

# FUZZY ADAPTIVE DYNAMIC FRICTION COMPENSATOR FOR ROBOT

J. Ohri, L.Dewan, M.K.Soni

**Abstract-** Intelligent tracking control design for robotic manipulators is proposed in this paper for dynamic friction compensation. A unified and a systematic procedure, which is based on adaptive fuzzy system, is employed for friction compensation and to drive the controller. Friction is highly nonlinear and dynamic effect and difficult to model. Friction of each joint of a manipulator impedes control accuracy. Therefore, friction has to be effectively compensated for in order to realize precise tracking control of robot manipulators. Adaptive Fuzzy compensator, which has the capability to approximate any nonlinear function over the compact input space, has been used in this paper to compensate the friction, based on Lyapunov function.

**Keywords--** robot, adaptive, fuzzy, control, and friction

## 1. INTRODUCTION

The tracking control is one of the fundamental problems in robot control. However, if friction of each joint of a manipulator is not compensated for enough, accuracy of the generated motion will deteriorate. Friction compensation techniques have been widely studied [1 - 5]. Most of them are model based feed forward compensation methods. It is difficult, however, to prepare a perfect friction model because of the complexity of static and dynamic characteristics of friction such as Stribeck effect, the Dhal effect, stick slip motion, and so on. In the case of multi-link robot manipulators, the effect of friction is more complicated. When the configuration of a robot manipulator is changed, the amount of joint friction is also changed because the moment of the gravity force acting on each joint varies. Therefore, an exact friction model is required to compensate for joint friction accurately and realize precise control.

From a control theoretical point it has been found that fuzzy logic can be a possible solution as a compensator for adaptive control schemes for any uncertainty. The basic idea of the adaptive fuzzy logic control arises from the fact a wide class of nonlinear system can be approximated to arbitrary closeness by a fuzzy logic system [5-6]. Adaptive fuzzy estimator provides a tool for making use of the fuzzy information in a systematic and efficient manner[7-9]. In this paper adaptive fuzzy estimators are used to compensate the friction. To reduce the error between the real friction function

and the compensator, simple robust adaptive laws based on Lyapunov stability theory are used.

This paper is organized as follows. Section 2 presents several properties of robot dynamics. In section 3 model based adaptive controllers with friction compensation are given. Stability of this algorithm is also given in this section. Section 4 presents MIMO-FLS, which is utilized to compensate the dynamic friction of the robot manipulator. Section 5 presents the simulation and results followed by conclusion in section 6.

## II. ROBOT DYNAMIC MODEL

A robot manipulator is defined as an open kinematics chain of rigid links. Each degree of freedom of the manipulator is powered by independent torques. Using the Lagrangian formulation, the equations of motion of an n-degree-of-freedom manipulator can be written as [10]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_f(\dot{q}) = T \quad (1)$$

Where  $q, \dot{q} \in R^n$  are respectively joint displacement and velocity,  $T \in R^n$  is the vector of applied joint torques,  $M(q) = M^T(q) (> 0) \in R^{n \times n}$  is the inertia matrix,  $C(q, \dot{q}) \dot{q} \in R^n$  is the vector of centripetal and Coriolis torques, and  $G(q) \in R^n$  is the vector of gravitational torques  $F_f(\dot{q}) \in R^n$  represents the dynamic friction.

The robot model (1) is characterized by the following structural properties.

**Property 1:** The mass matrix is bounded from above and below.

**Property 2:**  $\dot{M}(q) - 2C(q, \dot{q})$  is a skew symmetric matrix

**Property 3:** The dynamics of the robot without friction and disturbances can be linearly parameterized with respect to a specifically selected set of manipulator parameters, i.e.,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\beta \quad (2)$$

Where  $\beta$  is the vector of unknown manipulator parameters and  $Y$  is a known regressor matrix, which is a function of joint positions, velocities, and accelerations. [11, 12]

As opposed to classical static friction model, dynamic friction models attempt to incorporate a variety of friction characteristics such as stiction, zero slip displacement, stribeck effect etc. They also tend to capture effectively the changing friction characteristics that are caused primarily due to wear and aging. The early dynamic friction model was given by Dahl model [1] in the sliding regime by (3)

$$\dot{F}_d = \sigma_o \dot{q} - \sigma_o |\dot{q}| F_d / F_c \quad (3)$$

Since friction depends on sign function, hence it is

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discontinuous in nature and highly nonlinear effect.[1]

