Two dimensional dynamic modeling of a coaxial plasma opening switch with effect of Hall current and RLC equation

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Abstract—The plasma dynamic and the magnetic-field penetration in a coaxial plasma opening switch (POS) have been studied based on the two dimensional single-fluid magneto hydrodynamic (MHD) equation coupled with generalized Ohms law. MHD equations were discretized using Roe averaging method which is in essence a non-diffusive scheme. RLC equation is solved using Runge Kutta method to determine the current of the generator during the conduction phase. For this purpose, the voltage of the generator edge of the plasma has been obtained by integrating the electric field on the edge by Simpson method. The code was validated in two cases: firstly, assuming the boundary of magnetic field changes sinusoidal and secondly, obtaining magnetic field using calculated current from RLC equation. The contours of magnetic field, plasma density and plasma temperature are plotted in various time with initial plasma density $n_0 = 8 \times 10^{15}$ cm^{-3}.

Keywords—plasma opening switch, magneto hydrodynamic, simulation, RLC circuit.

I. INTRODUCTION

THE plasma opening switch (POS) has been used for pulse compression and power amplification in a variety of inductive energy storage systems with conduction times ranging from tens of nanoseconds to in excess of one microsecond. In such systems, the POS first acts as low series impedance while charging a vacuum inductor with energy from a capacitive store, and then rapidly transitions to a high impedance to transfer the energy quickly and at high power to a parallel load. The rate at which this opening occurs

determines the efficiency and the timescale of the energy transfer, and is directly related to the voltage developed by the switch [1]. Generally, the operation of a POS can be divided into three stages, namely, the conduction phase, the opening phase and the power-flow phase. During the conduction phase, a large amount of magneto electrical energy flows into the plasma from the generator edge, and the fast-magnetic-field penetration is a fundamental process. Large energy dissipation accompanies the penetration of the magnetic field [2]. In the previous studies [3-7], the interest was focused on the short conduction-time POS, in which the plasma density is low, and the characteristic scale length is less than the ion skin depth. Thus ions could be considered immobile and the mechanism of the magnetic field penetration is determined only by the electron dynamics that could be described by the electron-magneto hydrodynamic (EMHD) theory. It was found that the input energy flow penetrates into the plasma near the cathode and propagates radically into the plasma.

A large dissipated energy becomes the thermal energy of the Plasma [2]. If the electron collisionality is low (but not too low), so the electron inertia could be omitted. In this case, the dissipation is caused by the resistively of electrons, the rate of energy dissipation and the partitioning of the power are determined by the Hall field and the geometry [7]. If the collisionality is too low, resulting in a situation where the electron inertia could not be ignored, the heating of electrons might be small and the dissipated energy is then converted into the electron kinetic energy rather than the electron thermal energy.

For long conduction-time POSs, i.e., time scales being on the order of 1μs, the plasma density is high enough ($10^{14} - 10^{16}$ cm^{-3}). In this case, the ions should be considered, and the POS plasma dynamics can be described by MHD model [8]. It has been shown by the numerical simulations [2] that the plasma displacement and the magnetic-field penetration occur due to the combined influence of the Hall Effect and $\mathbf{j} \times \mathbf{B}$ force. During the penetration of the magnetic field into the plasma, the large magnetic-field-energy dissipation may occur. It has been shown that part of the dissipated magnetic-field energy can become the electron thermal energy. The previous theoretical investigations [9], [10] did not consider the energy exchange between the magnetic field and the

plasma, and assumed that the plasma temperature obeys the isothermal equation of state.

In recent simulation Xian Xu [11] considered the energy flow in a coaxial POS with the single-fluid MHD equations, but did not discussed how the current changes and assumed that boundary condition for magnetic field at generator edge, varies as tanh(t/taw)/r which taw being characteristic time.
In this paper, through marching in time, in each time step the voltage of the switch at generator edge was calculated by integrating the electric field E_r on edge and then the circuit equation was solved to determine the total current in new time. So, with this method it is possible to apply the boundary condition for magnetic field at generator edge using the calculated current.

