A heuristic algorithm for the constrained location - routing problem

L. Guerra, T. Murino and E. Romano

***Abstract*—**The logistics system of a firm deals with purchasing materials (acquisition logistics), controls work in process in each production phase (production logistics system), through the distribution management, it controls the flow of the products delivered to the customer (distributive logistics), defines the backtracking of a discarded, disused or damaged product, to reemploy its parts or materials (reverse logistics). The structures displaced within the logistics network must guarantee an opportune level of service and the cutback of logistics costs. Transport system performance are, therefore, of primary importance, as well as the location of the distribution centers and products distribution issues. In literature they are present accurate mathematical models and effective solution techniques to face location, allocation and distribution problems. The topic is still of extreme interest because of the increasing structural complexity of the models due to the constraints imposed by the "real systems" representation. Integrated location routing models are used to solve the facility location problem (FLP) and the vehicle routing problem (VRP) simultaneously so to reflect the interactions between the two decisions. In this paper, particularly, a possible approach (the *TSP-VRP* heuristic) will be proposed to optimize the routing phase in a Location-Routing Problem (LRP). Results are compared with those obtainable turning to other commonly adopted procedures.

***Keywords*—**Distributive Logistics, Location-Routing Problem, Traveling Salesman Problem, Vehicle Routing Problem.

I. Introduction

THE logistics system of a firm deals with purchasing materials (acquisition logistics), controls work in process in each production phase (production logistics system), through the distribution management, it controls the flow of the products delivered to the customer (distributive logistics), defines the backtracking of a discarded, disused or damaged product, to reemploy its parts or materials (reverse logistics).

The structures displaced within the logistics network must guarantee:

- an opportune level of service, with the purpose to locate the product as near as possible to the market (peripheral warehouses);
- the cutback of logistics costs. The consignments, related to different products, are gathered so to get meaningful economies of transport (distribution centres).

Transport system performance are, therefore, of primary importance: it must guarantee the mobility of the products among the various nodes of the system with high efficiency and punctuality, reducing, at the same time, the transport cost which, in particular cases, can weigh for 50% on the overall logistics costs. For example, according to Srivastava and Benton [26], the overall cost of transportation and warehousing accounts for over 20% of the GNP, therefore substantial savings can be achieved by improving distribution systems even by only a small amount.

Then, the location of the distribution centres (facilities or more simply warehouses) and connected products distribution issues represent some crucial questions [7, 18]. In different productive contexts these two aspects tightly appear interdependent, for such reason they must contemporarily be considered in the development of theoretical models and in the practical planning of the logistics network.

In practice, products are distributed from facilities to customers in two main ways:

- each vehicle serves only one customer on a straight-and-back basis on a given route. This is when a full truckload is requested;
- a vehicle stops at more than one customer on its route. This is when each customer requires less than a truckload.

In the first case, the delivery cost can be represented assuming the unit shipment cost from a facility to a customer to be independent of the route taken to visit the customer. Then the total delivery cost is the sum, considering customers and facilities, of the product of the unit shipment cost from the facility to the customer and the number of units delivered to that customer.

The delivery cost, instead, depends on the route of the delivery vehicles when the customer demands are less than a truckload: the previously proposed cost function ignores this interdependence between routing and location decisions. In these cases, the routing decisions should be incorporated in

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the location models to represent them realistically.

Naturally, real situations are even more tricky because there are many possible constraints to consider. For example, the customers have the given delivery demands and a vehicle on its route cannot serve more customers than its capacity permits. For each customer a time interval, often called the *time window* [32], and a time of service are defined. The aim could be establishing a set of routes which covers each customer exactly once, ensures that the service at any customer starts within the time window and preserves the vehicle capacity constraints. Furthermore the set of routes should minimize, firstly, the number of vehicle used, and secondly, the total distance traveled by vehicles.

Integrated location routing models are used to solve the *facility location problem* (FLP) and the vehicle *routing problem* (VRP) simultaneously so to reflect the interactions between the two decisions. Location-routing models are especially necessary for systems where the time horizon for the facility location decisions are not too long and location costs are comparable to the routing costs.

II. LITERATURE REVIEW

In literature there are accurate mathematical models and effective solution techniques to face location, allocation [18] and distribution problems [3, 16], that resort to the concept of integrated logistics systems and whose basis is constituted by a combined *location-routing model* (LRP) [15, 17].

Generally speaking, the location routing problem can be described as follows. There's a feasible set of potential facility sites, locations and expected demands of each customer are given. Each customer is associated to a particular facility which will supply his demand. Vehicles are dispatched from the facilities to carry out customers demand: they operate on routes that include multiple customers. There is a fixed cost associated with opening a facility at each potential site, and a distribution cost associated with any routing of vehicles that includes the cost of acquiring the vehicles used in the routing, and the cost of delivery operations. The last one is supposed to be linear in the total distance travelled by the vehicles. The LRP is to determine the location of the facilities and the vehicle routes from the facilities to the customers to minimize the sum of the location and distribution costs such that the vehicle capacities are not exceeded.

