{\partial}{\partial u_s} L_{F_1}^0 \varphi(z_1, u_s) = \frac{\partial z_1}{\partial u_s} = 0,$$

$$\frac{\partial}{\partial u_S} L_{F_1}^1 \varphi(z_1^0, u_S^0) = \frac{2a_2}{(a_3 z_1^0 + a_4)} u_S^0 \ne 0,\ u_S^0 \ne 0.$$

Equation (9) to determine the feedback linearizing control law takes the form

$$\upsilon_s = a_1 z_1 + \frac{a_2}{(a_3 z_1 + a_4)^2} u_s^2,.$$

whence we obtain

$$u_s = \sqrt{\frac{(-a_1 z_1 + \upsilon_s)(a_3 z_1 + a_4)^2}{a_2}}. \qquad (16)$$

The fast subsystem in this example has the form

$$\dot{z}_f = (a_3 z_1 + a_4) z_f + u_f.$$

The stabilizing control u_f for the fast subsystem according to (11) is presented in the form

$$u_f = -G_f(z_1) z_2 - \frac{G_f(z_1)}{a_3 z_1 + a_4} \cdot u_s, \qquad (17)$$

where function $G_f(z_1)$ is designed such that

$$\mathrm{Re}\,\lambda\big(a_3 z_1 + a_4 - G_f(z_1)\big) < 0, \ \forall z_1 \in D,$$

that is equivalent to the inequality

$$G_f(z_1) > (a_3 z_1 + a_4).$$

The expression for the composite control is obtained from (12)

$$u = \left(1 - \frac{G_f(x_1)}{a_3 x_1 + a_4}\right) \cdot u_s - G_f(x_1) x_2. \qquad (18)$$

Since for a sufficiently small ε the system (13) with control (18) can be approximately considered as

$$\dot{\omega} = \upsilon_s, i_2 = -\frac{u_s}{a_3 \omega + a_4},$$

that the external control υ_s is conveniently implemented as a proportional controller for tracking error, ie

$$\upsilon_s = k_p e, \ e = \omega_{ref} - \omega. \qquad (19)$$

The simulation results of the application of the composite controller (17)-(19) in the system (13) are shown below. Fig. 1 represents the angular velocity ω against the reference signal ω_{ref}, the transient response for a DC motor current i and the response for control voltage U.

Transient responses of closed loop system for different values of the controller coefficient k_p are shown in Fig. 2. The results show that the angular velocity corresponds to the reference signal ω_{ref}, wherein the tracking quality increases with the increase of the coefficient k_p.

To demonstrate the robust properties of the closed loop system with respect to the parameter ε the system is modeled for different values of perturbation parameter $\varepsilon = 0.0917$, $\varepsilon = 2.5$ and $\varepsilon = 10.5$. Fig. 3 shows transient response and indicates a good quality of system tracking for the reference signal with the presence of uncertainty in the system in the form of a parameter that corresponds to the results in [24].

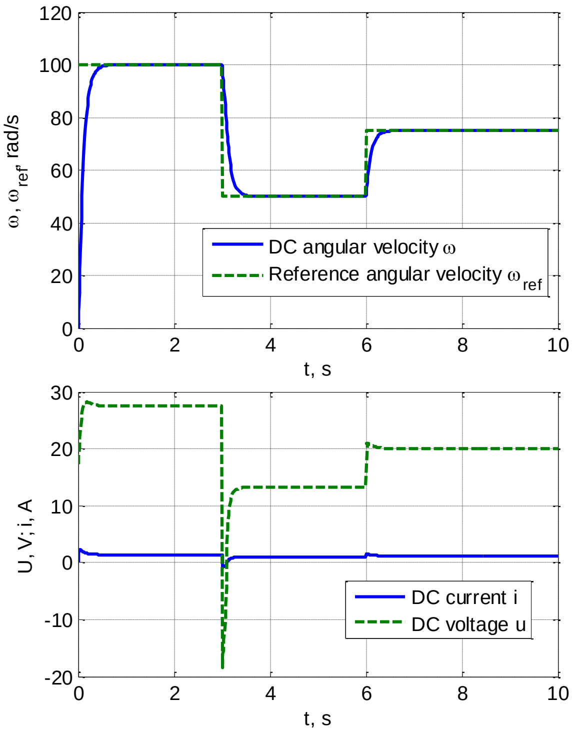

Fig. 1: The simulation results for $k_p = 10$: angular velocity ω, reference signal ω_{ref}, DC motor current i, control voltage U.