II. FORMULATION OF THE PROBLEM

A. Basic equations

It is seen that for a mega ampere current with a rise time of $\tau \approx 1\mu s$, the parameters of the POS plasma satisfy the inequalities $u << v_A$ and $\tau >> \omega^{-1}_H$, which u is current velocity, v_A is Alfven velocity and ω^{-1}_H is ion Larmor frequency.

$$u = \frac{J_0}{2\pi r_0 e \int n_e dz} \approx 5 \times 10^6 \qquad (1)$$

$$V_A = \frac{B_0}{\sqrt{\mu_0 n_i m_i}} \approx 10^8 \frac{cm}{s} \qquad (2)$$

$$\frac{1}{\omega_{H_i}} = \frac{m_i}{ZeB_0} \approx 7 \times 10^{-9} s \qquad (3)$$

where J_0 is current, e is elementary charge, n_e is line electron density, r_0 cathode radius, $B_0 = \mu_0 J_0/2\pi r_0$, m_i ion mass, n_i is ion density, z is ion charge state. The above condition is opposite to the EMHD regime conditions [12]. In this case, ensuring plasma quasi neutrality (the Debye radius is $\lambda_D = (kTe/4\pi e 2ne)^{1/2} \approx 10^{-2}$cm, and $T_e = 1$ eV), the electron velocity is equal to the ion velocity; i.e., we can neglect electron inertia. Note that the equality of the electron velocity to the ion velocity is provided by both collisions and the self-consistent magnetic field ($\Pi = (V_A / u)^2 = 4\pi Zne\ e^2L^2/Mac^2 \approx 50Z >> 1$). These conditions are those of the one-fluid MHD regime [10].
In the one-fluid MHD regime, the penetration of the magnetic field into plasma occurs via convection and diffusion. The Spitzer conductivity is:

$$\sigma = \frac{3(4\pi\varepsilon_0)^2(kT_e)^{3/2}}{4\sqrt{2\pi}\sqrt{m_e}e^2 Z\lambda} \qquad (4)$$

Where k is Boltzmann constant, m_e is electron mass ε_0 is permittivity of free space, the problem of calculating a POS has been formulated. The problem reduces to simultaneously solving both two-dimensional MHD equations with allowance
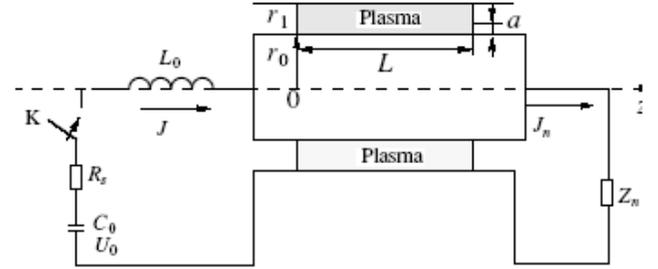
Fig. 1 geometry of the POS in simulation.

for the Hall current and the electric circuit equations. Schematic figure of the problem has been shown in Figure 1.
Above condition allow us to assume that during the conduction phase, the plasma is quasi-neutral, i.e., the electron density is equal to ion density. In addition, we assume that the electron and the ion temperature are also equal, and the plasma is an ideal gas. Thus the plasma dynamic can be accurately described by the MHD theory. In theory, the plasma density n, the fluid velocity u, and the plasma temperature T satisfy the following equations, i.e., the continuity equation:

$$\frac{\partial n}{\partial t} + \nabla.(nu) = 0 \qquad (5)$$

The momentum-balance equation:

$$\frac{\partial \rho u}{\partial t} + \nabla.(\rho uu) = -\nabla p + \frac{1}{c} J \times H \qquad (6)$$

And the energy balance equation:

$$\frac{\partial \varepsilon}{\partial t} + \nabla.(\varepsilon u) = -\nabla.(pu) - \nabla.q + J.E \qquad (7)$$

Where u is plasma velocity with two components (u_r, u_z), c is the light speed in vacuum, $p = 2nT$ is the plasma pressure, $\rho \approx m_i zn$ is the plasma mass density, m_i is the ion mass, and is the plasma total internal energy, i.e., the sum of plasma kinetic energy and plasma thermal energy. Here, it should be stressed that the off-diagonal part in the plasma pressure tensor has been ignored in the above MHD equations. In Eq. (7), q is the thermal flow vector. In a POS, the heat conduction coefficient is very small due to the strong magnetic field (about 70000 G), *so* the divergence of q is negligible in the simulations.