The main difference between a location-routing problem and a classical location-allocation problem is that, once the facility is located, in the former it's required that the customers are served along a route, while in the latter every customer is directly connected to the same facility (*radial distribution*) [9, 14]. Considering the first approach, the optimal facility location and the simultaneous construction of the routes leads to a considerable cutback of the overall costs. A LRP, generally speaking, can be assimilate to a vehicular scheduling problem (*vehicle routing problem*, VRP) in which the optimal number and location of the facilities are simultaneously determined with the vehicles scheduling and the circuits (*route*) release so to minimize a particular function (in general, the overall costs: costs of distribution, stocking and transportation) [5].

However, the LRP, considered as "the scheduling of locations taking into account route scheduling issues", is, clearly, NP-hard since it is constituted by two NP-hard problems and it's for this reason that simultaneous solution methods for locating and routing are limited to heuristics ([6], [17], [15]). On the other hand, location and routing problems can be seen as special cases of LRP:

- if each customer has to be directly connected to the facility, the LRP reduces to a classical location problem;
- if the centre location is settled, the LRP can be considered as a VRP.

Solution methodologies can be classified according to the way they create a relationship between the location and routing problems [19, 23].

In *Sequential methods* the location problem is first solved minimizing the distances between facility and consumers (radial distance), then a routing problem is faced. These methods don't allow a feedback from the routing phase to the location one so a sub-optimal design for the distribution system could be determined.

Clustering solution methods first divide and group the customers, then:

- for each cluster a facility is located and a VRP (or TSP) is executed [28];
- a *travelling salesman problem* (TSP) for each cluster is executed and then the facilities are located.

Iterative heuristics decompose the problem in two sub-problems which are iteratively solved moving the data from a phase to an other.

Although iterative methods are an improvement of sequential methods, when the location algorithm ends, it starts again receiving as input the new information coming from the routing algorithm. From a designing point of view, iterative heuristics give the same importance to these sub-problems.

Hierarchical heuristics consider, instead, the location as the main problem and the routing as a subordinate problem.

To solve a LRP it is possible to use multi-phase based procedures, which, breaking-up the problem, reduce its complexity. These ones include the combination of four algorithms:

- location-allocation first, route second;
- route first, location-allocation second;
- saving / insertion;
- routes improvement /exchange.

Among these, the last two are often used to solve vehicle routing planning problems within the LRP context [13].

In Min [14, 15] a problem concerning some terminals location (consolidation terminals) is considered. The products coming from different supply centers are first collected in a terminal and then dispatched to the consumers. Hierarchical network configurations with hub facilities have proven to be flexible and cost-effective as is evidenced, for example, by their increased use in the transportation and telecommunication industries. Although hub networks often increase the total distance travelled by passengers/freight/communications versus direct link networks, they can reduce total costs by more efficient vehicle/network infrastructure utilization by better matching capacity to demand. This issue is, however, somehow more complex than a LRP as there’s a certain number of supply centers and both the centers and the consumers must be assigned to the terminals. The consumers are clustered according to vehicles capacity and the “centroid” of each cluster is used in terminals location.

Barreto et al [1] used a cluster analysis procedure in a LRP heuristic approach (*route first, location second*). The consumers are clustered, a TSP for each cluster is executed and, finally, the facilities are located. Capacity constraints both for the vehicles and the distribution centres are considered (*capacitated location-routing problem*, CLRP).

In [31] is proposed a method for solving the *multi-depot location-routing problem* (MDLRP). Multiple depots, multiple fleet types, and limited number of vehicles for each different vehicle type are considered. The original problem is divided into two sub-problems, the location-allocation problem and the general vehicle routing problem. Each sub-problem is then solved in a sequential and iterative manner by an algorithm based on simulated annealing.

Tuzun and Burke [29] employed the two phase *tabu search algorithm* in both location and routing phases. In the location phase of the algorithm, a TS is performed on the location variables to determine a good configuration of facilities to be used in the distribution. For each of the location configurations visited during the location phase, another TS is run on the routing variables in order to obtain a good routing for the given configuration. The two searches are coordinated so that an efficient exploration of the solution space is performed.

Wu et al. [31] faced an extension of the LRP, considering multiple type of facilities and fleet with a limited number of vehicles for each different type of vehicle. The LRP is divided in a *location-allocation problem* (LAP) and a vehicle routing problem. To solve the LAP and the VRP, the authors developed some heuristic methods based on the *Simulated Annealing* (SA) technique.

Toth and Vigo [28] considered an extension of the capacitated VRP, known as the *vehicle routing problem with bckhauls* (VRPB), in which the set of customers is partitioned into two subsets: Linehaul and Backhaul customers. Each Linehaul customer requires the delivery of a given quantity of product from the depot, whereas a given quantity of product must be picked up from each Backhaul customer and transported to the depot. They presented a cluster-first-route-second heuristic which uses an original clustering method (based upon a modified TSP heuristic and inter-route and intra-route arc exchanges) which could also be used to solve problems with asymmetric cost matrix.