General friction model  $F_f$  is given as (4) as superposition of viscous friction and Dahl friction Units

$$F_f(\dot{q}) = \sigma_1 \dot{q} + F_d(\dot{q}) \quad (4)$$

where  $\sigma_0$  is stiffness coefficient,  $\sigma_1$  is viscous friction coefficient  $F_c$  is coulomb friction and  $F_d$  a dynamic friction term. Since friction is a local effect,  $F_f(\dot{q})$  is uncoupled among the joints and is dependent only on the angular velocity  $\dot{q}$ . This property is utilized in this paper in order to reduce the number of fuzzy rules in the fuzzy compensator.

### III. ADAPTIVE FUZZY DYNAMIC FRICTION COMPENSATOR

The considered tracking problem of robot manipulator is stated as follows: Knowing desired trajectories  $q_d$ ,  $\dot{q}_d$  and  $\ddot{q}_d$ , with some or all the manipulator parameter unknown, determine a control law T, such that, the tracking error  $e = q - q_d$  has a prescribed transient response and it goes to zero asymptotically as time approaches infinity

The following computed torque control law, for the above-mentioned tracking problem is chosen, based on the dynamic friction model

$$T = M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + F_f(\dot{q}_r) - A_v e_v \quad (5)$$

Where other quantities are defined as

$\dot{q}_r = \dot{q}_d - A_p e$ ,  $e_v = \dot{q} - \dot{q}_r$  and  $A_p$  and  $A_v$  are chosen positive definite gain matrices. The term  $A_v e_v$  is equivalent to a PD action on the error. In (5), it is assumed that the parameter matrices  $M$ ,  $C$ ,  $G$  are completely known except  $F_f$ . It is the dynamic friction, which is difficult to model and it will be compensated using fuzzy adaptive estimator  $\hat{F}_f(\dot{q}, t \parallel \beta_f)$  in this paper. Using this concept, (5) can be rewritten as (6)

$$T = M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + \hat{F}_f(\dot{q}, t \parallel \beta_f) - A_v e_v \quad (6)$$

Fuzzy adaptive estimator of friction  $\hat{F}_f$  can be written in the form as below

$$\hat{F}_f = \beta_f^T \psi(\dot{q}) \quad (7)$$

$\beta_f$  is assumed as unknown parameter matrix for friction and  $\psi(\dot{q})$  is known regressor matrix in adaptive control, which is a function of joint velocity  $\dot{q}$  and will be considered as fuzzy basis function in fuzzy compensation of friction.

Stability of the solutions of the closed-loop error dynamics (6) is shown by considering the following Lyapunov function candidate:

$$V = \frac{1}{2} e_v^T M(q) e_v + \frac{1}{2} \tilde{\beta}_f^T \Lambda_f^{-1} \tilde{\beta}_f \quad (8)$$

$\tilde{\beta}_f = \hat{\beta}_f - \beta_f$  and  $\Lambda_f$  is diagonal gain matrix which is strictly positive real constant. The time derivative of the Lyapunov

function candidate along the trajectories of the error dynamics is given by

$$\dot{V} = e_v^T M(q) \dot{e}_v + \frac{1}{2} e_v^T \dot{M}(q) e_v + \tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f \quad (9)$$

Using the skew symmetry property of the matrix  $[\frac{1}{2} \dot{M}(q) - C(q, \dot{q})]$  the (9) simplifies as (10)

$$\dot{V} = -e_v^T (M\dot{q}_r + C\dot{q}_r + G + F - T) + \tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f \quad (10)$$

By putting the value of T from (6) and substituting it into (10), we get

$$\dot{V} = -e_v^T [F_f(\dot{q}) - \hat{F}_f(\dot{q}) + \Lambda_v e_v] + \tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f \quad (11)$$