In Eqs. (6) and (7), the current density j, the magnetic field H, and the electrical field E can be determined by Faraday's law:

$$-\frac{1}{c}\frac{\partial H}{\partial t} = \nabla \times E \tag{8}$$

Ampere's law :

$$\nabla \times H = \frac{4\pi}{C}J \tag{9}$$

and the generalized ohm law :

$$E = \eta J - \frac{1}{c}u \times H + \frac{1}{c}\frac{J \times H}{ne} \tag{10}$$

with (8)-(10), one can further eliminate the electric field E and obtain the following equation:

$$\frac{\partial H}{\partial t} = \nabla \times (u \times H) - \nabla \times (\frac{c\nabla \times H}{4\pi ne} \times H)$$
$$-\nabla \times (\frac{\eta c^2}{4\pi}\nabla \times H) \tag{11}$$

and considering the switch in the circuit, the RLC equations are:

$$U_S = (R_0)I + (L_0)\frac{dI}{dt} + V_{pos} \tag{12}$$

and the capacitor voltage :

$$\frac{dU_S}{dt} = \frac{-I}{C_0} \tag{13}$$

This governs the evolution of the magnetic field in the Conduction-phase plasma. In equation (12), (13), U_S are capacitor voltage, R_0 is resistance of the circuit, I is the current, V_{pos} is the switch voltage and C_0 is the capacity. The right-hand side of (11) is composed of three terms: the convection term, the Hall term, and the diffusion term. In general, due to the value of the Spitzer resistively being typically 10^{-5} to 10^{-3} Ω m for a POS plasma, the last term is smaller than the first and second terms.

B. Boundary conditions

The plasma configuration used in our simulation of the POS is shown in Fig. 2; here, the electrodes are made of two coaxial cylinders. The inner electrode is the cathode with radius R_c and the outer is the anode with radius R_a. The length of the

POS is L. At the initial moment, the whole POS is full of plasma.
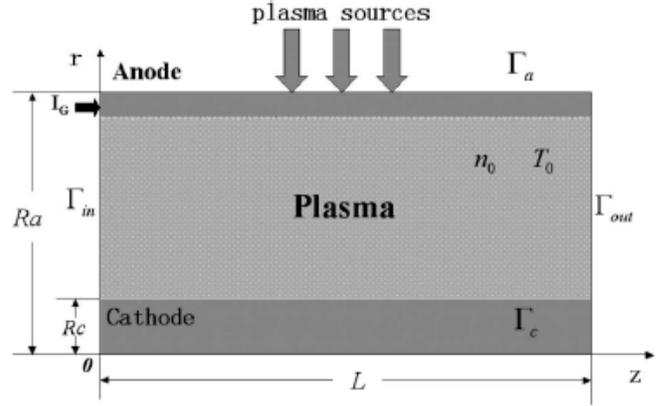
Fig. 2 domain of simulation.