Lin et al. [11, 12], a problem of location and distribution relative to a telecommunications service in Kowloon peninsula (Hong-Kong) is faced. The authors divide the LRP in three phases: facilities location, routing and loading. Each phase is treated applying heuristic or exact algorithms. An initial number of facilities is determined, then applying a specific algorithm [5] and considering capability (warehouses and vehicles) and routes length constraints, the initial routes are established. The routes are "reprocessed" by an improving algorithm (based on the travelling salesman problem) so to determine the optimal sequence of the nodes. To cut the routing costs, meta-heuristic techniques (*Threshold Accepting*, TA, and *Simulated Annealing*) are used and, at the end of the phase, every route is improved again through TSP to further reduce the distribution costs. Finally, different routes are allocated to a single vehicle until the overall route time doesn’t exceed the established temporal limit. At the end of the loading, a final solution is gotten for the considered number of facilities. If the recorded lowest cost results smaller than the opening cost for a further facility, the algorithm ends; otherwise, the procedure is repeated increasing by one the facilities number.

An iterative heuristic for a three-level LRP (factories, depots, customers) with capacitated routes and depots and a maximum duration per route was developed by Bruns and Klose [3]. One iteration begins by building clusters of customers fitting vehicle capacity. Then, an estimate of the routing costs is done for the allocation of the customers to depots. An heuristic, obtained by relaxing capacity constraints, determines which depots should be opened to minimize routing costs. Then, a routing phase, involving saving criteria, 2-Opt moves and exchanges of customers, is executed. Finally, the estimate for the routing costs is refined and the procedure iterates until the estimate is stabilized or a maximum number of iterations is reached.

In [4] is presented a meta-heuristic to solve the LRP with capacitated routes and depots. A first phase executes a GRASP (*Greedy Randomized Adaptive Search Procedure*), based on an extended and randomized version of Clarke and Wright algorithm. This phase is implemented with a learning process on the choice of depots. In a second phase, new solutions are generated by a post-optimization using a path relinking. The method is evaluated on sets of randomly generated instances, and compared to other heuristics and a lower bound.

In [7], the problem of a system characterized by a forward channel and a reverse channel is faced (soft drink industry where empty bottles have to be returned): operating the forward and reverse channel separately may result in an unnecessary vehicle utilization. This could be avoided by combining pick-ups at the customer locations, the pick-ups being destined for the depot, with deliveries, originating in the depot, being dropped off within the same vehicle routes. The heuristic proposed in [19] for this particular kind of VRP, the VRPSDP (*vehicle routing problem with simultaneous delivery and pick-up*), is modified to develop a graphical construction algorithm based on the "cheapest-insertion" concept. The idea is to successively insert customers into "growing" routes. In each step one customer is inserted. Either several routes can be constructed in a parallel way or routes can be filled consecutively.

III. PROPOSED APPROACH

In this paragraph an alternative approach will be presented to determine an optimal solution to the routing phase faced in a LRP. After a qualitative definition of the problem, it will be presented its analytical formulation. The CLRP will be faced solving the connected LAP and VRP. The results will be validated through some comparative tests.

A. Problem definition

A set of consumers and potential facility is given. If di is the demand of a consumer, each consumer with di>0 must be allocated to a facility so to completely satisfy di. The consignment is delivered through vehicles that depart from a facility and operate on circuits that include more customers. The set-up cost of a centre and the unitary distribution cost have been fixed. The vehicles and the potential centres have limited capacity. Facilities location and vehicles routes have to be determined so to minimize the overall costs (location and distribution costs).

The CLRP is constrained by the followings conditions:

- The demand of each customer must be satisfied;
- Each customer must be served by a single vehicle;
- The overall demand on every route must be smaller or, at the most, equal to the capacity of the vehicle allocated to the route;
- Each route begins and ends to the same facility.

It is assumed, moreover, that the vehicle fleet is homogeneous and there's no limit to its dimension.

B. Graph representation and objective function

Let $G = (N,A)$ be an oriented graph, where $N = \{\upsilon_1,\ldots,\upsilon_{m+n}\}$ is constituted by the nodes $D = \{\upsilon_1,\ldots,\upsilon_m\}$ (potential facilities locations) and by the nodes $I = \{\upsilon_{m+1},\ldots,\upsilon_{m+n}\}$ (demand centres). Each edge $\upsilon_i\upsilon_j \in A$ represents the existing link between the pair of nodes that defines it and it is associated with a distance, or cost, $c_{ij} > 0$. If some connections between nodes are forbidden it is still possible to consider a complete graph setting to ∞ the distance between them. It's assumed the graph to be symmetrical, therefore $c_{ij} = c_{ji}$. For each potential service node $\upsilon_i \in F$ it is known the maximum service capacity Q; for each demand node $\upsilon_j \in I$ it is known the service demand d_i. The deliveries are effected by a fleet of k vehicles characterized by a maximum capacity K.