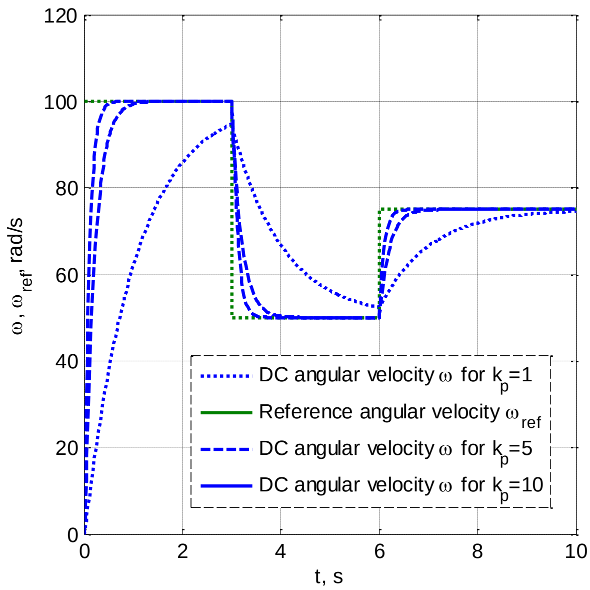

Fig. 2: Angular velocity for different values of the controller coefficient k_p.

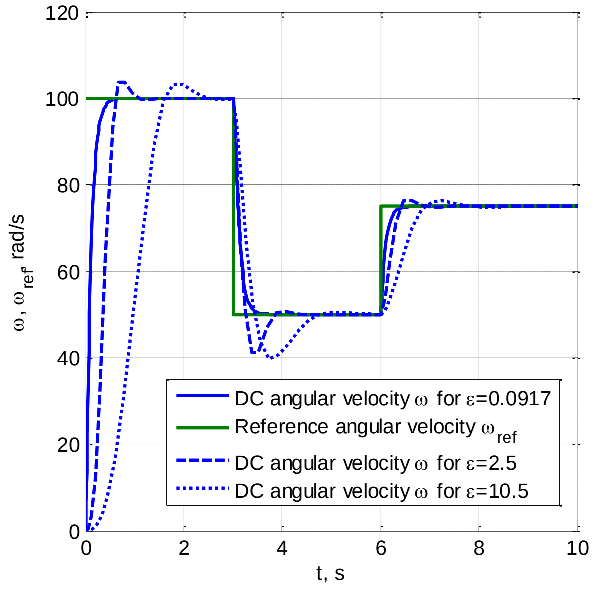

Fig. 3: Angular velocity for different values of the singular perturbation parameter ε .

6 Conclusions

In this paper, based on the idea of a composite control for a class of nonlinear SP systems we proposed a method for the synthesis of control by using of feedback linearization technique.

The advantage of the proposed method is to simplify the original feedback linearization problem for nonlinear SP system by its decomposition into two simpler problems: for the slow subsystem and for the fast subsystem. This fact is especially useful when the original nonlinear system is non-linearized via feedback. In this case the method of approximate feedback linearization is used [8-11]. If the original non-linearized system can be represented in the singularly perturbed form, the proposed method can be used as a means of synthesis of approximate feedback linearizing control.

The efficiency of the proposed method is demonstrated by an example of the tracking control system development for angular velocity of the DC series motor.

In future studies, for a closer approximation to reality in the mathematical description of the system we should take into account the presence of the interval uncertainty. In this case, for the synthesis of external linearizing feedback for uncertainties compensation, it is planned to use the techniques of parameter self-tuning and optimization [32-34].

7 Acknowledgements

The research was partially supported by the Russian Foundation for Basic Research (research project No. 15-08-06859 a.).

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