Due to the azimuthally symmetry of the geometry, the velocity has the radial and axial components, and the magnetic field has the azimuthally component only, i.e., $u = \{u_r, u_z\}$ and $H = \{0, -H_\varphi, 0\}$, where H_0 is the absolute value of the Magnetic field. Other boundary conditions are taken from the [11]. The left boundary Γ_{in} (z=0), the cathode Γ_c and the anode Γ_a are bounded by a solid surface. The right boundary Γ_{out} (z=L) is treated as outflow boundary. The boundary condition of the density are $\partial n/\partial r = 0$ on the boundaries Γ_c and Γ_a, and $\partial n/\partial z = 0$ on the boundaries Γ_{in} and Γ_{out}. The boundary conditions of the velocity are u_r=0 on the boundaries Γ_c and Γ_a, and $\partial u_z/\partial z = 0$ on the boundary Γ_{in} and Γ_{out}. It should be noted that Xu [11] used u_z=0 on Γ_{in} but he has mentioned that using Neumann condition conserve the energy. So, we applied the Neumann condition. For the magnetic field, it is assumed $\partial H/\partial r = 0$ on the boundaries Γ_c and Γ_a, and $\partial H/\partial Z = 0$ on the boundary Γ_{out}. the only difference to [11] is that the boundary for magnetic at the generator edge is calculated by obtaining the total current solving RLC circuit equation (12), (13) and integrating the electric field to find the voltage of the POS in conduction phase.

$$V_{pos}\big|_{z=0} = \int_{r=r_i}^{r=r_o} Eds \tag{14}$$

Then the boundary condition of the magnetic field on Γ_{in} is

$$H(R_c < r < R_a) = \frac{\mu_0 I}{2\pi R_c}\left(R_c/r\right) \tag{15}$$

In the POS, the ion and electron velocities are very large, and the plasma flow is highly convective. Therefore, (1)–(3), (5), and (7) are solved in a conservative form based on Roe

averaging first order method [14],[15] which is very suitable for inviscid and deep gradients simulations.

The most important characteristic of the Roe scheme as outlined by Roe [16] is that, it is essence a non-diffusive scheme. Any diffusive behavior that this scheme shows is due to the grid obliqueness with respect to the flow [14]. To use the non-diffusive property of the Roe scheme, it is desirable to align the grid with the flow as much as it is possible.

The RLC circuit equation has been solved by Runge Kutta forth order with initial condition of current I=0, u=u$_0$ for the capacitor charging voltage and C=C$_0$ is the capacity. Integration of electric field on the edge has been done by Simpson method.

C. Results and discussions

In section C-I the code has been tested for the case similar to [11] which does not consider circuit equation and assumes that current rises is $I = I_0 \sin(\frac{\pi t}{2\tau})$, I$_0$=720 kA τ=1.3μs .

In this case inverse z-pinch has been used as the plasma source [17]. It has been shown that these sources which use gas to produce plasma by a radio frequency (RF) breakdown produce almost pure and homogenize plasma and use less electrical energy than the other types. In common plasma sources producing plasma is based on flashover discharge on a polycarbonate surface which melt vaporize and ionize the material. In this method, the plasma has some impurities and mainly composed of two species: C^{++} and H$^+$. Experimental evidence shows that during the conduction phase light particles move fast and thus limit the switch operation [18]. So, due to these reasons, IP source was chosen to have more exact results. Details of IP plasma source can be fined in [10]. The plasma source use Argon and it will be considered in the simulation that at the beginning of generator current, the switch is full of Argon plasma (Ar^{++}) . Radius of cathode is R$_c$= 6.33 cm, anode is R$_{a=}$ 8.57 cm and switch length is L= 5cm and the mass of Argon ion is $m_i = 6.64 \times 10^{-23} g$ .The grids were meshed uniformly with a grid size $\Delta z = .05 cm$, $\Delta r = .056 cm$ which is larger than the plasma Debye length. At the initial plasma temperature $T_0 = 10^5$ K, the initial plasma density was varied from $n_0 = 2 \times 10^{14}$ cm^{-3} to n$_0$=9×10^{15}.

Figure 3 shows the conduction current versus plasma density. It has been assumed that when the magnetic field reaches the load edge of the switch, conduction phase finishes and switch impedance increases rapidly [11].

This is because at the end of conduction phase, under the action of the pressure of the magnetic field created by the current passing in the switch, near the cathode, plasma density decreases rapidly and a thin gap is created.