If D is the set of potential facilities, I is the customers set, V the vehicles set, the mathematical formulation of the problem is:

$$Min \sum_{i\in D} F_i y_i + \sum_{k\in V}\sum_{i\in D\cup U}\sum_{j\in D\cup U, j\neq i} C_{ij} x_{ijk} \tag{1}$$

subject to:

$$\sum_{k\in V}\sum_{i\in D\cup I} x_{ijk} = 1 \quad \forall j \in I \tag{2}$$

$$\sum_{i\in D}\sum_{j\in I} x_{ijk} \le 1 \quad \forall k \in V \tag{3}$$

$$\sum_{j\in D\cup I} x_{ijk} = \sum_{j\in D\cup I} x_{jik} \quad \forall k \in V, \forall i \in I \cup D \tag{4}$$

$$\sum_{j\in I}\sum_{i\in D\cup I} d_j x_{ijk} \le Q_k \quad \forall k \in V \tag{5}$$

$$\sum_{j\in I} d_j z_{ij} \le V_i y_i \quad \forall i \in D \tag{6}$$

$$\sum_{j\in I}\sum_{K\in V} x_{ijk} - y_i \ge 0 \quad \forall i \in D \tag{7}$$

$$\sum_{j\in I} x_{ijk} - y_i \le 0 \quad \forall i \in D, \forall k \in V \tag{8}$$

$$\sum_{i\in I} x_{ijk} = \sum_{i\in I} x_{jki} \quad \forall j \in D, \forall k \in V \tag{9}$$

$$v_i - v_j + (m+n)\cdot\sum_{k\in V} x_{ijk} \le m+n-1 \quad i,j \in I, i \neq j, k \in V$$

$$n = \text{number of costumers to be served} \tag{10}$$

$$m = \text{number of potential facilities locations}$$

$$x_{ijk} \in \{0,1\} \quad \forall i,j \in I \cup D \tag{11}$$

$$y_i \in \{0,1\} \quad \forall i \in D \tag{12}$$

$$z_{ij} \in \{0,1\} \quad \forall j \in I, \forall i \in D \tag{13}$$

where:

- F_i is the set-up cost for facility i, with $i \in D$;
- C_{ij} is edge i - j cost, with $i,j \in D \cup I$;
- d_j is the demand of the customer j, with $j \in I$;
- V_i is the capacity of the facility i, with $i \in D$;
- Q_k is the vehicle k capacity, with $k \in V$;
- $x_{ijk} = 1$ if vehicle k goes from node i to node j, with $i, j \in D \cup I, k \in V$;
- $S = \{D\} \cup \{I\}$ is the set of all possible facility locations and customers;
- $y_i = 1$ if a facility is set-up at node i, with $i \in D$;
- $z_{ij} = 1$ if customer j is allocated to facility i, with $i \in D$, $j \in I$.

The objective function minimizes the set-up costs of the facilities and the distribution costs. Equation (2) guarantees that each customer has been assigned to a single facility, (3) guarantees that each vehicle is sent by a single depository. Equation (4) assures that the very same vehicle enters and exits in each node i, (5) and (6) assure that vehicle and facilities capacities are not exceeded. Equations (7) and (8) assure that vehicles only come from opened facilities and (9) assures that a vehicle leaves and arrives in the same facility. Equation (10) guarantees the absence of sub-circuits (*Sub-Eliminator Constraints*, SEC), which guarantee that each tour must contain a depot from which it originates, i.e. none of the tours consists of customers only.

C. CLRP solution

To solve the CLRP a heuristic approach is proposed that divides the problem in:

- Location-allocation (LAP);
- Routing (VRP).

This two phase approach offers simple and natural representation, and the solution obtained in LAP phase is used as an input to the VRP phase.

In the first phase, the solution is a set of selected facilities and a project to allocate the customers to the facilities. In computing the distances each customer is directly connected to the nearest facility (*radial distance*). This solution will be used as input for the VRP, producing a set of admissible routes [5, 8, 10].

Because of the aggregative nature of the demand nodes, the various cost/time components in this problem (route cost/time and stopover cost/time in every node) must be transformed into node-to-node cost parameters. If s_i is the stopover cost/time in v_i node and c_{ij} is the travelling cost/time from node v_i to node v_j, than the transformed travelling cost/time, between node v_i and node is given v_j by:

$$c'_{ij} = c_{ij} + \frac{1}{2}(s_i + s_j) \quad \forall i \neq j \tag{14}$$

In this way, the problem is converted to the classical LRP with no stopover cost/time considered. The transformed cost/time will be simply indicated as c_{ij} .