If we define the optimal parameter matrix of the FLS as  $\beta_f^*$ , then we can define the minimum approximation error vector  $w_f$  as

$$w_f = F_f(\dot{q}, t) - \hat{F}_f(\dot{q}, t \parallel \beta_f^*) \quad (12)$$

Therefore using (12), (11) can be rewritten as below

$$\dot{V} = -e_v^T (\tilde{\beta}_f^T \Psi(\dot{q}) + w_f + \Lambda_v e_v) + \tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f \quad (13)$$

$$\dot{V} = -e_v^T \Lambda_v e_v - e_v^T w_f + \tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f - e_v^T \tilde{\beta}_f^T \Psi(\dot{q}) \quad (14)$$

where  $\tilde{\beta}_f = \beta_f^* - \beta_f$

Therefore, the adaptation law for dynamic friction parameters is chosen as (by putting  $\tilde{\beta}_f^T \Lambda_f^{-1} \dot{\tilde{\beta}}_f - e_v^T \tilde{\beta}_f^T \Psi(\dot{q}) = 0$  in (14))

$$\dot{\tilde{\beta}}_f = -\Lambda_f^{-1} e_v \Psi(\dot{q}) \quad (15)$$

Then

$$\dot{V} = -e_v^T \Lambda_v e_v - e_v^T w_f \quad (16)$$

If  $w_f = 0$ , that is, the searching space for  $\hat{F}_f$  is so big that  $F$  is included in it, then we have  $\dot{V} \leq 0$  and it vanishes if only if  $e_v = 0$ . The tracking error  $e$  converges to the sliding surface and is restricted to the surface for all subsequent time. Thus,  $V$  is a Lyapunov function, i.e.,  $V$  is positive definite and  $\dot{V}$  is negative definite. Hence, all the internal signals  $e$ ,  $e_v$ ,  $\hat{\beta}_f$  are bounded and the stability of algorithm is guaranteed.

It is seen from the section II that the friction is dependent on joint velocity and uncoupled among the joints. Therefore, we can use a SISO-FLS as a compensator that compensates the friction for each joint. So, the FLS in control law will consist of  $n$  basis function vectors and  $n$  regressor vectors for  $n$ -link robot manipulators. In this case, FLS needs  $n*k$  rules where  $n$  is the number of links of the robot and  $k$  denotes the number of fuzzy labels determined on each input variable.

$\dot{q} = (\dot{q}_1, \dots, \dot{q}_n)^T$  are the inputs and  $\hat{F}_f = (\hat{F}_{f1}, \dots, \hat{F}_{fn})$  are output vectors of the fuzzy system respectively.  $\beta_f$  is the parameter matrix,  $\psi(\dot{q})$  is the fuzzy basis function vector or the antecedent function vector [6].

The output of a SISO-FLS  $\hat{F}_f$  with centre-average defuzzifier, product inference, and singleton fuzzifier can be written as follows:

$$\hat{F}_f(\dot{q}) = \frac{\sum_{l=1}^k \mu_{A^l}(\dot{q}) \bar{F}_f^l}{\sum_{l=1}^k \mu_{A^l}(\dot{q})} = \sum_{l=1}^k \bar{F}_f^l \psi(\dot{q}) = \beta_f^T \psi(\dot{q}) \quad (17)$$

The regressor matrix for this case can be written as below

$$\psi_l(\dot{q}) = \frac{\mu_{A^l}(\dot{q})}{\sum_{l=1}^k \mu_{A^l}(\dot{q})} \quad (18)$$

where  $\mu_A$  is membership function.

The structure of FLS  $\hat{F}_f$ , is as follow

$$\hat{F}_f(\dot{q} \square \beta_f) = \begin{bmatrix} \hat{F}_{f1}(\dot{q}_1) \\ \hat{F}_{f2}(\dot{q}_2) \\ \vdots \\ \hat{F}_{fn}(\dot{q}_n) \end{bmatrix} = \begin{bmatrix} \beta_{f1}^T \psi^1(\dot{q}_1) \\ \beta_{f2}^T \psi^2(\dot{q}_2) \\ \vdots \\ \beta_{fn}^T \psi^n(\dot{q}_n) \end{bmatrix} \quad (19)$$

#### IV. SIMULATION STUDY

The control approaches developed in last sections is applied to the position control of a SCARA type two-link planar manipulator having revolute joints in this section.