In MHD regime, it is not possible to model these phenomena, because the continuity equation does not satisfy. Two dimensional (2D), particle-in-cell (PIC) simulations have shown that a gap of several mm can form in high density (10^{15} cm^{-1}) POS plasma.
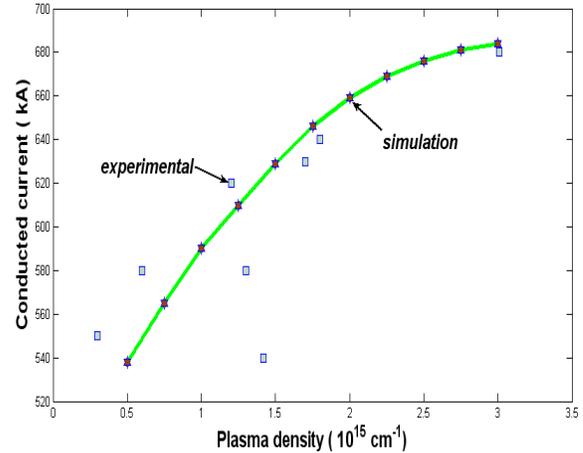
Fig. 3 conducted current versus plasma density.

Furthermore, interferometer measurements of a POS plasma (using an inductive, short-circuit load) have shown a local thinning of the POS plasma over a few mm radial region in conjunction with a radial redistribution of switch plasma [19]. After validation of the code, in the next section RLC equation (12), (13) has been coupled to the system of equations to calculate the current variation and switch impedance in conduction phase. The case which has been used for this simulation has taken from [18], anode radius is R$_a$ =12 cm, R$_c$=9cm and L=20cm. C=18μf, U=720 kV, L=120 n H.

For the case described above, the switch was simulated and the results are shown in Figure 3. The Figure shows that when the initial plasma density increases, theoretical model gives more exact results. It is also obvious that transferring large amount of energy through out the switch is possible when the plasma density is high. But, increasing the plasma density causes weak magnetic insulation and thus having low impedance when the switch opens. In   Figures 4-12, show the temporal evolution of the Argon plasma density (n/n$_0$), plasma temperature (T/T$_0$) and magnetic field (H/H$_0$) in coaxial switch using IP plasma source *at* different times: 0.43, .96, 1.43μs. One can see that, due to the role of the magnetic pressure, the plasma is pushed rapidly to both the load edge and anode, resulting in the rarefaction and compression of the plasma. At the end of the conduction phase, a rarefied gap is formed near the cathode where the density of the plasma has decreased more than one order of the magnitude compared with the initial density. From Figures, one can also find that the front position of the plasma compression coincides exactly with that of the magnetic-field propagation, which means that the plasma compression results mainly from the magnetic pressure rather than other pressures, for example, the thermal pressure.

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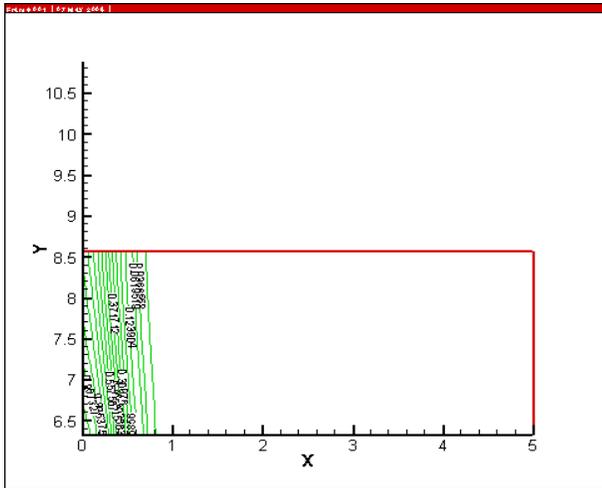
Fig. 4 magnetic field (H/H$_0$) , t=0.43μs

For the high-density plasma, the Hall Effect becomes negligible, and the penetration of the magnetic field in the POS plasma is mainly dominated by the convection effect. In this situation, the magnetic field is frozen in the plasma, i.e., moving with almost the same velocity with the plasma. Fig. 4-5-6 show the temporal evolutions of the magnetic field *(H/H$_0$)* at different times: 0.43, .96 and 1.36μs. It is obvious, that the magnetic field propagates rapidly along the cathode. Here, it should be pointed out that the POS coaxial geometry leads to the magnetic-field penetration starting from the cathode. For the geometry, the magnetic field at the boundary Γ in between the generator and the plasma is in inverse relative to r. As can be seen, although the gradients are very deep, but there is no oscillation in the results and this show that Roe scheme is so suitable for this simulation.