The procedure to solve the LAP consists in the following steps (Fig. 1):

- *Customers allocation to the potential facilities*. In computing the distances each customer will be directly connected to the nearest facility (*radial distance*) if its capacity constraint is not violated, otherwise the customer will be assigned to another facility minimizing the cost function. The output of the phase is an *incidence customer-facility matrix*. Each item of such matrix will be 1 if the customer i is connected to the facility j;
- *Customers distribution list determination*. A set where each item e_i is the number of customers assigned to facility i. The items will be sort in descending order in a following step.
- *Facilities number determination*. In this step a lower bound on the number of facilities is established (N_f). However, the actual distribution of the demand and supply nodes is not considered. Given N_f, it is possible to compute the combinations $\binom{m}{N_f}$.
- *L matrix definition*. Rows are sorted in descending order considering the number of customers assigned to each facility. In other words, the first line of the matrix contains that set of N_f facilities with the highest number of customers in the closeness. The solution obtained in this phase provides the minimum number of facilities to satisfy the whole demand of the customers and the potential facilities configurations; these ones will be the input for the VRP phase. Each item of L matrix will be 1 if the corresponding facility column is open, 0 otherwise. Therefore the sum of each row is the $\binom{m}{N_f}$ combination.

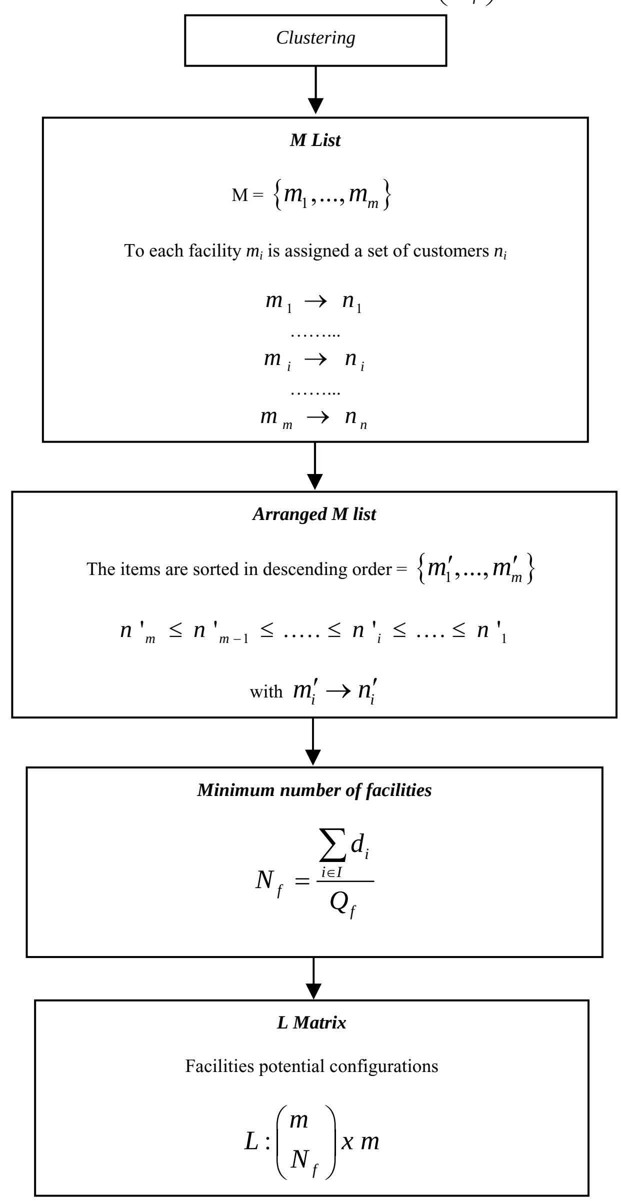

Figure 1 - LAP solving procedure

Given the L matrix, the set of the potential configurations is constituted by all the sets of facilities previously determined and, therefore, every line of the matrix defines a potential configuration of facilities to enable. The procedure to solve the VRP consists in the following steps (Fig. 2):