The dynamic equation matrices for two-link manipulator are of following form [10]

$$M(q) = \begin{bmatrix} (m_1 + m_2) l_1^2 & m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2 (c_1 c_2 + s_1 s_2) & m_2 l_2^2 \end{bmatrix} \quad (20)$$

$$C(q, \dot{q}) = m_2 l_2 l_1 \begin{bmatrix} \dot{q}_2 & 0 \\ 0 & \dot{q}_2 \end{bmatrix} \quad (21)$$

$$G(q) = \begin{bmatrix} -(m_1 + m_2) l_1 g s_1 \\ -m_2 l_2 g s_2 \end{bmatrix} \quad (22)$$

where link masses  $m_1=1.0$  Kg,  $m_2=0.8$  Kg and link lengths  $l_1=1$  m,  $l_2=0.8$  m. are chosen.

Short hand notations  $c_1 = \cos(q_1)$  and  $s_1 = \sin(q_1)$  etc. are used. The sampling time  $t=0.01$  sec is chosen.

The desired trajectories chosen are written below and shown in Fig.1 and Fig. 2 respectively

$$\begin{aligned} q_{d1} &= 0.3 \sin(0.7t - \pi/2) + 0.3 \sin(0.1t - \pi/2) + 0.7; \\ q_{d2} &= 0.5 \sin(0.9t - \pi/2) + 0.5 \sin(0.1t - \pi/2) + 1.1; \end{aligned} \quad (23)$$

This type of desired trajectory will have changing velocity, which is very important for different types of friction characteristics.

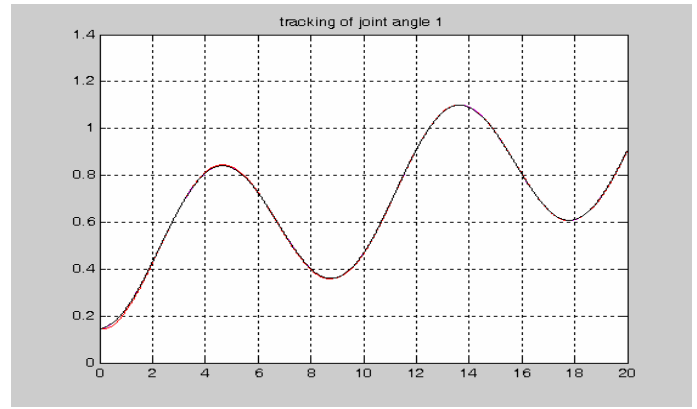


Fig 1 Desired trajectory of joint angle1

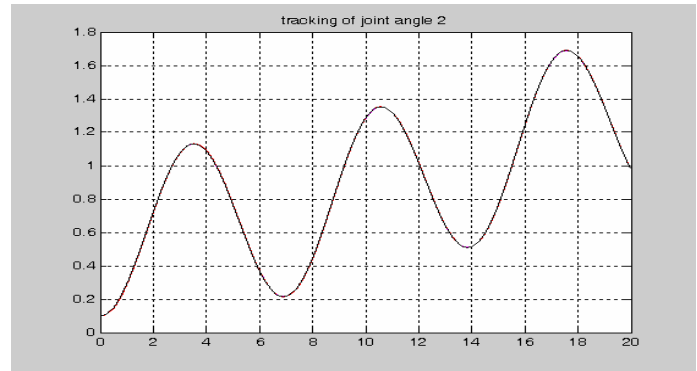


Fig 2 Desired trajectory of joint angle2

Friction parameters for two link planar robot, taken from the literature are as following

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad F_c = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

The following gain parameters are chosen as  $\Lambda p = \text{diag}(10, 10)$ ,  $\Lambda v = \text{diag}(40, 40)$ ,  $\Lambda_f = \text{diag}(0.03, 0.05, 0.1, 0.3)$ .

The initial values of the estimated friction parameters are set as  $\beta_f = (0, 0, 0, 0)^T$  and the initial values of estimated parameter vector are calculated based on initial conditions.

The fuzzy control rules are chosen according to section - 4. The linguistic variable  $A_i^l, B_j^l, i=1, \dots, 5$  and  $j=1, 2$  whose five fuzzy levels are PB, PS, ZO, NS, NB on the universe of discourse of the each input variable, are chosen.

Here Gaussian membership function is chosen as

$$\mu_{A_i^l}(x_i) = \exp \left[ - \left( \frac{x_i - \bar{x}_i^l}{\pi / 2.4} \right)^2 \right]$$

Where  $\bar{x}_i^l$  are  $-\pi/6, 0, \text{and } \pi/6$  as  $A_i$  are PB, ..., NB respectively.