Fig. 5 magnetic field (H/H0) , t=96μs

Fig. 7 density (ρ/ρ0) , t=0.43μs

Fig. 6 magnetic field (H/H$_0$), t=1.36μs

Fig. 8 density (ρ/ρ0) , t=0.96μs

Fig. 9 density (ρ/ρ0) , t=1.36μs.

Fig. 11 temperature (T/T0) , t=. 96 μs.

In order to demonstrate clearly that the magnetic pressure or snow-plow model dominates the behavior of the plasma in the high-density POS, the temporal evolutions of the plasma density, and the temperature are plotted in Fig. 7-12.

From figures, it can be seen that the density and temperature of the plasma change a lot with the penetration of the magnetic field. Before the magnetic field comes to the point, the plasma density and the temperature at the point are almost the same as the initial values. But, with the penetration of the magnetic field, the density and temperature of the plasma increase rapidly. After the penetration of the magnetic field, the plasma density and the temperature drop fast.

Fig.12 temperature (T/T0) , t=1.36 μs.

It is clear that, for the high-density plasma, the compression effect dominates in the variation of the plasma temperature, while the Joule heating effect is not significant. In addition, one can see that, because of the presence of the strong magnetic field, the role of the heat conduction is completely suppressed. For the high-density plasma, since the plasma temperature increases significantly in the shock region and is redistributed greatly, it is naturally to expect that the temperature gradient force $-n\nabla T$ would have a certain effect on the penetration of the magnetic field.

 For the next section (C=II), the second case, the RLC circuit equation (12), (13) was considered to determine the current during conduction phase. In this case, in each time step integration (14) was solved with Simpson method and the POS voltage was determined. Solving circuit equation i.e., (12), (13) by Runge Kutta method gives the current in new time step. Using this current to determine the magnetic field on the

Fig. 10 temperature (T/T0) , t=.43 μs.

Fig. 13 calculated current against experimental Marx Generator current.

was updated. The calculated current has plotted against experimental Marx current. As can be seen from Figure 13 the simulated current matches well with experimental current.

boundary, it is possible to advance in time and solve equation (11). Calculated current was plotted in Figure13. It is seen that the result is in good agreement with experimental profile. Conduction time in experimental is 1.1μs but in simulation is .91 μs. the maximum conducted current in experimental is 3. 6 MA and simulation current is 3.8 MA. It is assumed that when the magnetic field reaches the load edge of the plasma the conduction phase finishes and the next phase begins. Because of this, the simulated current, which has been plotted against the experimental current does not continue after t=9.1μs.

III.   CONCLUSION

In this paper, 2D MHD simulation was done for two cases. In the first one, the current was assumed to vary sinusoidal as similar to [2]. In this method, the boundary condition for magnetic field does not depend on the switch voltage. The current was plotted against plasma density to validate the results. As can be seen in Figure 3 the results are in good agreement with experimental data. For this case, magnetic shock prorogation in within plasma was discussed and was pointed out those variables gradients was highly influenced by magnetic filed gradients. It was also found that thermal pressure against magnetic pressure is negligible. Although this method is good to estimate the characteristic of the switch but it does not discuss how the currant changes. The switch which was chosen for simulation used IP (inverse z-pinch) plasma source. As was discussed this type of plasma source uses less electrical energy and produce almost pure plasma that helps to have more exact simulation.

In the next section, the RLC equation for circuit was coupled to the system of MHD equations. The voltage of POS was determined by integrating electric field E_r on generator edge of the plasma in each time step. With this method in each time step the current was calculated and boundary condition for magnetic field on generator edge of plasma opening switch

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