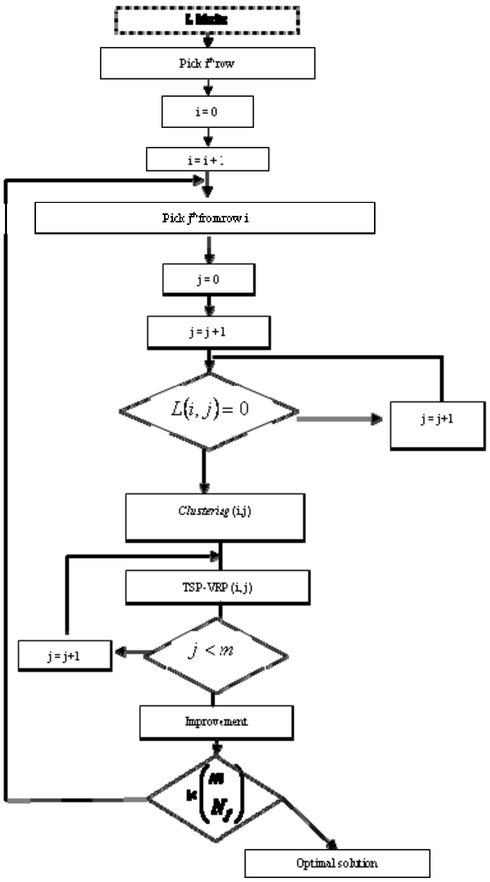

Figure 2 – VRP solving procedure

- *Customers allocation to facilities*. In this step the consumers are reassigned to the considered facilities. This step doesn't differ from the analogous one in the LAP procedure: within every cluster a VRP will be performed to determine the necessary routes, satisfying the whole demand of all the customers.
- *Resolution algorithm*. At first a TSP is resolved with no vehicles capacity constraint, then the same TSP is modified to take into account this constraint (*TSP-VRP*). This could be result in an inadmissible solution of TSP for the VRP, and therefore the initial route is modified, producing a set of routes.

In Fig. 3 the algorithm to solve the TSP-VRP is graphically reproduced. The same procedure is performed for both the directions.

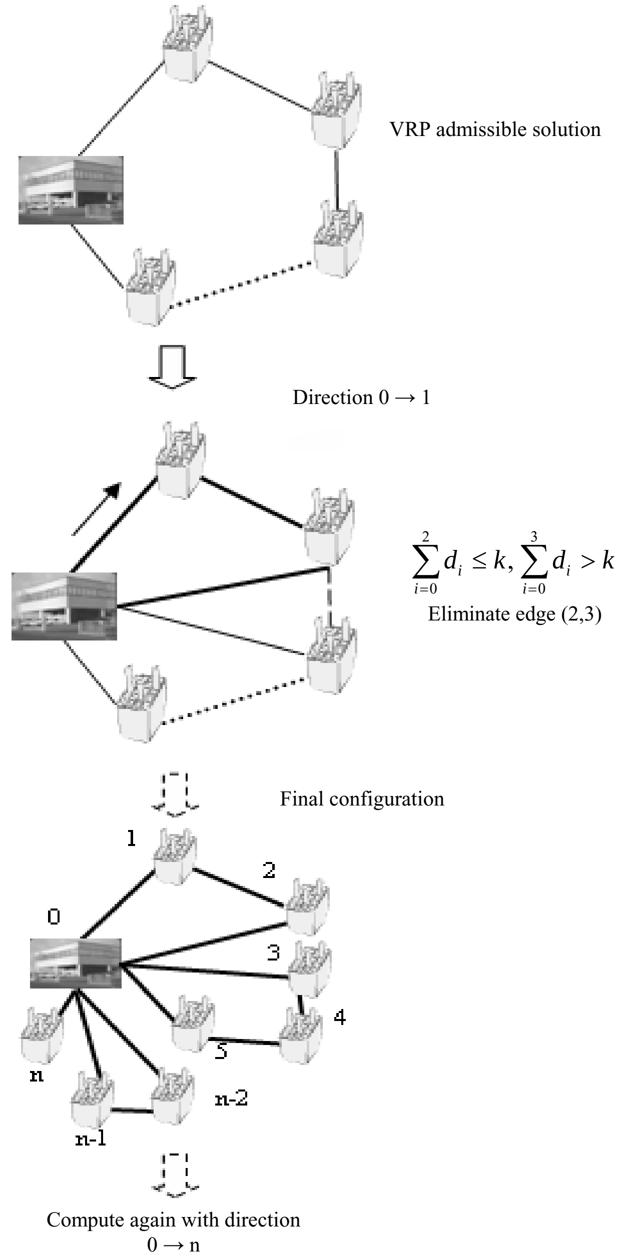

Figure 3 – Edges elimination

The steps to solve the TSP-VRP are:

1. A set of *n* demand nodes and one facility *0* are given. Requested demand is satisfied by vehicles of capability *K*;
2. The TSP is performed with no vehicles capacity constraints to determine an initial route containing all the nodes (0-1-2-…-n-0);
3. Considering vehicles capacity constraint the route is modified as it follows:

   3.1 Set R → Macro route,
   3.2 Initial node = facility;
   3.3 Possible travelling directions: 0 → 1; 0 → n;
   3.4 Choice of one direction (choosing one direction is due to c_{ij} = c_{ji} hypothesis, symmetrical TSP and

VRP);

3.5 Travel across the edges till the demand of the nodes doesn't exceed K. If $\sum_{i=1}^{n} d_i \leq K$ than R is admissible for the VRP, otherwise 3.6;

3.6 Let s and t be two generic consecutive nodes belonging to the circuit (0-1-2-…-s-t-…-n-0) such that $\sum_{i=1}^{s} d_i \leq K$ and $\sum_{i=1}^{t} d_i > K$. Since the capacity of the vehicle is overcome, the edge (*s, t*) is eliminated and the edges (*s,0*) and (*0,t*) are established. In such a way two routes are determined: $R' : (0-1-...-s-0)$ and $R'' : (0-1-...-s-0)$. Record R' ;

3.7 Set $R = R''$ and go to step (3.1).

The optimal VRP solution is the one with the lowest cost in the set of found solutions.