The simulation results for the tracking error  $e$  in joint angle 1 and angle 2 are shown in Figs.3 to 4

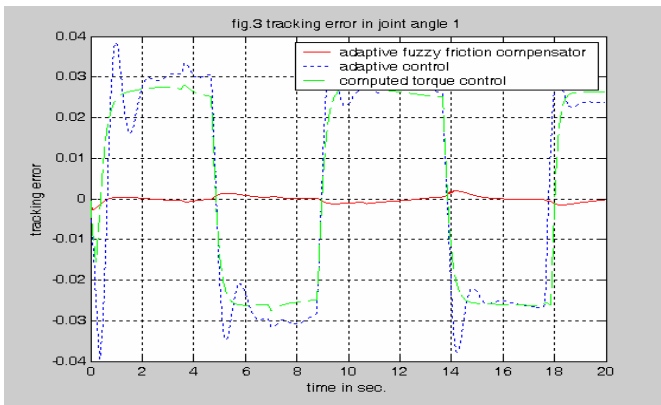


Fig 3 Tracking error in joint angle 1

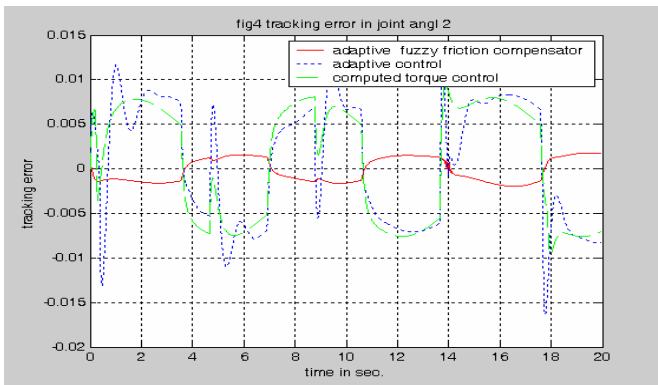


Fig 4 Tracking error in joint angle 2

Case1: The adaptive control law of (2) without friction compensation is applied for tracking control of manipulator whose dynamic model is given by (1).

Case2: The Computed torque control law of (5) having the friction compensation term that uses exact friction models of (3) and (4) is applied for tracking control of robot, whose dynamic model is given by (1).

Case3: Lastly, the computed torque control law of (6) with adaptive fuzzy friction compensation of (19) is applied for tracking control of same dynamics.

Results show the significant improvement in the performance with adaptive fuzzy friction compensation as seen from Figs. 3-4. The comparison of these results is shown in Table I.

TABLE I

Type of controller	2 norm of the tracking error $e$ in		Max. error	
	joint 1	Joint2	joint 1	Joint2
Case1 Adaptive Control	1.2052	0.3119	0.0385	0.0385
Case2 Computed torque law	1.1097	0.2970	0.0286	0.0094
Case3 Adaptive Fuzzy friction compensation	0.0354	0.0607	0.0020	0.0018

2 norm of the tracking error  $e$  for the above three cases are shown in Figs 5-6