The solution for the routing phase, is obtained performing, at first, a TSP and determining a route characterized by the lowest "travelling" cost. The route passes through every node just one time (*Hamiltonian circuit*). In the following phase, when more strength constraint are considered, a set of optimal routes to serve the customers is defined, guaranteeing the optimality of the nodes sequence inside each route. So, the best solution in the whole solutions set is characterized by an optimal number of routes and each route is characterized by the best sequence (in terms of time/costs) of the served demand nodes.

The TSP-VRP algorithm starts from a macro-route which is divided, in a second phase, in a certain number of sub-route. The nodes sequence, nevertheless, doesn't change, it's the optimal one suggested by the resolution of the TSP.

IV. Computational results

Two main group of tests have been conducted.

Initially, obtained results have been examined applying the algorithm to problems characterized by a limited number of nodes. In this phase it has been possible to adopt technique and tools for the exact solution (global optimum) of the problem, both for the conventional VRP procedure and for the TSP-VRP. For the latter one, particularly, an exact procedure to determine the Hamiltonian circuit is used. This circuit will be modified to take in account vehicles and facilities constraints. Obtained results allowed a verification of the TSP-VRP model.

Subsequently, some problems have been examined characterized by an increased number of nodes. Heuristics or meta-heuristics have been applied, to random instances or to instances proposed in literature, because of the increased computational difficulties. Particularly, tests has been conducted applying Clarke and Wright's algorithm [5] both in the case of VRP and TSP. The choice of this algorithm is due to its robustness and simplicity. In this phase, obtained results allowed to calibrate the TSP-VRP model. Turning to heuristic or meta-heuristic techniques to solve the VRP, if high number of nodes are considered, would result in sub-optimal solutions. Particularly it wouldn't be able to be respected, within each route, the optimal demand nodes travelling sequence. In order to find an optimal travelling sequence (least route cost and/or minimum route time), a procedure of local improvement applied to each circuit is carried on. The TSP-VRP model, nevertheless, allows to bypass such a problem because it stars from the configuration provided by the TSP.

An initial set of nodes is assigned. The VRP and the TSP-VRP have been executed.

To solve the VRP, and to find a solution of the TSP in the TSP-VRP, the Branch and Bound model has been used implemented in LINGO® 10. The number of nodes is equal to 7 (the facility node is included), and the cost matrix of all links is assigned. The VRP provided a cost function value equal to 178. Applying the same model to solve the TSP, considering the very same number of nodes, resulted in a Hamiltonian circuit whose overall cost is equal to 74. The heuristic "cut" procedure has then determined the number of needed sub-routes to comply with capacity constraints. Obtained result is, as for the VRP, equal to 178.

The same problem has been resolved with the heuristic procedures by Clarke and Wright (savings algorithm). The initial star-shaped solution of the VRP provided a cost function value equal to 208. Considering the savings matrix this solution is optimized and a final value of 178 is obtained. The substantial difference with respect to the TSP-VRP algorithm, nevertheless, is the growth of the iterations number and of the computational speed.

The TSP-VRP robustness is proved by the results obtained varying some input factors. Particularly, as concern vehicles capacity variation, obtained results are in Table I and in Table II.

TABLE I – INPUT DATA

Node	Demand
1	*Facility*
2	3
3	2
4	4
5	3
6	2
7	2

TABLE II - SOME EXPERIMENTAL RESULTS (DISTANCES AND/OR COSTS) VARYING VEHICLES CAPACITY

	Vehicles capacity										
	k	4	5	6	7	8	9	10	11	12	13
Instances	VRP	574	471	395	433	433	410	395	395	395	343
	TSP - VRP	574	471	395	433	433	408	395	395	395	343

In these cases, the TSP almost always provides the optimal initial solution for the VRP. Therefore, vehicles capacity constraint doesn't generally distort the optimal sequence of the nodes provided by the TSP resolution. The model has then been tested on an increased number of nodes (Table III).

TABLE III - SOME EXPERIMENTAL RESULTS (COSTS AND/OR DISTANCES) VARYING NODES NUMBER

		Nodes number			
	n	***14***	***12***	***10***	***25***
Instances	***VRP***	988	760	670	1356
	TSP-VRP	902	742	670	1325

It can be noticed that the solution provided by the TSP-VRP, is always better than the solution provided by Clarke and Wright's algorithm for the VRP. This result shows the better improvement capabilities of the TSP-VRP.

Table 4 gives computational results for the four heuristic and branch and bound algorithm.