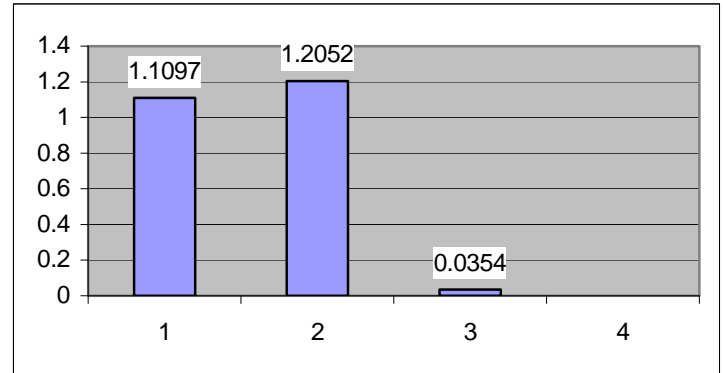


Fig.5. 2 norm of tracking errors in joint angle 1 for above three cases

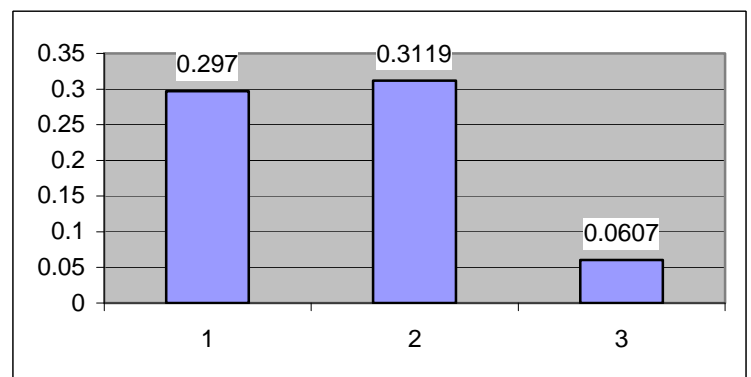


Fig.6. 2 norm of tracking errors in joint angle 2 for above three cases

Hence from these comparisons it is clear that on introducing adaptive fuzzy system for friction compensation in control, there is a significant improvement in the tracking performance. As reported in the literature friction is a very difficult phenomena to model and it has varying nature .So Fuzzy compensator proves to be a perfect solution to estimate the dynamic friction. Many simulations were performed for different cases to support the above results.

## VI CONCLUSION

In this paper, an adaptive fuzzy estimator is presented to compensate the effect of dynamic friction in robot manipulator. The design method relies on the use of Lyapunov method to ensure the stability of perturbed system. The advantage of use of fuzzy logic systems as compensators for friction of robot manipulator is that, the controller designer need not derive the regressor matrix and tune the parameters, as friction is a varying nonlinear phenomenon, and very difficult to model. Results are compared with computed torque control and adaptive control without friction compensation, which shows that friction compensation, is essential for obtaining low trajectory tracking errors. Further uncertainties in robot model parameters and gravitation can be compensated using fuzzy logic system.

## REFERENCES

1. B. Armstrong-Helouvry, P. Dupont and C. Canudas, "A Survey of Models, analysis tool and compensation methods for the control of machines with friction". *Automatica*, vol.30, no.7, 1994, pp 1083-1138
2. S.C. Southward, C.J. Raclis and MacCluer, "Robust nonlinear stick-slip friction compensation", *ASME J. of Dynamic Syst. Measurement and control*, vol.113, pp.639-645, 1991.
3. C. Canudas de Wit, P. Noel, A. Aubin and B. Baogliato, "Adaptive friction compensation in robot manipulators: low velocities," *Int. J. Robotic Res.*, vol.10, no.1, 1991, pp35-41.
4. B. Armstrong, "Friction: Experimental determination, modeling and compensation," *IEEE International Conference on Robotics and Automation*, Philadelphia, 1988, pp. 1422-1427.
5. Chen, Juang, "A robust friction control scheme of robot manipulators", *Proceedings of IEEE international conference on robotics and automation*, 2003, pp345-567
6. L.X. Wang, "Stable adaptive fuzzy control of nonlinear systems", *IEEE Tran. Fuzzy Systems*, vol.1, no.21, 1993, pp. 146-155
7. L.X. Wang, *Adaptive fuzzy systems and control*, Prentice-Hall, 1994
8. Tianyou Chai, Shaocheng Tong, "Fuzzy direct adaptive control for a class of nonlinear systems", *Fuzzy Sets and Systems* 103, 1999, pp 379-387.
9. D.T. Pham, D. Karaboga, "Self-tuning fuzzy controller design using genetic optimisation and neural network modelling", *Artificial Intelligence in Engineering*, Volume 13, Issue 2, April 1999, pp 119-130
10. Euntai Kim, "Output feedback tracking control of robot manipulators with model uncertainty via adaptive fuzzy logic", *IEEE Tran. Fuzzy Syst*, 12, 2004, pp 368-378
11. J.J. Craig, *Introduction to Robotics-Mechanics and Control* Addison Wesley, 1988.
12. Slotine and Li, *Applied Nonlinear control* Englewood Cliffs, NJ: Prentice Hall, 1991.
13. Ortega and Spong, "Adaptive motion control of rigid robots: A tutorial," *Automatica*, vol.25 (6), 1987, pp 877-888.
14. Slotine and Li, "On the adaptive control of robot manipulators" *Int. J. Robotics Res.* 16, 1987, pp49-59.

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