TABLE IV - COMPUTATIONAL RESULTS COMPARISON

Problem (n° of nodes)	**Solution (total costs)**			**CPU time (s)**		
	B&B (LINGO 10)	VRP	TSP-VRP	B&B (LINGO 10)	VRP	TSP-VRP
1 (10)	670	670	670	5	7	5
2 (12)	742 (feasible solution)	760	742	7	7,8	4,8
3 (14)	1031 (feasible solution)	988	902	10	12	6,4
4 (25)	1467 (feasible solution)	1356	1325	13	13	7

There are three problems in which the branch and bound algorithm does not reach an optimal solution in a fixed time limit: in these cases the best obtained solution within the time limit is reported. When it exists the optimal solution, the TSP-VRP heuristic solution coincide with the optimal/best solution in the B&B. There's, however, a substantial tradeoff in the running time.

In Fig. 4 and Fig. 5 solutions generated by the TSP-VRP heuristic and by the Clarke and Wright's algorithm are, respectively, reported considering a vehicle capacity of five units. It must be noticed that the first solution is characterized by non-overlapping routes which is synonymous of an effective optimization.

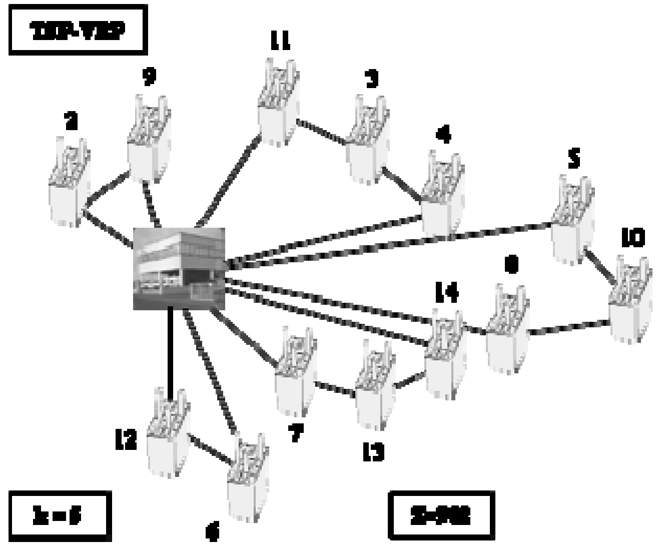

Figure 4 - *TSP-VRP* final solution

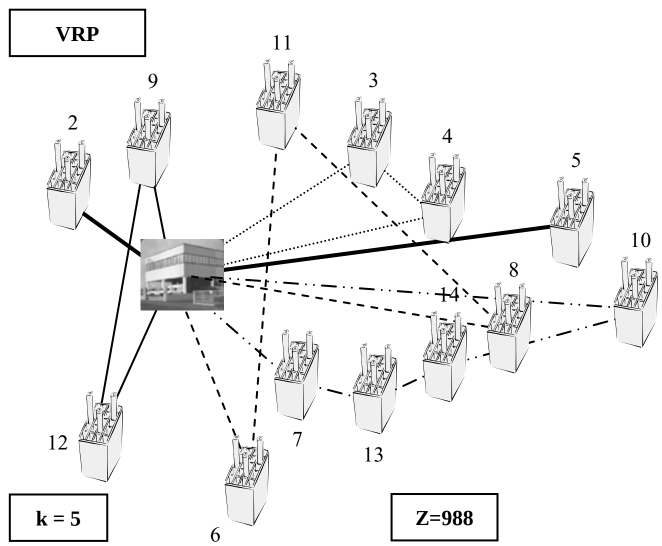

Figure 5 - Clarke and Wright's algorithm final solution for the *VRP*

V. CONCLUSIONS

As known the lower bound for a VRP is provided by a kTSP [2, 20], where *k* is the minimum number of vehicles to fulfil the overall demand, while an upper bound is generally provided by an heuristic solution of the problem.

Comparing the solutions provided by TSP and VRP to the same instance, it is noticed that in 95% of cases, the routes provided by the VRP derive from the macro-route by TSP. The TSP-VRP algorithm supplies, therefore, more than satisfactory results. Moreover, the macro-route from which each solution of the VRP derives is always the global optimal one, as the Hamiltonian circuits has been deduced applying exact algorithms. The solutions of the various VRPs are not always optimal, but however they derive from the TSP modifying the macro-route with respect to capacity constrains.

The TSP-VRP model, finally, provides results which depends on:

- the quality of the TSP solution;
- the number of nodes considered;
- vehicle capacity.

Due to complexity of the location routing problem, heuristic approaches are a promising way to find good solutions for medium and large problems. In this paper, an heuristic modelling to determine the feasible and/or local optimum routes, that uses simple procedures was presented.

Using heuristic algorithms to solve the TSP and applying the TSP-VRP, provided a 3% decrease in costs, proving the effectiveness of the model. Despite these encouraging results, there are yet many opportunities ahead. The work is in progress and improvements are expected. Particularly, the construction of groups (cluster) with limited capacity is a crucial step, so it will be necessary to improve the way the method performs it.

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